

BIPOLAR FUZZY SETS IN SWITCHBOARD AUTOMATA AND
OPTIMISATION PROBLEMS

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To my late mother and my late father: the signs of love, support and encouragement.

Al-Fatehah...



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In the name of Allah, the Most Gracious, the Most Merciful.

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ABSTRACT

Bipolar fuzzy sets can be extended to triangular bipolar fuzzy number and are applied in optimisation problems, specifically critical path problem and reliability system of an automobile. Some of the properties of triangular bipolar fuzzy numbers are introduced and used in critical path problems to find a bipolar fuzzy critical path. As a result, acceptance area and rejection area could be recognised successfully. By using a tree diagram, triangular bipolar fuzzy number is then applied to a reliability system of an automobile in order to find the failure rate to start of an automobile that is based on the ideas of circuits which are connected to the system. An illustrative example is presented and the tolerance level of acceptance (positive membership value) and tolerance level of rejection (negative membership value) could be determined successfully in a reliability system of an automobile. In automata theory, the decomposition theorem for bipolar fuzzy finite state automata and its transformations semigroups are initiated and discussed in order to enrich the structure of algebraic properties in bipolar fuzzy finite state automata. Furthermore, the idea of bipolar general fuzzy finite switchboard automata and asynchronous bipolar general fuzzy switchboard automata is initiated. In particular, the algebraic properties of bipolar general fuzzy switchboard automata are discussed in term of switching and commutative by proving the theorems that are related into these concepts. Finally, the notion of the switchboard subsystems and strong switchboard subsystem of bipolar general fuzzy switchboard automata are initiated. As a result, it can be concluded that every switchboard subsystem is a strong switchboard subsystem throughout the proven theorems. As an application, a concept of Lowen fuzzy topology is induced in switchboard subsystem of bipolar general fuzzy switchboard automata by using Kuratowski closure operator.

ABSTRAK

Teori set kabur boleh dilanjutkan kepada nombor segitiga dwikutub kabur dan diaplikasi dalam masalah pengoptimuman khususnya masalah laluan kritikal dan sistem kebolehpercayaan sebuah automobil. Sebahagian sifat-sifat nombor segitiga dwikutub kabur yang diperkenalkan dan digunakan untuk menyelesaikan masalah laluan kritikal untuk mencari laluan kritikal dwikutub kabur dimana kawasan penolakan dan kawasan penerimaan dapat ditentukan. Dengan menggunakan gambarajah pokok, nombor segitiga dwikutub kabur kemudiannya diaplikasi untuk menyelesaikan masalah kadar kegagalan menghidupkan sebuah automobil berkonsepkan idea litar yang disambungkan kepada sistem tersebut. Tahap toleransi penerimaan (nilai keahlian positif) dan tahap toleransi penolakan (nilai keahlian negatif) dapat ditentukan dengan jayanya dalam sistem kebolehpercayaan sebuah automobil. Di dalam teori automata, teorem penguraian untuk dwikutub kabur keadaan mesin terhingga dan perubahan subkumpulan diperkenalkan dan dibincangkan untuk memperkayakan struktur sifat algebra dalam papan suis dwikutub terhingga automata kabur. Kajian ini juga menggabungkan idea papan suis di dalam automata umum kabur untuk diperkenalkan kepada papan suis dwikutub umum automata kabur dan *asynchronous* papan suis dwikutub umum automata kabur. Secara khususnya, sifat algebra dwikutub umum papan suis automata kabur dibincangkan. Akhir sekali, konsep subsistem papan suis dan subsistem papan suis dwikutub umum automata kabur kuat diperkenalkan. Hasil kajian menunjukkan bahawa setiap papan suis subsistem itu adalah papan suis subsistem kuat dan sebaliknya. Sebagai aplikasinya, konsep topologi kabur Lowen diinduksi dalam subsistem dwikutub umum papan suis automata kabur menggunakan operator tertutup Kuratowski.

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LIST OF SYMBOLS AND ABBREVIATIONS

μ	-	A (strong) subsystem of \tilde{F}^*
δ	-	A bipolar transition function in $Q \times \Sigma \times Q$ of \tilde{F}
c_α	-	A closure operator
S	-	A finite semigroup
ϖ	-	A restricted cascade product of BGFSAs
(f, g)	-	A strong homomorphism of BFTS
\circ	-	A wreath product of BGFSAs
$G(V, E)$	-	An acyclic network
\approx	-	An equivalence relation on semigroup S
\odot	-	Binary operation on semigroup $S(\mathcal{M})$
$\mathcal{Q}_{act}(t_i)$	-	Bipolar fuzzy set of all active states at time t_i , $\forall i \geq 0$
φ	-	Bipolar fuzzy sets in X
ρ	-	Bipolar fuzzy subset of $Q \times S \times Q$
\mathcal{N}	-	Bipolar submachine of \mathcal{M}
\mathcal{T}_{\approx}	-	Faithful bipolar fuzzy transformation semigroup
D_j	-	j^{th} fuzzy path length
μ_A	-	Membership function of set A
ν_A	-	Non-membership function of set A
\mathbb{R}	-	Real numbers
X	-	Set of input symbol in bipolar fuzzy set
Q	-	Set of the states in bipolar fuzzy set
ω	-	The function mapping from Q to Z in \tilde{F}
D_{max}	-	The fuzzy longest path
d_{tu}	-	The length necessary to transverse from node t to node u

μ^+	-	The membership value of a predecessor of BGFA
$\varphi^-(x)$	-	The negative membership degree of set X
$\varphi^+(x)$	-	The positive membership degree of set X
L_j	-	The possible path lengths
Π	-	The sum used in reliability system
δ^+	-	The weight of transition of BGFA
ABGFSA	-	Asynchronous bipolar general fuzzy switchboard automata
BFFSM	-	Bipolar fuzzy finite state machine
BFFSA	-	Bipolar fuzzy finite state automata
BGFA	-	Bipolar general fuzzy automata
BFTS	-	Bipolar fuzzy transformation semigroup, \mathcal{T}
BFCP	-	Bipolar fuzzy critical path
DFA	-	Deterministic finite automata
FSM	-	Finite state machine
FSA	-	Finite state automata
FFSM	-	Fuzzy finite state machine
FFSA	-	Fuzzy finite state automata
$F(\mathfrak{R})$	-	Fuzzy quantities
GFSSM	-	Genetic fuzzy finite state machine
F_1	-	Membership assignment functions
MVF	-	Membership value function
F_2	-	Multi-membership resolution functions
F_1	-	Membership assignment functions
F_2	-	Multi-membership resolution functions
$NMVF$	-	Non-membership value function
PDF	-	Probability density function
R_{SS}	-	Reliability series system
R_{PS}	-	Reliability parallel system
R_{PSS}	-	Reliability parallel-series system
R_{SPS}	-	Reliability series- parallel system
V	-	Set of vertices in an acyclic network
E	-	Set of edges an acyclic network

$ER(D_j)$	-	The Euclidean ranking of D_j
$ER(D_j)$	-	The Euclidean ranking of D_j
TBFN	-	Triangular bipolar fuzzy number
F_{FIF}	-	The failure to start an automobile due to fuel injection failure
F_{FPF}	-	The failure to start an automobile due to fuel pump failure
F_{FS}	-	The system failure to start an automobile
F_{FSF}	-	The failure to start an automobile due to fuel supply failure
F_{BLC}	-	The failure to start an automobile due to low charged battery
F_{IBS}	-	The failure to start an automobile due to internal battery shortage
F_{IF}	-	The failure to start an automobile due to ignition failure
F_{LBF}	-	The failure to start an automobile due to low battery fluid
F_{SPF}	-	The failure to start an automobile due to spark park failure
F_{WHF}	-	The failure to start an automobile due to wire harness failure
$R(A_\alpha)$	-	α -cut ranking technique for TBFN of A



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CHAPTER 1

INTRODUCTION

1.1 Background and motivation of research

In 1965, Zadeh proposed the fuzzy set theory (Zadeh, 1965). The idea of fuzzy sets seemed to receive much attention from the researchers in a variety of fields such as engineering, management and decision-making (Zimmermann, 2001), classification, control, forecasting and function approximation (Dubois & Prade, 1980; Wang & Mendel, 1992; Su & Stepanenko, 1994; Castro, 1995; Wang *et al.*, 2004, 2005). Fuzzy sets are the type of a set in set theory that is imprecise and has no boundaries (Klir & Yuan, 1995). Subsequently, fuzzy set theory could be considered as a generalisation of either classical set theory or of a classical dual logic (Zimmermann, 1999) since the fuzzy set only deals with the degree of memberships or in other words, it only allows elements to be partially in a set, in which crisp sets could not address and handle. Since then, fuzzy sets have become an effective tool in order to manage uncertainties and vagueness for a better understanding to the real-life problems.

In fuzzy set theory, the membership of an element lies between 0 and 1. However, in reality, fuzzy set may not forever be true since there may be some hesitation due to the degree of non-membership degree of an element in a fuzzy set is equal to 1 minus the membership degree. With this intention, fuzzy set theory has been extended to type-2 fuzzy sets (Zadeh, 1975), intuitionistic fuzzy sets (Atanassov, 1986), fuzzy multisets (Yager, 1996), neutrosophic sets (Smandarache, 1999), nonstationary fuzzy sets (Ozen *et al.*, 2004), hesitant fuzzy sets (Torra, 2010),

Pythagorean fuzzy sets (Yager, 2014) and bipolar fuzzy sets (Zhang, 1994).

As a matter of fact, bipolarity tends to occur in our real life. Due to this problem, Zhang (1994) introduced bipolar fuzzy sets as a generalisation of fuzzy sets in the interval of $[-1, 1]$. Subsequently, Reza *et al.* (2018) proposed triangular bipolar fuzzy number (TBFN) to overcome the problem that cannot be handled by triangular intuitionistic fuzzy number (TIFN) proposed by Burillo *et al.* (1994). In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property. The membership degree $(0, 1]$ of an element somewhat satisfies the property and the membership degree $[-1, 0)$ of an element indicates that the element somewhat satisfies the implicit counter-property. The positive information represents what is granted to be possible, while negative information is considered to be impossible or forbidden, or surely false (Jun & Kavikumar, 2011; Kavikumar *et al.*, 2012; Benferhat *et al.*, 2004; Dubois & Prade, 2004; Isabelle, 2009)

In particular, triangular fuzzy number is one of the extensions of bipolar fuzzy sets theory. By using triangular bipolar fuzzy number (TBFN) (Reza *et al.*, 2018), a unified approach to polarity and fuzziness could be formalised in decision analysis, bipolar clustering and coordination, bipolar fuzzy aggregation, composition, linguistic description and mathematical computation, provides bipolar cognitive modelling and multivalent decision analysis, captured the bipolar or doubled sided (negative and positive, or effect and side effect) nature of human perception and cognition (Kavikumar *et al.*, 2012). For this reason, by using bipolar relations, the common interests and the conflicts between two countries can be naturally characterised as bipolar fuzzy, which has both positive and negative poles (Zhang, 1999). This is because the logical values in the two classical logical models, which are Boolean logic and fuzzy logic, are unipolar models in nature and they lie in the positive interval $[0, 1]$ (Zhang, 1998).

Notably, optimisation problem is an intrinsic part of life and human activity. By using optimisation method, the complex systems could be simplified to allow a fuller exploitation of the advantages inherent to complex systems. Optimisation problem works better than traditional “guess-and-check” methods and also it can reduce the cost of building and testing which is relatively less expensive. Intensive researches have been made in various fields such as project scheduled and cost

performance (Ghanbari *et al.*, 2017), business (Mazlum & Güneri, 2017), project cash flow (Mohagheghi *et al.*, 2016), decision makers' risk attitude (Oladeinde & Oladeinde, 2014) and other optimisation problems as stated in Chapter 2 in Section 2.3.1 and Section 2.3.2, by using triangular fuzzy number and triangular intuitionistic fuzzy number.

However, the problems still arise since the bipolarity is not considered to exist in real life. As a consequence, data become insufficient and indeterministic. Under those circumstances, TBFN could become a good model to real world applications in the bipolarity environment. In this research, the emphasis is made on the optimisation problems so that it could enhance the idea of bipolarity in the real field that is limited to critical path problem and reliability system of an automobile. Since the range of TBFN lies in the interval of $[-1, 1]$, the problems can be determined successfully by using its positive membership value (acceptance area) and non-membership values (rejection area) of triangular bipolar fuzzy numbers.

On the other hand, automata theory is the general system theory, which provides mechanisms for the formulation and solution of general problems that can be applied to real-world problems in the future. There are a variety of conventional automata, such as Deterministic Finite State Automata (DFA), Non-Deterministic Finite State Automata (NFA), Probabilistic (stochastic) Automata (PA) and Fuzzy Finite State Automata (FFSA). Over the years, DFA have been applied in many applications (Doostfateme & Kremer, 2005). Therefore, the automata theory is of great use to engineers, scientists and also mathematicians.

However, there are still lacking in literature studies the automata theory in bipolar environment. For that reason, the properties of bipolar fuzzy finite state automata (BFFSA) and transformation semigroups inspired by Jun & Kavikumar (2011) could be discussed. Meanwhile, due to the fact that mathematical ideas of BFFSA are still in the early stage, the algebraic properties and their various applications in the real world phenomenon seem to be disputing. Thus, by incorporating the idea of bipolar set theory and switchboard automata, the properties of BGFFSA could be studied and its topological properties could also be discussed. It is hoped that BFFSA will become well known in numerous researches as the interval of bipolar fuzzy sets have been extended from -1 to 1 .

1.2 Problem statements

In many real-life problems, some of the information around the behaviour of the systems encounters uncertainties and vagueness. Sometimes the evaluation of non-membership values do not fulfill our satisfaction since it is indeterministic in nature and fuzzy set theory is not suited to deal with such problems. In this research, the elongation of the bipolar fuzzy set, which is known as triangular bipolar fuzzy number is applied to solve the critical path problem for project network and is very well known in operational research. Since bipolar fuzzy number is a powerful tool to handle the satisfaction degree to counter-property, it can overcome the problems of triangular intuitionistic fuzz number. In probability assumption, the system is fuzzily characterised in the context of probability measures. Since the data are imprecise and inaccurate, the prediction becomes very difficult to be adapted in the systems. The rejection area cannot simply be defined by using fuzzy sets and the binary sets. In this research, the triangular bipolar fuzzy number is applied to solve the reliability system of an automobile. The idea that lies behind this explanation is connected with bipolar information, which is positive information and negative information about the given set. Positive information represents what is conceded to be possible, while the negative information represents what is considered to be impossible. Hence, through the mathematical calculation, the rejection evidence (negative area) and acceptable evidence (positive area) of the reliability system of an automobile could be configured successfully.

Many applications especially in mathematics and sciences face problems because most of the information is not dealt with bipolarity. Since the bipolar fuzzy sets are the extension of fuzzy sets and the interval are enlarged to $[-1, 1]$, bipolar information is crucial for many applications and domains. In this research, the concept of decomposition theorem of fuzzy automata and transformations semigroups in bipolar environment are studied inspired by Jun & Kavikumar (2011). Next, the study of algebraic automata is extended as a truth structure in bipolar setting.

A switchboard automaton is a special kind of finite automata. The characterisation of algebraic structures and its classification of finite state automata with switchboard, that described the connection of switching and commutative were

established by Sato & Kuroki (2002). Subsequently, Kavikumar et al., (2012) introduced bipolar fuzzy finite switchboard state automata (BFFSSA) and investigated some of algebraic properties. However, the algebraic approach is still lacking. Hence, it is necessary to study the algebraic properties of the general fuzzy switchboard automata in bipolar fuzzy environment based on the idea of Doostfatemeleh & Kermer (2005). Moreover, on that point there are some possibilities of topological concepts that are available for bipolar general fuzzy switchboard automata.

1.2 Research objectives

The objectives of this research are as follows:

- (i) To investigate the critical path problem and reliability system of an automobile by using the triangular fuzzy number in the bipolar fuzzy context.
- (ii) To establish decomposition theorem and introduce the properties of transformation semigroup in bipolar fuzzy finite state automata.
- (iii) To investigate the algebraic and topological properties of bipolar general fuzzy switchboard automata.

1.3 Scope of the research

Bipolar fuzzy sets are the yardstick for this research. Triangular bipolar fuzzy number (TBFN), which is the extension of bipolar fuzzy sets theory limited to be applied in optimisation problems which cover critical path problems and reliability system of an automobile as motivation studies in bipolarity environment. The scope of the research is limited to the decomposition theorem and transformation semigroups of bipolar fuzzy finite state automata. Meanwhile, in understanding the bipolar general switchboard automata, the properties of switching and commutative, strong subsystems and its topological properties are studied.

1.4 Significance of study

Some of the significances of study that can be referred to this research are given as follows:

1. The investigation is expected to be useful for the development of bipolar fuzzy number and automata theory in real life problems.
2. Critical path problems could be determined by using triangular bipolar fuzzy numbers successfully.
3. In a reliability system of an automobile, the triangular bipolar fuzzy numbers could be applied efficiently.
4. The new concept of the idea of mathematics, which is known as bipolar general fuzzy switchboard automata could be seen through its algebraic properties and topological properties applied in computer and science disciplines. Therefore, the research is of some significance and worthy of effort.
5. The concept of switching and commutative in bipolar general switchboard automata allows the flow of information in the system to be more efficient in bipolar environment. Thus, it has important use in many fields such as artificial intelligence, optimal control and also production theory.

1.5 Organisation of thesis

The thesis consists of eight chapters including the Introduction chapter. The other remaining seven chapters are organised as follows.

In Chapter 2, the literature review on the development of automata theory is presented and discussed. Some applications in the real-life situations that relate to the bipolar fuzzy sets are reviewed. The literature of optimisation problems that are focused on critical path problem and a reliability system of an automobile are also discussed.

In Chapter 3, the discussion on the critical path problem by using the triangular bipolar fuzzy numbers (TBFN) is introduced. In addition, the algorithm is also proposed and the illustrative example for a project network is presented for better understanding in order to find the critical path problem in terms of a bipolar environment by using α -cut technique and Euclidean ranking technique for ordering the critical path problem inspired by Elizabeth & Sujatha (2015). Throughout this chapter, the new term so-called bipolar fuzzy critical path (BFCP) is introduced.

In Chapter 4, the triangular bipolar fuzzy number (TBFN) is further used to solve the reliability system of an automobile and could be represented as a system

and the cause and sub-cause of the system that could be represented as subsystems. Some of the properties of triangular bipolar fuzzy numbers in this optimisation problem are discussed. The series, parallel, series-parallel and parallel-series system of the reliability circuit, which adhered approximation triangular bipolar fuzzy number, is then focused. Theorems and calculations of the fault-tree to start an automobile that were proposed by Shaw & Roy (2012) and Mahapatra & Roy (2013) are discussed. An illustrative example is given in this chapter. As a result, the reliability system of an automobile can be modelled by using triangular bipolar fuzzy number in order to find the rejection area (negative area) and the acceptance (positive area) through the given illustrative example.

In Chapter 5, the decomposition theorem and transformation semigroups of bipolar fuzzy finite state automata are discussed. Some of the algebraic properties are investigated. The theorems and proving are also discussed. In this chapter, the decomposition property can be extended to bipolar submachines since every fuzzy finite state automaton can be decomposed to primary submachines. Transformation semigroups of bipolar fuzzy finite state automata are introduced by considering the state membership as bipolar fuzzy sets. Inspired by Jun & Kavikumar (2011), the concepts of decomposition of bipolar fuzzy finite state automata and bipolar fuzzy transformation semigroups have been generalised as a truth structure of the transition in bipolar studies of algebraic automata in the interval $[-1, 1]$.

In Chapter 6, the idea of bipolar general fuzzy switchboard automata is discussed. For the purpose of the study of algebraic automata, the concept of product and covering, which was inspired by Sato & Kuroki (2002), Horry (2016), Kavikumar *et al.*, (2012) and Doostfatemeleh & Kermer (2005) is presented. In order to show whether the bipolar general fuzzy state automata are switching and commutative, the examples are presented for better understanding. In addition, an algorithm for constructing bipolar general fuzzy switchboard automata is presented. The properties of BGFSAs in terms of switching and commutative are studied. Subsequently, the notion of asynchronous bipolar general fuzzy switchboard automata is introduced and its onto-switching homomorphic image is studied.

In Chapter 7, the discussion on the subsystem of bipolar general fuzzy switchboard automata is presented. Next, the concept of switchboard subsystem, strong switchboard subsystem and homomorphism of bipolar general fuzzy switchboard automata is introduced, and the idea of a switchboard is incorporated in

the bipolar general fuzzy switchboard automata is initiated. In this chapter, the properties of switchboard subsystem are also discussed in order to show that every switchboard subsystem is a strong switchboard subsystem. Finally, the concept of topology on bipolar general fuzzy switchboard automata in terms of these characterisations is formulated, as an example of its application.

Finally, Chapter 8 summarises the findings presented in the previous chapters and some conclusions are drawn from them. The chapter is ended with some recommendations for the future research. Summary of the above outline or map of the thesis is presented in Figure 1.1.



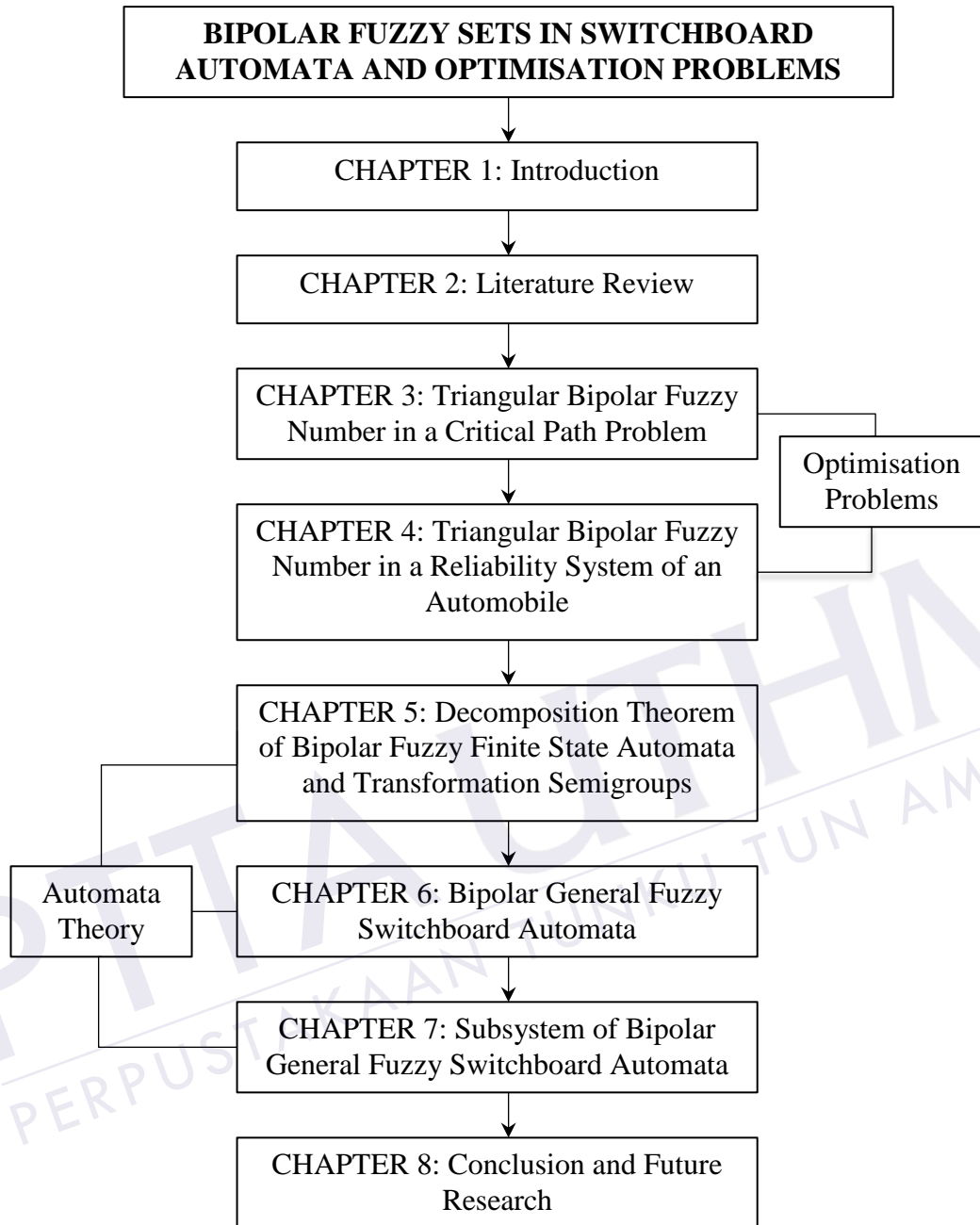


Figure 1.1: Summary of the thesis

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Due to uncertainties and vagueness encountered in some problems that are modelled by the real-life problems in the optimisation problems and automata theory, bipolar fuzzy set theory is considered as the interest and the main theme of investigation within this domain. The literatures that are reviewed in this section show the relationship of the works and ideas of mathematics by the previous researchers in order to accommodate justification to the consequent work in the progress of the research process. The discussion helps to understand the nature of mathematical essence and its relation to the research work in this report.

In this chapter, definitions of bipolar fuzzy sets, applications in the real life and some useful results are given in this preliminaries section, namely Section 2.2. Subsequently, some relevant concepts and results are discussed are also discovered in the same section. Next, the development automata theory is revised in Section 2.3. After that, the optimisation problems are reviewed on critical path problems and reliability systems by the previous researchers as motivation of study in bipolar environment are discussed in Section 2.4.1 and Section 2.4.2 respectively.

2.2 Preliminaries

In this section, definition of bipolar fuzzy sets and its applications to the real-life problems are discussed in order to give a good portrayal to the flow of the thesis. Some of the main concepts and results that are useful for this research are presented.

Motivating examples of real-life problems of bipolar fuzzy sets

Example 1: Bipolar fuzzy set “young”

In Figure 2.1, a bipolar fuzzy set “young” is shown. It is manifested that 50 is an irrelevant age to the property of young and 98 is more apart from the property young than 50. We can say that 98, is a contrary age to the property young.

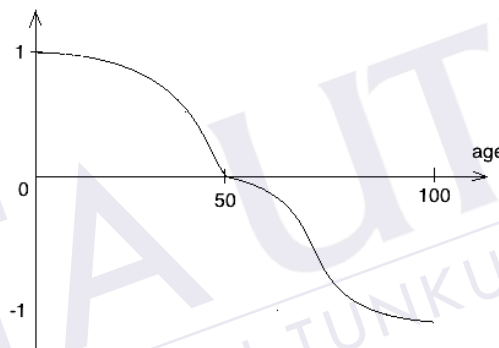


Figure 2.1: Bipolar fuzzy set “young”

Example 2: A bipolar fuzzy sets “food”

Let sweetness of “food” be a bipolar fuzzy set, which denotes the positive degree membership of value (element somewhat satisfies the property) and the bitterness of food denotes negative degree of membership value (element somewhat satisfies the implicit counter-property). The other tastes of food (for example, salty, sour, pungent) is irrelevant to the corresponding property and taken as 0 as membership values.

Example 3: A bipolar fuzzy set of frog's prey

Let the set of frog's prey become a bipolar-valued fuzzy set as such that frog's prey = {(mosquito [1, 1]), (fly [0.3, 0.7]), (elephant [0, 0]), (snake [0, -1])}. In this set, mosquito and fly somewhat satisfy the property of frog's prey and an elephant is irrelevant property to the frog's prey. Snake does not satisfy the property corresponding to the frog's prey, despite the snake is a predator of the frog. Snake somewhat satisfies some counter-property in respect to frog's prey.

Example 4: Spatial information in the image processing

In the image processing area, bipolarity tends to occur too. When we assess the position of an object in a space, we may have positive information expressed as a set of possible places and negative information expressed as a set of impossible places. Human beings consider left and right as opposite directions, but this does not mean that one of them is the negation of the other. The semantics of "opposites" captures a notion of symmetry rather than a strict complementation. In particular, there may be positions, which are considered neither to the right nor to the left of some reference object, thus leaving some room for indetermination. The idea of the union of positive and negative information does not cover the whole space (Kavikumar *et al.*, 2012). Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are essentially different sets (Lee, 2004).

Definition 2.2.1: (Zhang, 1994)

A bipolar fuzzy set φ in X is an object having the form

$$\varphi = \{(x, \varphi^-(x), \varphi^+(x)) | x \in X\} \quad (2.1)$$

where $\varphi^-: X \rightarrow [-1, 0]$ and $\varphi^+: X \rightarrow [0, 1]$ are mappings. The positive membership degree $\varphi^+(x)$ denotes the satisfaction degree of an element x to the property

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