SHORT COMMUNICATION

Thermal and solutal stratification on MHD nanofluid flow over a porous vertical plate

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Thermal and solutal stratification; Nanofluid; Magnetic field; Nanoparticle volume fraction; Porous vertical plate

Abstract
Nanoparticles have the highest credibility to develop the thermal properties compared to conventional particle fluid suspension. Thermal and solutal stratification on heat and mass transfer induced due to a nanofluid over a porous vertical plate is analyzed. The transport equations engaged in the study include the effect of Brownian motion and thermophoresis particle deposition. The nonlinear governing equations and their related boundary conditions are initially looked into dimensionless forms by similarity variables. The resulting equations are solved numerically utilizing the fourth-fifth order Runge–Kutta–Fehlberg method with shooting technique (MAPLE 18). It is investigated that the temperature of the nanofluid and the concentration fraction decelerate with increase in thermal and solutal stratification.

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1. Introduction

Nanofluids are engineered by suspending nanoparticles with average size below 100 NM in traditional heat transfer fluids such as water, oil, and ethylene glycol. Thermal conductivity of water, oil, and ethylene glycol with nanoparticles plays an important role in the heat transfer coefficient between the heat and mass transfer medium and the heat and mass transfer surface. The roses in thermal conductivity are substantial in improving the heat transfer behavior of fluids. Many authors have predicted that the nanofluids have better convective heat transfer capability than that of base fluids. Thermal and solutal stratification has many practical applications on heat and mass transfer, Gan and Qiao [1], Senthilraja et al. [2], Chen and Eichhorn [3], Kulkarni et al. [4], Angirasa and Srinivasan [5], Chen and Eichhorn [6] Ishaq et al. [7], Mukhopadhyay et al. [8] and Srinivasacharya and Ram Reddy [9–11].

In practical situations, it is interesting to investigate the effect of thermal and solutal stratification on the convective transport of the nanofluids.

The thermal stratification of lakes refers to a change in the temperature at different depths in the lake, and is due to the change in water’s density with temperature. Cold water is denser than warm water and the epilimnion generally consists of water that is not as dense as the water in the hypolimnion, Lackey and Robert [12]. Thermal stratification is uncommon in warm, shallow bodies of water found in southern regions. The tendency of water to form horizontal layers is also disrupted by extremely low temperatures. When the temperature

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of a lake or pond drops below the freezing point, the coldest layer moves to the top. Cold bodies of water that are above freezing are called isothermal, because the temperature of the water becomes relatively even at all depths. Effect of stratification is an important aspect in heat and mass transfer analyses. Stratification of fluids occurs due to temperature variations (thermal stratification), concentration differences (solutal stratification) or the presence of different fluids of different densities. When the heat and mass transfer are present simultaneously then it is important to analyze the effect of double stratification on the convective flows in different situations, Chang and Lee [13], Cheng [14], Srinivasacharya and Reddy [15], Srinivasacharya and Upendar [16], Ibrahim and Makinde [17] and Srinivasacharya and Surender [18]. The impact of thermal and solutal stratification on heat and mass transfer analysis in the boundary layer nano fluid flow over a stretching surface has key role in the industrial and engineering applications, for example, manufacturing of plastic and rubber sheets, annealing and thinning of copper wires, drawing on stretching sheets through quiescent fluids, boundary layer along a liquid film condensation process, damage of crops due to freezing, desalination, refrigeration and air conditioning, compact heat exchangers, solar power collectors, human transpiration and many others, Sajid et al. [19] and Hayat et al. [20].

Particularly, vehicle cooling/ heating has an essential technological meaningful as it is directly or indirectly associated with thermal and solutal stratification performance. Depending on the size and shape of the nanoparticles in the presence of thermal and solutal stratification, the nanofluid may be able to alter its viscosity proportion to the strength of the magnetic field assigned to it. Choi [21] and Khan and Aziz [22] investigated that the nanoparticles inclusion in a base fluid has changed the transport mechanisms and thermal and diffusion characteristics of a convectional base fluid. Nanofluid has been proved both experimentally and theoretically to get advanced heat and mass transport resources and better energy performance in a collection of thermal exchange system for various industrial applications, Kuznetsov and Nield [23], Khan and Pop [24], Rosmila et al. [25], Wubsheh and Makinde [17], Ibrahim and Shanker [26] and Anbucedzhian et al. [27]. The investigation of mixed convection in a thermal and solutal stratified medium of nanofluid is an essentially interesting and significant problem because it has a large amount of industrial application, Rathish Kumar and Shalini [28], Murthy et al. [29] and Lakshmi Narayana and Murthy [30,31]. Particularly, the strength of the thermal and solutal stratification plays a dominant role in dying industries. Recently, the convective fluid flow in a porous medium has contributed one of the best schemes for heat and mass transfer theory and it has a comfortable theoretical and practical interest and has been broadly investigated [32–35].

MHD flow and heat transfer of nanofluids against different types of plate in porous medium with variable stream conditions in the presence of water based nanoparticles are analyzed

<table>
<thead>
<tr>
<th>Nomenclature</th>
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<tbody>
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<td>$B_0$</td>
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<tr>
<td>$A, B$</td>
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<td>$C_w$</td>
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<td>$T_\infty$</td>
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<td>$T_{\infty,0}$</td>
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<tr>
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<td>$u, v$</td>
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<td>$V_0$</td>
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<th>Greek symbols</th>
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<td>$\beta$</td>
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<tr>
<td>$\rho$</td>
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<td>$\rho c_p$</td>
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<td>$\sigma$</td>
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<tr>
<td>$\mu$</td>
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<td>$\epsilon_1$</td>
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<td>$\lambda$</td>
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<td>$f$</td>
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<tr>
<td>$\theta$</td>
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Current analysis is to study the MHD convective nanofluid flow on a porous vertical plate with thermal and solutal stratification. The effect of thermal and solutal stratification with variable stream conditions is investigated and presented through graphs. The effects of governing parameters on fluid velocity, temperature and nanoparticle volume fraction have been examined and shown graphically and in tables. The results are correlated and found to be a good agreement.

2. Mathematical formulation

Consider the steady two-dimensional convective nanofluid flow along a porous vertical plate with thermal and solutal stratification. Let x-axis is along the porous vertical plate and the y-axis is normal to the plate. The inelastic magnetic field of strength $B_0$ is activated in the positive direction of the x-axis. The ambient temperature and concentration of the nanofluid are considered as $T_\infty = T_{\infty,0} + Ax$ and $C_\infty = C_{\infty,0} + Bx$ where the body surface is kept at a constant temperature $T_w$ and nanofluid concentration $C_w$ (Fig. 1). $A$ and $B$ are constants and the depth of stratification in the medium can be considered as $T_{\infty,0}$ and $C_{\infty,0}$. Based on the boundary-layer approximation, the physical significance of the governing equations can be written as

$$
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}
$$

$$
u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} = \left( \frac{v}{K} + \frac{\sigma B_0^2}{\rho} \right) u + (1 + C_{\infty,0}) \rho_f \alpha \beta (T - T_{\infty,0}) - (\rho - \rho_{f,0}) g (T - T_{\infty,0}) \tag{2}
$$

$$
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = 1 \frac{\partial^2 T}{\partial y^2} + \tau \left( D_b \frac{\partial C}{\partial y} + D_f \frac{\partial^2 T}{\partial y^2} \right) \tag{3}
$$

$$
\frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} = D_b \frac{\partial^2 C}{\partial y^2} + D_f \frac{\partial^2 T}{\partial y^2} \tag{4}
$$

Boundary conditions

$$
u = 0, \quad v = -V_0, \quad T = T_{w}, \quad C = C_{w} \quad \text{at} \ \eta = 0; \quad u = 0, \quad T = T_{\infty}, \quad C = C_{\infty} \quad \text{and} \ \eta \rightarrow \infty \tag{5}
$$

The local Rayleigh number and the stream functions are explained as

$$Ra_c = \frac{(1 - C_{\infty,0}) \beta g (T_w - T_{\infty,0})}{\nu} \Rightarrow u = -\frac{\partial \phi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{6}
$$

Submit the similarity transforms

$$
\eta = \frac{y}{x} Ra_c^{1/4}, \quad \psi = Ra_c^{1/4} \zeta f(\eta), \quad \theta(\eta) = \frac{T - T_{\infty,0}}{T_w - T_{\infty,0}}, \quad \varphi(\eta) = \frac{C - C_{\infty,0}}{C_w - C_{\infty,0}} \tag{7}
$$

$C_w - C_{\infty,0} = nx, \quad T_w - T_{\infty,0} = nx, \quad m, n$ are constants.

Eqs. (1)–(4) are committed to the non-dimensional form

---

**Table 1** Comparison of the result of $-\theta(0)$ for mixed convection along a vertical plate [23,26].

<table>
<thead>
<tr>
<th>Pr</th>
<th>Kuznestov and Nield [23]</th>
<th>Ibrahim and Shanker [26]</th>
<th>Present result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.401</td>
<td>0.401</td>
<td>0.40085723</td>
</tr>
<tr>
<td>10.0</td>
<td>0.463</td>
<td>0.463</td>
<td>0.46269571</td>
</tr>
<tr>
<td>100.0</td>
<td>0.485</td>
<td>0.481</td>
<td>0.48275198</td>
</tr>
<tr>
<td>1000.0</td>
<td>0.491</td>
<td>0.484</td>
<td>0.48902638</td>
</tr>
</tbody>
</table>

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**Table 2** Comparison of skin friction, Nusselt number and Sherwood number for various values of $M$ with $Pr = 6.2, Le = 5.0, Nb = Nt = Ec = 0.1, Ec = 0.0, S = 0.0, n = 2.0, \epsilon_1 = \epsilon_2 = 0$, Mabood et al. [40].

<table>
<thead>
<tr>
<th>$M$</th>
<th>$-f'(0)$</th>
<th>$-\theta'(0)$</th>
<th>$-\varphi'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.1010083094</td>
<td>1.0671888952</td>
<td>1.077193973</td>
</tr>
<tr>
<td>0.5</td>
<td>1.309887243</td>
<td>1.0436527961</td>
<td>1.010901861</td>
</tr>
<tr>
<td>1.0</td>
<td>1.4891197029</td>
<td>1.0233716833</td>
<td>0.954948470</td>
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</table>
Eqs. (8)–(10) are extremely nonlinear coupled equations and numerical solutions are achieved using (fourth-fifth order Runge–Kutta–Fehlberg method with shooting technique) MAPLE 18 and the Maple worksheet is listed in Appendix A. The computational outputs are pictured in terms of dimensionless velocity, temperature and nanoparticle volume fraction with fixed values of \( \text{Nr} = 0.5 \), \( \text{Pr} = 10.0 \), \( \text{Nb} = \text{Nt} = 0.5 \), \( \lambda = 1.0 \), \( \varepsilon_1 = \varepsilon_2 = 0.2 \) and, correspond physically to the nanofluid. In order to validate our method, we have compared the results of \(-\theta'(0)\) with those of Kuznestov and Nield [23] and Ibrahim and Shanker [26] and the results of \(-f'(0), \theta'(0)\) and \(-\phi'(0)\) with those of Mabood et al. [40] and found them in excellent agreement, Tables 1 and 2. Thus the present results are more accurate than their results.

The velocity profiles for various values of \( \text{Nr} \) with \( M = 0.0 \) (solid line of Fig. 2) are exactly correlated with Fig. 4 of Ibrahim and Makinde [17].

Figs. 2 and 3 present the velocity, temperature and concentration of the nanofluid with \( M = 1.0 \) and \( M = 0.0 \). In the presence of reliable Brownian motion of the nanoparticle, the velocity decreases \( 0 \leq \eta < 4.18/0 \leq \eta < 5.07 \) and then increases \( \eta > 4.18/\eta > 5.07 \) for \( M = 1.0/M = 0.0 \) whereas both \( M = 1.0 \) and \( M = 0.0 \), the temperature and the concentration of the nanofluid increase with increase in buoyancy ratio \( \text{Nr} \). The momentum boundary layer thickness for \( M = 1.0 \) is weaker than that of \( M = 0.0 \) (Fig. 2) but the thermal and diffusion boundary layer thickness for \( M = 1.0 \) is stronger than that of \( M = 0.0 \) (Fig. 3) with increase in the buoyancy ratio because of the combined effect of nanoparticle mass density and thermal coefficient expansion of the base fluid with Lorentz force plays a powerful role on the nanofluid flow field. Both \( M = 1.0 \) and \( M = 5.0 \), the temperature and concentration increase with increase in thermophoresis parameter and the boundary layer thickness for \( M = 5.0 \) is higher compared to \( M = 1.0 \) because of the joint effects of the density and electric conductivity of the nanofluid, Fig. 4. However, for high strength of the magnetic field, the thermal and diffusion effect is deeply correlated with the convection effect and hence the thermophoretic force is expected to modify the thermal and the diffusion boundary layer significantly.

In the presence of magnetic field \( M = 1.0 \) and \( M = 5.0 \), the velocity and temperature of the nanofluid decrease with increase in thermal stratification strength, whereas the velocity increases and the concentration of the nanofluid decreases with increase in solutal stratification strength due to the decrease of

\[
\begin{align*}
\frac{3}{4Pr} (f'' - f^2) + \theta \phi - (M + \lambda) u &= 0 \\
\frac{3}{4} f'' - f^2 - \frac{1}{4} N \phi + Nt \theta^2 &= 0 \\
\frac{3}{4} \phi'' + \frac{1}{4} Le f' \phi' - Le f' \phi^2 + \frac{Nt}{Nb} \theta' &= 0
\end{align*}
\]

\[
\begin{align*}
\frac{3}{4} f'' &= S, \quad u(0) = 0, \quad v(0) = z, \quad \theta(0) = 1 - \frac{\varepsilon_1}{4}, \\
\phi(0) &= 1 - \frac{\varepsilon_2}{4}, \quad p(0) = \beta, \quad q(0) = \tau, \\
&\theta(\infty) = 0, \quad \phi(\infty) = 0
\end{align*}
\]

\[\begin{align*}
\varepsilon_1 &= \text{porous parameter}, \quad M = \frac{a E^2 \nu^2}{\mu Re_x} - \text{magnetic parameter}. \\
\eta &= 0: f(0) = S, \quad u(0) = 0, \quad \theta(0) = 1 - \frac{\varepsilon_1}{4}, \quad \phi(0) = 1 - \frac{\varepsilon_2}{4} \\
\eta &= \infty: f'(0) \rightarrow 0, \quad \theta'(0) \rightarrow 0, \quad \phi(\infty) \rightarrow 0, \quad \chi(\infty) \rightarrow 0
\end{align*}
\]

\[S = \frac{4V_0 x}{\text{Re}^{1/2}}, \quad S > 0 \text{ corresponds to suction}; \\
S < 0 \text{ corresponds to injection.}
\]

If \( \varepsilon_1 = \varepsilon_2 = 0 \), the problem is unstratified, Kuznetsov and Nield [23]. Physical quantities are \( C_f = \frac{\partial u}{\partial \eta} \) – skin friction coefficient, \( Nu_t = \frac{\partial \theta}{\partial \eta} \) – local Nusselt number, \( Sh_t = \frac{\partial \phi}{\partial \eta} \) – local Sherwood number. \( \tau_v, \nu_1, \) and \( \phi_0 \) are defined as

\[\begin{align*}
C_f (Re_x)^{1/2} &= f'(0), \quad \frac{Nu_t}{Re_x^{1/2}} = -\theta'(0), \quad \frac{Sh_t}{Re_x^{1/2}} = -\phi'(0)
\end{align*}
\]

\[\begin{align*}
Re_x &= \frac{u_{\infty} \eta^{1/2}}{\nu} \quad \text{the local Reynolds number.}
\end{align*}
\]

3. Numerical solution

Eqs. (8)–(11) are reduced into the system of first order ordinary differential equations as

\[\begin{align*}
f'(\eta) &= u(\eta) \\
u'(\eta) &= v(\eta) \\
v'(\eta) &= -\frac{3}{4Pr} (f(\eta)v(\eta) - u(\eta)^2) - \theta(\eta) + Nr \cdot \phi(\eta) + (M + \lambda) u(\eta)
\end{align*}
\]

\[\begin{align*}
\phi'(\eta) &= \rho(\eta) \\
\phi'(\eta) &= -\frac{3}{4} f(\eta)p(\eta) + \frac{\varepsilon_1}{4} u(\eta) - Nb \cdot q(\eta)p(\eta) - Nt \cdot p(\eta)^2
\end{align*}
\]

\[\begin{align*}
\phi'(\eta) &= q(\eta) \\
\phi'(\eta) &= -\frac{3}{4} Le \cdot f(\eta)q(\eta) + Le \cdot \frac{\varepsilon_2}{4} u(\eta)
\end{align*}
\]

\[\begin{align*}
-\frac{Nt}{Nb} \left( \frac{3}{4} f(\eta)p(\eta) + \frac{\varepsilon_1}{4} u(\eta) - Nb \cdot q(\eta)p(\eta) - Nt \cdot p(\eta)^2 \right)
\end{align*}
\]
Figure 2  Comparison of velocity profile for \( Nr \) with Fig. 4 of Wubshet Ibrahim and Makinde [17].

Figure 3  Temperature and concentration profiles for different values of \( Nr \).

Figure 4  Temperature and concentration profiles for different values \( Nt \).
Figure 5  Effect of thermal stratification on velocity and temperature profiles.

Figure 6  Effect of solutal stratification on velocity and concentration profiles.

Figure 7  Effects of Brownian motion with velocity and concentration profiles.
In particular, the thermal and diffusion boundary layer thickness for $M = 5.0$ being more advanced than $M = 1.0$ due to the Brownian motion of the nanoparticles in the magnetic field. Brownian motion is the accidental motion of particles in the base fluid and the movement of the particles undergoing Brownian motion is performed by solving the diffusion equation under convenient boundary conditions. Both $M = 1.0$ and $M = 5.0$, the velocity increases and the concentration of the nanofluid decreases (Fig. 7) with increase in Brownian motion of the nanoparticle whereas the momentum boundary layer thickness of the nanofluid for $M = 1.0$ is higher than $M = 5.0$ because the rate of movement of nanoparticles is associated with the viscosity and concentration of the nanofluid.

Due to the mixed effect of permeability of the porous medium and the kinematic viscosity of the nanofluid, it is shown that the velocity of the nanofluid decreases and the temperature of the nanofluid increases with increase in porosity, Fig. 8. The momentum boundary layer thickness for $M = 1.0$ being monotonically stronger than $M = 5.0$ because of the strength of the density of the nanofluid and permeability of the porous medium. Fig. 9 presents the velocity and temperature for different values of magnetic strength in the presence of Prandtl number $Pr = 1.0$ and $Pr = 10.0$ with uniform injection $S = -0.5$. Due to the uniform injection with Prandtl number $Pr = 10$, the velocity of the nanofluid firstly decreases $0 \leq \eta \leq 5.56$ and then increases, whereas the temperature of the nanofluid increases with increase in magnetic strength because of the Lorentz force falls down the motion of the fluid and to enhance the temperature profiles.

5. Conclusion

Thermal and solutal stratification boundary condition in the presence of magnetic field makes this study novel one. The important results of this investigation are listed as follows:

The buoyancy ratio in the presence of magnetic field plays a dominant role in the temperature and a nanoparticle volume fraction of the nanofluid.

The Brownian motion and thermophoretic particle deposition in the presence of the high strength of the magnetic
field attain a significant role on concentration because of the random motion of particles suspended in a fluid. The temperature and concentration of the nanofluid decrease with increase in thermal and solutal stratification respectively. It implies that the displacement of temperature and the concentration depends on the strength of thermal and solutal stratification.

The momentum boundary layer thickness of the nanofluid reduces/raises with increase in thermal/solutal stratification whereas the thickness for $M = 5.0$ being lower compared to $M = 1.0$. This is because of the high strength of the magnetic field in the presence of permeability of the porous medium plays an important role on the flow field.

The velocity/temperature of the nanofluid (Pr = 10) decreases/increases with increase in magnetic parameter since the electromagnetic force to the viscous force of the viscous force of the nanofluid.

The thermal and solutal stratification in the presence of nanofluid with high strength of the magnetic field can be treated as an effective transport technique using the conservation of energy and diffusion principles together with the second law of thermodynamics and Fick’s law of diffusion. The impact of thermal and solutal stratification in the presence of nanofluid has been attracting interest in several thermal applications, e.g. active and passive solar heating, water heating, cooling and air-conditioning applications.

Appendix A. Numerical solution

Eqs. (10) and (11) subjected to the boundary condition (12) are converted into the following simultaneous system of first order differential equations as follows:

$$
A_1 = (1 - \zeta)^{2.5} \left(1 - \zeta + \frac{\rho_s}{\rho_f}\right),
A_2 = (1 - \zeta)^{2.5} \left(1 - \zeta + \frac{\sigma_s}{\sigma_f}\right),
A_3 = \left(1 - \zeta + \frac{(\rho C_p)_s}{(\rho C_p)_f}\right),
A_4 = (1 - \zeta),
A_5 = \left(\frac{k_{nf}}{k_f} + \frac{Pr R}{k_f A_3}\right)
$$

$$
f'(\eta) = u(\eta),
\ v'(\eta) = v(\eta),
\ u(0) = 0, \quad p(0) = -\frac{k_f}{k_{nf}};
\ u(L) = 1,
\ \theta(L) = 0, \quad v(0) = x, \quad \theta(0) = \beta
$$

where $x$ and $\beta$ are priori unknowns to be determined as a part of the solution.

This software uses a fourth-fifth order Runge–Kutta–Fehlberg method with shooting technique as default to solve the boundary value problems numerically using the Dsolve command MAPLE 18. The values of $x$ and $\beta$ are determined upon solving the boundary conditions $v(0) = x$, $\theta(0) = \beta$ with trial and error basis. The numerical results are represented in the form of the dimensionless velocity and temperature in the presence of water, ethylene glycol and engine oil based CNTs, Cu and Al$_2$O$_3$.

Appendix B. MAPLE 18 software
\[ IC := \{ f(0) = s, u(0), \theta(0) = 1 - \frac{q_1}{r}, \varphi(0) = 1 - \frac{q_2}{r} \} \]

\[ \{ v(0) = x, r(0) = t, h(0) = \zeta \} \]

\[ >L := 6; L := 6 \]
\[ >BC := \{ u(L) = 0, \theta(L) = 0, \varphi(L) = 0 \} \]
\[ >S := \text{shoot}(ODE, IC, BC, FNS, [x = 0.4245, \tau = -0.211667, \zeta = -0.494458] \]

References

[13] W. Ibrahim, B. Shanker, Thermophoresis and Brownian motion effects on boundary layer flow of nanofluid in presence of thermal


