MAGNETOHYDRODYNAMIC DUSTY NANOFLUID FLOW OVER A MOVING PLATE WITH TWO DIFFERENT BOUNDARY CONDITIONS

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For my beloved family, supervisors and friends.

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ABSTRACT

Currently, there are different issues related to low thermal conductivity in the conventional heat transfer fluid, such as water, ethylene glycol and oil, in engineering electronic devices. Aiming to overcome this defect in conventional fluid, this research focuses on nanofluid. Furthermore, dust is considered because in real world, impurities exist and it may affect the flow. Therefore, this research studies the flow and heat transfer characteristics of a dusty nanofluid over a moving plate in the presence of magnetohydrodynamic (MHD). Three types of nanoparticles namely Copper Oxide (CuO), Aluminium Oxide (Al_2O_3) and Titanium Oxide (TiO_2) are considered. The governing partial differential equations are converted into a system of non-linear ordinary differential equations using a similarity transformation, then the non-linear ordinary differential equations are solved using bvp4c program in MATLAB software. The influence of non-dimensional governing parameters such as magnetic parameters and nanoparticle volume fraction on the velocity and temperature profiles for fluid and dust phases of dusty nanofluids are discussed. Then, the results obtained are analysed by comparing two cases of boundary conditions, which are constant surface temperature and convective boundary condition in terms of efficiency. The results show that CuO has the lowest velocity but highest heat transfer rate on both fluid and dust phase compared to Al_2O_3 and TiO_2. Besides, the flow with prescribed surface temperature has better heat transfer rate than the flow with convective boundary condition.

ABSTRAK

Pada masa kini, terdapat pelbagai isu berkenaan dengan kekonduksian terma yang rendah dalam bendalir konvensional seperti air dan minyak yang terdapat di peralatan elektonik. Untuk mengatasi kekurangan dalam bendalir konvensional tersebut, kajian ini ditumpukan pada nanobendalir. Debu turut dipertimbangkan kerana bendasing wujud di dunia sebenar dan keadaan ini boleh menjejaskan aliran. Oleh itu, kajian ini dijalankan ke atas aliran dan pemindahan haba nanobendalir berdebu ke arah plat bergerak dengan kehadiran magnetohidrodinamik (MHD). Tiga jenis nanozarah iaitu Kuprum Oksida (CuO), Aluminium Oksida (Al_2O_3) dan Titanium Oksida (TiO_2) dipertimbangkan. Persamaan pembezaan separa menakluk ditukarkan kepada sistem persamaan pembezaan biasa tak linear dengan menggunakan penjelmaan keserupaan, kemudian persamaan pembezaan biasa ini diselesaikan menggunakan program bvp4c di software MATLAB. Pengaruh parameter menakluk tanpa matra seperti parameter megnetik dan pecahan isipadu nanozarah pada profil halaju dan suhu untuk fasa bendalir dan debu bagi nanobendalir berdebu dibincangkan. Hasil yang diperoleh dianalisis dengan membandingkan dua kes syarat sempadan, iaitu suhu permukaan ditetapkan dan syarat sempadan olakan dari segi kecekapan. Keputusan menunjukkan CuO mempunyai halaju terendah tetapi kadar pemindahan haba tertinggi pada kedua-dua fasa bendalir dan debu berbanding dengan Al_2O_3 and TiO_2. Selain itu, aliran dengan suhu permukaan ditetapkan mempunyai kadar pemindahan haba yang lebih baik daripada aliran dengan syarat sempadan olakan.

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LIST OF SYMBOLS AND ABBREVIATIONS

Nomenclature

Al_2O_3	-	Aluminium Oxide
Bi	-	Biot number
B_0	-	Induced magnetic field
C_f	-	Skin friction coefficient
C_p	-	Specific heat of the fluid particles
C_m	-	Specific heat of the dust particles
CuO	-	Copper Oxide
Ec	-	Eckert number
f	-	Dimensionless stream function of the nanofluid
F	-	Dimensionless stream function of the dust particles
h_f	-	Heat transfer coefficient
k	-	Thermal conductivity
K	-	Stokes resistance
m	-	Mass of dust particles
M	-	Magnetic field parameter
MHD	-	Magnetohydrodynamic
N	-	Number density of dust particle
N_1	-	Density of particle phase
Nu_x	-	Local Nusselt number
Pr	-	Prandtl number
q_w	-	Surface heat flux
Re_x	-	Local Reynolds number
RKF45	-	Runge-Kutta Fehlberg method

T - Temperature of the nanofluid

T_p - Temperature of the dust particles

T_f - Wall temperature

T_∞ - Ambient temperature

TiO_2 - Titanium Oxide

U_w - Plate velocity

U_∞ - Free stream velocity

(u,v) - Velocity components of the nanofluid in the (x,y) directions respectively

(u_p,v_p) - Velocity components of the dust particles in the (x,y) directions respectively

Greek symbols

α - Mass concentration of the dust particles

β - Fluid particle interaction parameter for velocity

β_T - Fluid particle interaction parameter for temperature

γ - Ratio of the specific heat of the fluid to dust particles

η - Similarity variable

θ - Dimensionless temperature of the nanofluid

θ_p - Dimensionless temperature of the dust particles

λ - Velocity ratio parameter

ρ - Density

σ - Electrical conductivity

τ_T - Thermal equilibrium time

τ_v - Relaxation time of the dust particles

τ_w - Surface shear stress

ϕ - Volume fraction of nanoparticles

ϕ_p - Volume fraction of dust particles

ψ - Stream function

Superscripts

$'$ - Differentiate with respect to η

Subscripts

f - fluid

nf - nanofluid

s - solid

w - Condition at the surface

∞ - Condition at infinity

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CHAPTER 1

INTRODUCTION

1.1 Background of study

Boundary layer is a thin layer adjacent to the surface of the body. The viscous effects in this thin layer cannot be ignored. Back in 1904, Ludwig Prandtl introduced a theory, which pointed out that the flow of liquid can be divided into two parts; the inviscid flow represents the main part, while the other part is viscous flow, which is a thin layer adjacent to the surface of the object. In this thin layer, the frictional force must be considered, whereas the frictional force in the areas outside this layer can be ignored since it is very small (Schlichting and Gersten, 2000). The thickness of the boundary layer is a function of the ratio between the inertial force and viscous force known as the Reynolds number. In the case of a low Reynolds number, the viscous force governs the entire boundary layer with laminar flow, while in the case of a high Reynolds number, the inertia forces allocate the boundary layer, making the fluid to become turbulent (Kakac *et al*., 2014).

Heat transfer is a process that concentrates on temperature and heat flow; each indicates the movement of thermal energy from one site to another. Heat transfer is the energy change caused by the temperature difference inside a medium or between media. The heat transfer rate in a specific direction depends on the magnitude of the temperature gradient, which is known as the rate of temperature change in that specific direction. The greater the temperature gradient, the higher the rate of heat transfer. The variation of temperature may exist inside the fluid due to the temperature difference between the boundaries and the ambient fluid. Such variation also appears from several causes, such as radioactivity, absorption of thermal radiation and the release of latent heat (as the vapour of the fluid condenses). Heat transfer cannot be stopped but can be slowed down. Three methods cause heat transfer, namely conduction, convection and radiation (Schlichting and Gersten, 2000).

Convection is the heat transfer that occurs between a surface and a moving fluid at different temperatures. The transfer of heat by convection affects the transfer of energy from the surface to the fluid on a molecular scale, while the volume mixing due to the fluid motion causes the diffusion of heat through the fluid. Unlike conduction, the current flow of liquid is greatly involved in the process of convection. This motion occurs in liquid and cannot take place in solid. In a solid, the molecules maintain their relative situations. As a result, the flow cannot occur, thus preventing convection. Convection occurs in two forms, which are natural convection and forced convection.

In natural convection, the fluid surrounding the heat source receives heat and becomes less dense and rises. The cooler fluid then moves to replace it and is subsequently heated. This process continues and forms a convection current. Buoyancy is the driving force of natural convection as the result of differences in fluid density in the presence of gravity or any type of acceleration in the system. Forced convection occurs when instruments are used to push the fluid and create an artificially induced convection current. Forced convection is sometimes referred to as heat advection. In certain systems of heat transfer, both natural and forced convection contribute significantly to the rate of heat transfer, which creates mixed convection.

Convective boundary condition corresponds to the presence of convection heating or cooling at the surface and is acquired from the surface energy balance. The equation of this condition is given by

$$-k\frac{\partial T}{\partial x}=h(T_\infty-T)\ , \tag{1.1}$$

where $\partial T/\partial x$ is the temperature gradient at the surface; k is the thermal conductivity of the material; T is the surface temperature of the wall; T_∞ is the temperature in the surrounding far away from the surface; h is the convective heat transfer coefficient.

Convective boundary condition is the most common boundary condition in practical settings since most heat transfer surfaces are exhibited to a convective environment at indicated parameters. In such conditions, the heat conduction at the surface of the material matches that at the surface in the same direction. Since the boundary cannot store energy, the net heat entering the surface from the convective side must leave the surface from the conduction side.

Nanofluid is a type of fluid that contains nanometre-sized particles, also known as nanoparticles. It comprises metals, oxides, carbides or carbon nanotubes. The fluids are designed for the colloidal suspension of nanoparticles in a base fluid. Water, ethylene, glycol and oil are some common examples of base fluids. Nanofluid has novel properties that make it widely useful in heat transfer since it exhibits enhanced thermal conductivity and convective heat transfer coefficient compared to the base fluid.

When base fluid is mixed with dust particles that contain millimetre-sized or micrometre-sized particles, it turns into dusty fluid. Dusty fluid aids in improving the thermal conductivity of the base fluid. Dusty fluid model flows have been a main part of special interest in recent studies owing to their two-phase nature. This phenomenon happens in fluid (liquid or gas) flows containing a distribution of solid particles. For example, the motion of the dusty air in fluidisation problems and the chemical process in which raindrops are formed by compounding small dust particles. Cosmic dust, which is formed due to the mixing of dust particles and gas, is the main guide for planetary systems. The production of tails of comet 238 is due to the discharge of ionised gas and dust particles from the comet's body. The application of the dusty fluid can also be pretended in processes such as nuclear reactor cooling, atmospheric fallout, powder technology, rain erosion, sedimentation, dust collection, acoustics, paint spray, performance of solid fuel rock nozzles, as well as guided missiles. These facts have boosted the consideration of solving, modelling and analysing the flow of dusty fluids.

Moving plate has a motion consisting of buoyancy forces generated by their thickness variations according to the distribution and size of their deduced parts. The pattern of motion may be dominated by different types of heat transfer. The torque balance is maintained by steady plate motion and heat transfer over a long time. The movements may be divergent, convergent, or parallel. The detailed structure and progress of all these aspects depend on a wide variety of specific parameters that differ according to the speed of relative motion.

Magnetohydrodynamic (MHD) is an academic discipline that concentrates on the macroscopic interactions of electrically conducting fluids within a magnetic field. Hannes Alfvén introduced MHD back in 1942, for which he received the Nobel Prize in Physics in 1970 (Jacob *et al.*, 2012). The idea of MHD is that magnetic fields can induce currents in a moving conductive fluid, which then create forces on the fluid and change the magnetic field. When a conducting fluid moves through a magnetic field, a Lorentz force acts on the fluid and modifies the motion. Nevertheless, when the nanofluid is under the influence of a magnetic field, a retarding force acts on the flow. This force moves in the opposite direction of the flow and decelerates the velocity of the fluid motion.

Many natural phenomena and engineering issues are subject to the analysis of MHD. Since the magnetic field exists all around the world, phenomena related to MHD occur whenever the conducting fluids are accessible. Electrically conducting fluids are plentiful although their conductivities change greatly. MHD has special technical significance owing to its regular occurrence in many industrial applications, such as MHD generators, plasma confinement, pumps, geothermal energy extractors, thermal insulators, nuclear waste disposal, heat exchangers, liquid-metal cooling of nuclear reactors, petroleum and polymer technologies, as well as heat transfer involving metallurgical processes.

1.2 Problem statement

Many applications involve fluid flow and heat transfer towards the moving plate in industrial and engineering areas. Currently, there are different issues related to low thermal conductivity in the conventional heat transfer fluid, such as water, ethylene glycol and oil, in engineering electronic devices. Aiming to overcome this defect in conventional fluid, studies have focused on mixing nanometre-sized particles in base fluid, creating nanofluid. Besides, other studies have attempted mixing millimetre-sized or micrometre-sized conducting dust particles in the base fluid, creating dusty fluid. These methods help to enhance the thermal conductivity of the base fluid. Nanofluid without impurities has been the focus of numerous studies. However, realistically, the presence of impurities affects the flow toward the moving plate including heat dissipation to the ambient surroundings, which reduces their efficiency, pressure of the flow and the temperature differential that will not transfer energy. Besides,

As a result, studies have focused on exploring the flow and heat transfer behaviour of either nanofluid or dusty fluid (Sastry *et al.*, 2016). MHD is also considered due to the exerted magnetic field that controls the suspended particles and rearranges their concentration in the fluid, which extremely changes the heat transfer characteristics of the flow. Based on the argument, this current study develops a mixture of millimetre-sized or micrometre-sized conducting dust particles in nanofluid in the presence of magnetic field, which is termed MHD dusty nanofluid, to analyse its flow and heat transfer characteristics.

1.3 Research questions

The research is to identify the following problems:

1. What is the mathematical model that best describes the flow of MHD dusty nanofluid towards a moving plate?
2. What is the method used to solve the mathematical model numerically?
3. How do magnetic parameter and nanoparticle volume fraction affect velocity profile, temperature profile, skin friction coefficient and Nusselt number?

1.4 Objectives of study

The specific objectives of this study are presented in the following:

1. To develop a mathematical model for the flow of MHD dusty nanofluid towards a moving plate with constant surface temperature and convective boundary condition.
2. To solve the mathematical model numerically using MATLAB boundary value problem solver, bvp4c program.
3. To examine the effects of magnetic parameter and nanoparticle volume fraction on the velocity profile, temperature profile, skin friction coefficient and Nusselt number of the flow.

1.5 Scope of study

The scope of this study is outlined as follows:

1. Two-dimensional flow, steady, incompressible and electrically conducting boundary layer flow towards the moving plate in a dusty nanofluid.

ii. Focusing on MHD dusty nanofluid, this study incorporated the effects of non-dimensional governing parameters, namely magnetic parameters and nanoparticle volume fraction, on the velocity and temperature of the nanofluid.
iii. This study employs the MATLAB boundary value problem solver, bvp4c program to solve the mathematical model.
iv. This study uses water (H_2O) as a base fluid.
v. This study utilises three types of nanoparticles, namely copper oxide (CuO), aluminium oxide (Al_2O_3) and titanium oxide (TiO_2).
vi. Two different cases of boundary conditions involving the flow of MHD dusty nanofluid towards the moving plate, namely constant surface temperature and convective boundary condition, are compared in terms of efficiency.

1.6 Significance of study

Fluid heating and cooling are important aspects of science and engineering areas. Thus, an effective cooling technique for any type of high-energy device is required. However, fluids have limited heat transfer capabilities to act as a medium for heat transfer. As a result, it is encouraged to find fluids with enhanced thermal properties, specifically with advanced heat transfer capabilities and higher conductivity.

Many engineering disciplines can apply fluid heating and cooling such as thin film solar energy collector device, transpiration cooling and climate control. These aspects are not only suitable for extended ranges of temperature control based on fluid selection but also have smooth continuous temperature control with no gaps.

1.7 Thesis organisation

This thesis is divided into five chapters, including the introductory chapter. The remaining four chapters are organised as follows:

Chapter 2 discusses the findings of prior studies. The heat transfer in nanofluid, flow of dusty fluid and nanofluid, magnetohydrodynamic fluid flow, fluid flow towards the moving plate, fluid flow with convective boundary conditions as well as numerical method for solving fluid flow are reviewed in the chapter. The chapter is then concluded with the identified research gap.

Chapter 3 describes the current study's method to solve the mathematical model of MHD dusty nanofluid towards the moving plate. The governing equations of flow and heat transfer are covered in the chapter. Similarity variables are also introduced, which are used to convert the governing equations into a system of non-linear ordinary differential equations. The numerical method to solve the system of non-linear ordinary differential equations is discussed in the chapter. Following that, the chapter describes the applied numerical method to solve the equations with the use of mathematical software.

Chapter 4 presents the numerical solutions to the problem in tables and figures. Moreover, two cases of boundary conditions of the problem are compared in terms of efficiency to determine the best heat transfer rate of MHD dusty nanofluid towards the moving plate.

Chapter 5 concludes the study's results after comparing the numerical solutions obtained between the two cases. All results were referred to the study's objectives to determine whether or not the objectives have been achieved. The limitations of this study and recommendations for future research are also presented in the chapter.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter reviews the findings of prior related studies. In particular, Section 2.2 deals with heat transfer in a nanofluid. Section 2.3 then discusses the flow of dusty fluid and nanofluid. After that, Section 2.4 introduces the MHD fluid flow. The fluid flow towards the moving plate is discussed in Section 2.5, while the fluid flow with convective boundary condition is explained in Section 2.6. Furthermore, the numerical method for solving the fluid flow is reviewed in Section 2.7. Lastly, Section 2.8 concludes the identified research gap based on the literature review from Section 2.2 to Section 2.7.

2.2 Heat transfer in nanofluid

Li and Xuan (2002) investigated the convective heat transfer and flow characteristics of nanofluid in a tube utilising copper water nanofluid for the experimental system. Based on the results, the suspended nanoparticles significantly increased the convective heat transfer coefficient of the base fluid. There was almost no change in the friction factor of the sample nanofluid with the low volume fraction of nanoparticles. Meanwhile, Koo and Kleinstreuer (2005) studied the heat transfer of nanofluid flow in micro-heat-sinks and revealed that the presence of nanoparticles significantly increased the heat transfer capability of the micro-heat-sinks. The study highlighted the advantages of high thermal conductivity of nanoparticles, justifying the benefits of adding nanoparticles.

Kwak and Kim (2005) considered the viscosity and thermal conductivity of copper oxide nanofluid for the dispersion in ethylene glycol. The results showed that a significant enhancement in thermal conductivity pertaining to particle concentration can only be achieved when the particle concentration is below the dilute limit. Chon *et al.* (2005) reported the correlation for the case of thermal conductivity of aluminium oxide nanofluid as the effect of nanoparticle size on temperature. High temperature increases the nanofluid thermal conductivity and subsequently, the Brownian motion of nanoparticles. Evans *et al.* (2006) explained the role of the Brownian motion hydrodynamics on the thermal conductivity of nanofluid where Kinetic theory was used to analyse the heat flow in fluid suspension of the nanofluid, which revealed only a minor effect of the Brownian motion hydrodynamics on the thermal conductivity of nanofluid.

In another study, Kang *et al.* (2006) estimated the thermal conductivity of nanofluid using experimental effective particle volume, thus indicating the critical need to consider the heat transfer mechanisms, such as phonon transport and electron transport, to estimate the exact thermal conductivity of nanofluid. Meanwhile, Jang and Choi (2007) analysed the effects of various parameters on the thermal conductivity of nanofluid and discovered the major effects of nanoparticle size, volume fraction, and temperature on the conductivity of nanofluid. However, these parameters only exhibited minor effects on the ratio of the thermal conductivity of nanoparticles to that of the base fluid. The study further noted the emerging field of nanofluids, with high potential and challenges. Hence, any new concepts should be confirmed tentatively, and more feasible models should be developed in the future.

Yu *et al.* (2008) reviewed and compared nanofluid thermal conductivity and its heat transfer enhancement. The study reported that the heat transfer enhancement for nanofluid was within the range of 15% to 40%. Nevertheless, studies of laminar and turbulent flow still need to be developed substantially in future research. Anoop *et al.* (2009) examined the characteristics of the convective heat transfer of nanofluid in the developing region of tube flow with constant heat flux. Two particle sizes were used in the study, namely the average particle sizes of 45 nm and 150 nm. Based on the results, the heat transfer characteristics of both nanofluids were higher than that of the base fluid, and the nanofluid with a particle size of 45 nm had a higher heat transfer coefficient compared to that with a particle size of 150 nm.

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