

BOUNDARY LAYER FLOW AND HEAT TRANSFER OF DUSTY  
NANOFLUID OVER HORIZONTAL SHEET WITH VARIOUS CONDITIONS

NURUL AISYAH BINTI JOHAN

A thesis submitted in  
fulfillment of the requirement for the award of the  
Degree of Master of Science

Faculty of Applied Sciences and Technology  
Universiti Tun Hussein Onn Malaysia

FEBRUARY 2023

## ACKNOWLEDGEMENT

First of all, Alhamdulillah, sincere praise to Allah the Almighty because with His Power and Will, this thesis has proved that the author already completed the Master's degree successfully.

The author would like to express the deepest gratitude to her supervisor, Dr Syahira Binti Mansur for the encouragement and guidance throughout the journey of the research. Thank you for the willingness to spend time giving ideas, instructions, support, and motivations. To the examiners of the research, thank you for the time and advice to fix the research's flaws. Appreciation is also given to the Research Management Centre (RMC) of Universiti Tun Hussein Onn Malaysia for granting the author the opportunity to be a graduate research assistant, which helped her financially.

Special appreciation goes to the author's beloved parents and family, which have always motivated and supported her throughout the entire project research. Last but not least, the contribution would not be a success without the help from everyone involved directly or indirectly towards the completion of this thesis. Therefore, the author would like to extend her sincere thanks to all of them.



## ABSTRACT

The study of boundary layer flow has gained the interest of researchers as the ability of fluid flow on increasing machines productivity. This study presents the analysis and discussion of various conditions of boundary layer flow with dusty nanofluid. This research discusses two problems where dusty nanofluid flow over stretching surface with partial slip effects and towards stretching/shrinking sheet with heat and suction. The analysis involves three types of nanoparticles namely copper (Cu), aluminium oxide ( $\text{Al}_2\text{O}_3$ ) and titania ( $\text{TiO}_2$ ). Hence, the effect of the volume fraction of nanoparticles has been examined besides the volume fraction of dust particles, velocity slip, and thermal slip. Meanwhile, the second part is to study the effect of parameters namely stretching/shrinking sheet, suction, and heat generation/absorption. The governing equations for both problems were transformed into non-linear ordinary differential equations using similarity transformation, which were then numerically solved using the boundary value problem solver, bvp4c program of MATLAB R2019b software. The parameters involved were computed, analysed, and discussed. The numerical solutions for skin friction coefficients, local Nusselt number, velocity and temperature profiles are presented graphically. In addition, a comparison of present results with the existing study has achieved excellent agreement. It was found that nanoparticles act with good thermal conductivity. Besides,  $\text{Al}_2\text{O}_3$  and  $\text{TiO}_2$  showed significant effects on the velocity of fluid and dust phases. Suction enhanced heat transfer rate while minimising momentum and thermal boundary layer. Furthermore, the heat transfer rate improved by heat generation and absorption over the stretching/shrinking sheet.

## ABSTRAK

Kajian aliran lapisan sempadan menarik minat para penyelidik kerana kemampuan aliran bendalir yang mampu meningkatkan produktiviti mesin. Kajian ini membentangkan analisis dan perbincangan pelbagai syarat terhadap aliran lapisan sempadan dengan bendalir nano berdebu. Penyelidikan ini membincangkan dua masalah iaitu aliran bendalir nano berdebu di atas permukaan meregang bersama kesan gelinciran separa dan terhadap kepingan meregang/mengecut bersama haba dan sedutan. Analisis melibatkan tiga jenis zarah nano iaitu tembaga (Cu), aluminium oksida ( $\text{Al}_2\text{O}_3$ ) dan titania ( $\text{TiO}_2$ ). Justeru, kesan bagi isipadu zarah nano dikaji disamping isipadu zarah-zarah debu, gelinciran halaju dan gelinciran haba. Sementara itu, bahagian kedua adalah kajian tentang kesan-kesan parameter seperti kepingan meregang/ mengecut, sedutan dan penghasilan/penyerapan haba. Persamaan menakluk bagi kedua-dua masalah ditukar menjadi persamaan pembezaan biasa tidak linear melalui penjelmaan keserupaan yang kemudiannya diselesaikan secara berangka menggunakan penyelesaian masalah nilai had, program bvp4c yang terdapat di dalam perisian MATLAB R2019b. Parameter-parameter yang terlibat dikira, dianalisis dan dibincangkan. Hasil berangka bagi pekali geseran kulit, nombor Nusselt tempatan beserta profil-profil halaju dan suhu dipersembahkan dalam bentuk graf. Tambahan lagi, perbandingan antara kajian terkini dengan hasil kajian sedia ada mendapati terdapat persamaan. Kajian ini mendapati bahawa zarah nano bertindak dengan terma konduktiviti yang baik. Selain itu,  $\text{Al}_2\text{O}_3$  dan  $\text{TiO}_2$  menunjukkan kesan yang ketara pada halaju bagi fasa bendalir dan debu. Sedutan meningkatkan kadar pemindahan haba sambil meminimumkan lapisan sempadan momentum dan haba. Selain itu, kadar pemindahan haba bertambah baik dengan penghasilan and penyerapan haba pada permukaan meregang/mengecut.

## CONTENTS

<b>TITLE</b>	<b>i</b>
<b>DECLARATION</b>	<b>ii</b>
<b>ACKNOWLEDGEMENT</b>	<b>iii</b>
<b>ABSTRACT</b>	<b>iv</b>
<b>ABSTRAK</b>	<b>v</b>
<b>CONTENTS</b>	<b>vi</b>
<b>LIST OF TABLES</b>	<b>ix</b>
<b>LIST OF FIGURES</b>	<b>x</b>
<b>LIST OF SYMBOLS AND ABBREVIATIONS</b>	<b>xiv</b>
<b>LIST OF APPENDICES</b>	<b>xvii</b>
<b>CHAPTER 1 INTRODUCTION</b>	<b>1</b>
1.1 Research background and rationale of study	1
1.2 Problem statement	3
1.3 Research objectives	3
1.4 Scopes of study	4
1.5 Significances of study	5
1.6 Thesis outline	5
<b>CHAPTER 2 LITERATURE REVIEW</b>	<b>7</b>

2.1	Nano-sized particles in based fluid	7
2.2	Dust particles in fluid	8
2.3	Stretching and shrinking surface	10
2.4	Factors effected fluid flow behaviour	11
2.4.1	Heat generation/ absorption towards the flow	11
2.4.2	Slip at the wall boundary	12
2.4.3	Suction	13
2.5	Summary	13
<b>CHAPTER 3 PROBLEM FORMULATION</b>		<b>15</b>
3.1	Flowchart of research	16
3.2	Mathematical model	16
3.3	Governing equations of fluid dynamic	17
3.3.1	The continuity equation	17
3.3.2	The momentum equation	18
3.3.3	The energy equation	19
3.4	Assumptions on the governing equations	20
3.5	The governing equations on two-dimensional boundary layer of dusty nanofluid flow	21
3.6	Numerical solution	22
<b>CHAPTER 4 BOUNDARY LAYER FLOW OF DUSTY NANOFLUID OVER STRETCHING SHEET WITH PARTIAL SLIP EFFECTS</b>		<b>24</b>

4.1	Introduction	24
4.2	Governing equations	25
4.3	Reduced to first-order system	29
4.4	Results and discussion	30
4.5	Conclusion	41

**CHAPTER 5 DUSTY NANOFLUID FLOW OVER STRETCHING/  
SHRINKING SHEET WITH SUCTION AND HEAT  
GENERATION/ ABSORPTION** **42**

5.1	Introduction	42
5.2	Governing equations	43
5.3	Reduced to first-order system	46
5.4	Results and discussion	47
5.5	Conclusion	62

**CHAPTER 6 CONCLUSION AND RECOMMENDATIONS** **64**

6.1	Summary and conclusion	64
6.2	Recommendations for future research	66

**REFERENCES** **67**

**APPENDICES** **73**

**VITA** **96**

## LIST OF TABLES

4.1	Thermophysical properties of nanoparticles and base fluid	31
4.2	Comparison results for the dimensionless temperature gradient $-\theta'(0)$ in the case of $\varepsilon = 1, Pr = 2, \phi = \phi_d = \omega = \beta_T = \alpha = \beta = 0$	31
5.1	Comparison results for the dimensionless temperature gradient $-\theta'(0)$ in the case of $\varepsilon = 1, \phi = \phi_d = \delta = \omega = \beta_T = \alpha = \beta = 0$	47
5.1 (continued)		48





## LIST OF FIGURES

3.1	Flowchart of research	16
4.1	Physical model and coordinate system	25
4.2	Velocity profile of fluid and dust phases for $\phi$ (volume fraction of nanoparticles parameter) on different types of nanoparticles.	32
4.3	Temperature profile of fluid and dust phases for $\phi$ (volume fraction of nanoparticles parameter) on different types of nanoparticles.	32
4.4	Velocity profile of fluid and dust phases for $\phi_d$ (volume fraction of dust particles parameter) on different types of nanoparticles.	33
4.5	Temperature profile of fluid and dust phases for $\phi_d$ (volume fraction of dust particles parameter) on different types of nanoparticles.	34
4.6	Velocity profile of fluid and dust phases for $\delta$ (velocity slip parameter) on different types of nanoparticles.	35
4.7	Temperature profile of fluid and dust phases for $\delta$ (velocity slip parameter) on different types of nanoparticles.	36
4.8	Temperature profile of fluid and dust phases for $\omega$ (thermal slip parameter) on different types of nanoparticles.	36
4.9	Effects of $\phi$ (volume fraction of nanoparticles parameter) on skin friction coefficients, $f''(0)$ for different types of nanoparticles.	37

4.10	Effects of $\phi$ (volume fraction of nanoparticles parameter) on local Nusselt number, $-\theta'(0)$ for different types of nanoparticles.	38
4.11	Effects of $\delta$ (velocity slip parameter) and $\phi_d$ (volume fraction of dust particles parameter) on skin friction coefficients, $f''(0)$ for different types of nanoparticles.	39
4.12	Effects of $\delta$ (velocity slip parameter) and $\phi_d$ (volume fraction of dust particles parameter) on local Nusselt number, $-\theta'(0)$ for different types of nanoparticles.	39
4.13	Effects of $\omega$ (thermal slip parameter) on local Nusselt number, $-\theta'(0)$ for different types of nanoparticles.	40
5.1	Physical model and coordinate system	43
5.2	Velocity profile of fluid and dust phases for $S$ (suction parameter) on different types of nanoparticles.	48
5.3	Temperature profile of fluid and dust phases for $S$ (suction parameter) on both stretching/ shrinking cases on different types of nanoparticles.	49
5.4	Effects of $S$ (suction parameter) on local Nusselt number, $-\theta'(0)$ for different types of nanoparticles with stretching and shrinking cases.	50
5.5	Effects of $S$ (suction parameter) on skin friction, $f''(0)$ for different types of nanoparticles with stretching and shrinking cases.	50
5.6	Velocity profile of fluid and dust phases for $\varepsilon$ (stretching and shrinking parameter) on different types of nanoparticles.	51

5.7	Temperature profile of fluid and dust phases for stretching parameter on different types of nanoparticles.	52
5.8	Temperature profile of fluid and dust phases for shrinking parameter on different types of nanoparticles.	52
5.9a	Temperature profile of fluid and dust phases for heat source, $A^*$ with stretching case on different types of nanoparticles.	53
5.9b	Temperature profile of fluid and dust phases for heat sink, $A^*$ with stretching case on different types of nanoparticles.	54
5.10a	Temperature profile of fluid and dust phases for heat source, $A^*$ for shrinking case on different types of nanoparticles.	55
5.10b	Temperature profile of fluid and dust phases for heat sink, $A^*$ for shrinking case on different types of nanoparticles.	55
5.11a	Effects of $A^*$ (heat source) on local Nusselt number, $-\theta'(0)$ for different types of nanoparticles with stretching and shrinking cases.	56
5.11b	Effects of $A^*$ (heat sink) on local Nusselt number, $-\theta'(0)$ for different types of nanoparticles with stretching and shrinking cases.	57
5.12a	Temperature profile of fluid and dust phases for heat source, $B^*$ for stretching case on different types of nanoparticles.	58
5.12b	Temperature profile of fluid and dust phases for heat sink, $B^*$ for stretching case on different types of nanoparticles.	58

- 5.13a Temperature profile of fluid and dust phases for heat source,  $B^*$  for shrinking case on different types of nanoparticles. 59
- 5.13b Temperature profile of fluid and dust phases for heat sink,  $B^*$  for shrinking case on different types of nanoparticles. 59
- 5.14a Effects of  $B^*$  (heat source) on local Nusselt number,  $-\theta'(0)$  for different types of nanoparticles with both stretching and shrinking cases. 60
- 5.14b Effects of  $B^*$  (heat sink) on local Nusselt number,  $-\theta'(0)$  for different types of nanoparticles with both stretching and shrinking cases. 61
- 5.15 Effects of  $A^*$  and  $B^*$  (heat source and sink) on skin friction,  $f''(0)$  for different types of nanoparticles with both stretching and shrinking cases. 62



## LIST OF SYMBOLS AND ABBREVIATIONS

$a$	-	Acceleration
$A^*$	-	Space-dependent internal heat generation/ absorption
$Al_2O_3$	-	Aluminium oxide/ Alumina
$B$	-	Proportionality constant
$B^*$	-	Temperature-dependent internal heat generation/ absorption
BVP	-	Boundary value problem
$c_p$	-	Specific heat at constant pressure
$c_m$	-	Specific heat of dust particles
$Cu$	-	Copper
$CuO$	-	Copper oxide
$D/Dt$	-	Material derivative operator
$Ec$	-	Eckert number
$F$	-	Net force
$f(\eta)$	-	Dimensionless stream function
$f$	-	Transverse velocity of fluid phase
$f'$	-	Horizontal velocity of fluid phase
$f''$	-	Skin friction
$h$	-	Specific enthalpy
$K$	-	Stoke resistance
$k$	-	Thermal conductivity
$L$	-	Slip length
$\ell$	-	Characteristic length
$m$	-	Mass of solid/ dust particle
$\dot{m}$	-	Mass flow rate
MHD	-	Magnetohydrodynamic
nm	-	Nanometers
$N/ N_1$	-	Number density of dust particles

$Nu_x$	-	Local Nusselt number
ODE	-	Ordinary differential equations
$P$	-	Pressure
$Pr$	-	Prandtl number
PST	-	Prescribed Power law Surface Temperature
PHF	-	Prescribed Power law Heat Flux
$q'''$	-	The space and temperature dependent heat generation/ absorption (non-uniform heat source/ sink)
$q_w$	-	Surface heat flux
$Re_x$	-	Reynold number
$S$	-	Suction parameter
$T$	-	Temperature
$T_\infty$	-	Ambient temperature
$T_w$	-	Temperature near the surface
$TiO_2$	-	Titania
$u, v$	-	Velocity component on $x$ and $y$ axis
$U_w(x)$	-	Ambient fluid velocity
$V(x, t)$	-	Velocity vector of fluid
$V_p(x, t)$	-	Velocity vector of dust particles
$\nu_f$	-	Kinematic viscosity
$x$	-	x-axis
$y$	-	y-axis

#### Greek symbol

$(\rho c_p)_{nf}$	-	Nanofluid heat capacity
$v_w$	-	Suction velocity
$\beta_T$	-	Fluid-particle interaction for temperature
$\tau_t$	-	Thermal equilibrium time
$\tau_v$	-	Dust particle relaxation time
$\tau_w$	-	Surface shear stress

$\alpha$	-	Mass concentration of dust particles/ stretching rate
$\beta$	-	Fluid-particle interaction for velocity
$\gamma$	-	Ratio of specific heat of fluid to dust particles
$\delta$	-	Velocity slip parameter
$\varepsilon$	-	Velocity ratio parameter, stretching/ shrinking parameter
$\eta$	-	Similarity variable
$-\theta(0)$	-	Temperature gradient
$\mu$	-	Viscosity
$\rho$	-	Density
$\psi$	-	Stream function
$\omega$	-	Thermal slip parameter
$\phi$	-	Dissipation function/ volume fraction of nanoparticles

#### Subscript

$cv$	-	Control volume
$d$	-	Dust particles
$f$	-	Fluid properties
$nf$	-	Nanofluid phase
$p$	-	Particle phase
$s$	-	Solid particles



PTTA UTHM  
PERPUSTAKAAN TUNKU TUN AMINAH

**LIST OF APPENDICES**

<b>APPENDIX</b>	<b>TITLE</b>	<b>PAGE</b>
A	Derivation of equations	73
B	Computer programming for Chapter 4	92
C	Computer programming for Chapter 5	94



**PTTA UTHM**  
PERPUSTAKAAN TUNKU TUN AMINAH



## CHAPTER 1

### INTRODUCTION

#### 1.1 Research background and rationale of study

The study of fluid mechanisms is crucial to understanding the flow of every motion. Fluid is defined as a substance that deforms to continually flow under applied shear stress ignoring molecular effects (Kraus, Welty & Aziz, 2011). Liquids and gases are categorised into the fluid system as both states can easily be deformed and can fill a volume of any container either by force or pouring.

Meanwhile, non-fluid solid can resist applied shear stress and eventually stop deforming at some fixed strain angle. Solid, liquid, and gas are known as single-phase flow, while multi-phase flow is the combination of two or more phases that co-exist in motion. The difference between the matters can be differentiated by their thermodynamic state called phase or multiple chemical components (Kraus *et al.*, 2011).

Scientists have brought forward the technology of hydrodynamic to another level along with technological changes. To compete in today's modernisation, industries create more advanced and high-quality mechanisms to fulfil demands. Hence, engineering functional systems involving nano-sized materials known as nanotechnology had gained the attention of researchers since decades ago. Since then, various new materials and mechanisms have been designed regarding the nano-sized concept.

Nanofluid is known as a single-phase flow fluid containing particles with an average size below 100 nanometers (nm). The nanoparticles are usually made of metals, metal oxide, carbides, or carbon nanotubes with common base fluid either water, ethylene glycol or oil then mixed to form nanofluid. Since the beginning of the

20<sup>th</sup> century, nanotechnology becomes the main topic in computational fluid dynamic studies.

Nanofluid is called a smart fluid as its heat transfer can be reduced or enhanced at will. They are very stable elements and carry non-Newtonian behaviour. Problems such as sedimentation, erosion, and pressure drop can be avoided due to the tiny size and low volume fraction of nano-elements based on Bachok, Ishak & Pop's (2011) study. Hence, the thermal and transport properties of the fluid flow can be altered due to the suspended nanoparticles.

The combination of fluid and solid particles is considered as two-phase flow, hence introducing the study of dusty fluid. Dust particles are referred to as impurities that may exist in a fluid. Researchers such as Manjunatha, Gireesha & Bagewadi (2012) and Sulochana & Sandeep (2016) have shown interest in dusty flow as the results of experiments showed good outcomes to improve the thermal conductivity of fluid and heat transfer rate.

However, researches have covered on the multi-phase flow of dusty nanofluid with milli- or micrometre-sized dust particles as impurities mixing into a base fluid of nanoparticles. There are a lot of journals and researches proved that dusty nanofluid flow would bring advantages towards the study of fluid dynamic. For example, researches by Naramgari, Sulochana & Kumar (2016) and Siddiq *et al.* (2018). The outcome ensures an increase in thermal conductivity and provides an effective way to significantly improve their heat transfer characteristics.

Industries especially in the manufacturing process emphasise the type of surface to create products. Therefore, there are several studies considered on stretching/shrinking sheets either horizontally or vertically and rotating stretchable disks. A few applications to be mentioned are manufacture wire drawing, extrusion of polymer, glass blowing, electric-power generating system, and air cleaning machines.

According to Bachok *et al.* (2011), the rate of heat transfer at the surface will determine the quality of final products. Hence, the involvement of heat transfer characteristics by the surface will affect the outcomes of products. For example, wire drawing is a metalworking process used to reduce the cross-section of a wire by pulling it through a series of dies (Kastner, 2013).

The characteristic of metals' shape can be changed as the temperature rises according to required diameters. Technology has allowed machines to be more

effective in mass production. Thus, researchers studying metallurgy have improved the machines in creating wires with vary of diameters. This shows how a surface with velocity, either being stretched or shrunk, becomes a variable to be analysed through this research.

The development of the manufacturing industry proves that the studies on nanomaterials are crucial as they are impossible to be neglected or avoided. Some of the applications are related to the industry of aerodynamics, material process, glass industry, and industrial thermal management. Due to their effectiveness of thermal conductivity, nanomaterials with based fluid and inclusion of impurities have huge advantages towards science and technology.

## **1.2 Problem statement**

Most conventional heat transfer fluids, such as water, ethylene glycol, and engine oil have limited capabilities due to low thermal properties. Meanwhile, most metal solids have a higher value of thermal conductivity than fluids. Then, it is expected that fluid-containing metal solid particles may significantly increase their conductivity (Bachok *et al.*, 2011). The fluid contains solid particles known as nanofluid have garnered much attention from researchers due to demands and application of nanotechnology.

However, many studies consider nanofluids as free of impurities. In reality, fluid contains impurities including dust particles, known as dusty nanofluid which will affect fluid flow. The inclusion of dust particles will alter the fluid flow as expected to improve heat transfer and thermal conductivity in machines' productivity. Hence, the study will focus to investigate the boundary layer flow and heat transfer of dusty nanofluid. As consideration for other real-life situations, factors such as heat generation/ absorption, stretching/shrinking sheet, suction, and partial slip effects are taken into account.

## **1.3 Research objectives**

This study aims at solving the boundary layer flow of dusty nanofluid with various conditions. The objectives are:

- i. To modify a mathematical model for the related problem.
- ii. To reduce the partial differential equations into ordinary differential equations using similarity transformation.
- iii. To obtain numerical solutions by developing computational programming through bvp4c in MATLAB R2019b.
- iv. To analyse the dimensionless physical parameters' effects such as volume fraction of dust particles, the volume fraction of nanoparticles, velocity slip, thermal slip, stretching/shrinking, heat generation/absorption and suction on velocity, temperature, skin friction coefficient, and local Nusselt number.

Therefore, the objectives would be achieved through two problems:

1. Boundary layer flow of dusty nanofluid over a stretching sheet with partial slip effects.
2. Dusty nanofluid flow over the stretching/shrinking sheet with suction, heat generation/source and heat absorption/sink.

#### 1.4 Scopes of study

This study is limited to:

- i. Two-dimensional horizontal permeable sheet.
- ii. The study will be on incompressible and steady boundary layer flow.
- iii. The surface studied in this research is a sheet with stretching and shrinking ability of motion.
- iv. This research comes forward to analyse the Tiwari-Das model regarding the volume fraction of nanoparticles' influence on boundary layer flow.
- v. The dust particles involved are in spherical shape, uniform size and equally distributed in a water-based nanofluid.
- vi. The suspension of particles namely copper (Cu), titania ( $\text{TiO}_2$ ), and aluminium oxide/alumina ( $\text{Al}_2\text{O}_3$ ) with spherical shape of nanometer-sized.

## 1.5 Significance of study

The significances of this study are:

- i. This research shows a study on the effectiveness of fluid flow that can improve the heat transfer rate in a machine or mechanism. Since the study of nanofluid has gained the interest of researchers, especially among the previous five years, this study includes dust particles mixed in the fluid as the flow can be useful for future research.
- ii. This study discusses a surface with the ability to stretch and shrink, which will affect fluid motion. The research on fluid flow over stretching/ shrinking sheets will bring advantages to be applied in industrial processes and benefit the future, especially in fluid dynamics scope. This research gets interesting as external factors such as slip effects, suction, and heat are also included.

## 1.6 Thesis outline

The study of heat transfer characteristics for fluid dynamic consist of six chapters. This thesis is divided into three parts. The first part involves the discussion of fluid properties, reviewed articles and mathematical models in chapters 1, 2 and 3, respectively. Meanwhile, the main research of related problems is deliberated in chapters 4 and 5. Lastly, Chapter 6 presents the third part on conclusion for the whole research.

Chapter 1 is an introductory chapter with a brief explanation of fluid properties. Dust particles that are considered impurities embedded into nanofluid are also discussed in the chapter. In addition, the characteristics of boundary layer wall that plays a role in heat transfer are deliberated. This chapter also includes the problem statement, scope of the study, significance of the study and thesis outline.

Chapter 2 reviews past researches related to the current study. These articles were cited as references to guide the research and study the thermal conductivity of boundary layer flow. This chapter reviews articles related to nanofluid, dust particles, and stretch/shrink sheets. Factors considered in boundary conditions are also discussed such as heat generation/absorption, partial slip effects, and suction.

Chapter 3 discusses the development of the mathematical model. To understand the fundamental of fluid flow, this chapter shows the governing equations of boundary layer flow consisting of three principles, namely conservation of mass, momentum, and energy. To adjust the governing equations with considered problems, a few assumptions have been made. Moreover, the bvp4c program in MATLAB R2019b software was used to obtain the numerical solution.

Two different boundary layer problems are developed and discussed in chapters 4 and 5. As the governing equations of the two-dimensional boundary layer are obtained from Chapter 3, few conditions are applied related to the problems, especially towards the boundary conditions. The analysis of results is on velocity fluid, temperature fluid, skin friction and heat transfer rate near the surface.

Chapter 4 develops the first problem of this study on dusty nanofluid flow over stretching sheets. The non-dimensional parameters considered are the volume fraction of nanoparticles and dust particles. Besides, there are two types of slips that appear at the surface to be investigated namely velocity and thermal slip. The study discusses three types of nanoparticles namely copper (Cu), titania (TiO<sub>2</sub>), and aluminium oxide/alumina (Al<sub>2</sub>O<sub>3</sub>).

Besides, Chapter 5 explains the study of dusty nanofluid flow over stretching and shrinking sheets. Since the surface can elongate and shrink, the results will be different from Chapter 4. The non-dimensional parameters taken into account are stretching sheet, shrinking sheet, heat source, heat sink and suction. The chapter also examines the three types of nanoparticles.

As of conclusion, Chapter 6 presents an overview of related boundary layer flow's study. The fluid flow has shown various reactions by the effects of the parameters and different types of nanoparticles. Surfaces with two different mechanisms also play a role. In addition, the study also proposes a few suggestions for the future research of boundary layer flow.

## CHAPTER 2

### LITERATURE REVIEW

It is important to review other existing studies regarding the topic of dusty nanofluid over the stretching/shrinking sheet as well as factors that may affect the flow. This review will bring significant knowledge and information to the project. Hence, the objective of this chapter is to survey previous investigations that had been carried out among other researchers to gain more information related to the project.

#### 2.1 Nano-sized particles in based fluid

The nanofluid term was introduced by Choi & Eastman (1995) as a new class of engineered fluids. The fluids were created with a high value of thermal conductivity on suspension of metallic particles with an average size of 10 nanometers (nm) in industrial heat transfer fluids. Nanofluid possesses superior properties compared to conventional heat transfer fluids or fluids filled with micrometre-sized particles (Choi & Eastman, 1995). This is owing to the heat transfer that most likely takes place at the large surface area of nanoparticles. With the discovery, analysts have focused their attention and found that this type of fluid is more efficient in solving thermal problems such as overheating in gadgets, cars, and electronic circuits. Nanofluid has advantages with minimal clogging inflow passages, long-term stability, consistency, and high thermal conductivity according to Krishnamurthy *et al.* (2016).

In early 2000, there are just a few research related to nanofluid. Xuan & Li (2003) have built an experimental system to investigate convective heat transfer and flow in a tube. The researchers discovered that under the same Reynold number, nanofluid has a larger heat transfer coefficient than that of the base fluid. Likewise, the study of Oztop & Abu-Nada (2008) obtained the same result where the increase of

volume fraction of nanoparticles influenced heat transfer. Besides, copper-nanoparticle had shown the highest heat transfer enhancement when compared to alumina and titania. Research by Bachok *et al.* (2011) also acknowledged the results in which copper-nanoparticle has the highest values of local Nusselt number and skin friction coefficient compared to others. From the observation, the type of nanoparticles is also key to heat transfer enhancement.

Yacob & Ishak (2012a) and Aurangzaib *et al.* (2016) studied micro-sized particles embedded in the fluid. The studied micropolar fluid dropped the heat transfer rate at the boundary and the thermal boundary layer became thicker. This shows that replacing micro-sized with nano-sized particles can bring great advantages towards the technology of fluid dynamics. Animasaun *et al.* (2017) has analysed the magnetohydrodynamic (MHD) mixed convection stagnation-point flow of nanofluid embedded with dust particles, which was solved numerically by the Runge-Kutta based shooting method. Until recent years, the researches have expanded and the method of solving has also improved. A numerical study on nanofluid flow along an exponentially stretching surface has been solved utilising the Quasilinearization technique as well as the implicit finite difference scheme by Patil, Kulkarni & Hiremath (2020). The outcome had determined that the appearance of nanoparticles could enhance the skin friction and reduce the heat transfer rate from the hot wall to the cold fluid.

## 2.2 Dust particles in fluid

The inclusion of dust particles promotes heat transfer characteristics towards the flow. The analysis of two-phase fluid with the presence of impurities has been widely applied. For instance, environmental pollution, sedimentation, and blood rheology. Vajravelu & Nayfeh (1992) carried out a study to investigate the accumulation and impingement of the particles on the surface. The researchers analysed the dusty fluid flow on hydromagnetic and discovered that numerical solution has a close agreement with the analytic solution. Meanwhile, Awartani & Hamdan (1999) discussed the possible geometries for the effect of permeability surface on dusty fluid flow. The research had concluded that possible flows are in parallel straight lines and radial only.



## REFERENCES

- Abel, M. S. & Mahesha, N. (2008). Heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation. *Applied Mathematical Modelling*, 32(10), pp. 1965-1983.
- Abu-Nada, E. (2008). Application of nanofluids for heat transfer enhancement of separated flows encountered in a backward facing step. *International Journal of Heat and Fluid Flow*. 29(1), pp. 242-249.
- Aladdin, N. A. L., Bachok, N. & Pop, I. (2020). Cu-Al<sub>2</sub>O<sub>3</sub>/water hybrid nanofluid flow over a permeable moving surface in presence of hydromagnetic and suction effects. *Alexandria Engineering Journal*, 59(2), pp. 657-666.
- Aman, F., Ishak, A. & Pop, I. (2013). Magnetohydrodynamic stagnation-point flow towards a stretching/shrinking sheet with slip effects. *International Communications in Heat and Mass Transfer*, 47, pp. 68-72.
- Anderson Jr, J. D. (1995). *Computational Fluid Dynamics: The Basic with Applications*. New York: McGraw-Hill, Inc.
- Animasaun, I. L., Prakash, J., Vijayaragavan, R. & Sandeep, N. (2017). Stagnation flow of nanofluid embedded with dust particles over an inclined stretching sheet with induced magnetic field and suction. *Journal of Nanofluids*, 6(1), pp. 28-37.
- Aurangzaib, Uddin, M. S., Bhattacharyya, K. & Shafie, S. (2016). Micropolar fluid flow and heat transfer over an exponentially permeable shrinking sheet. *Propulsion and Power Research*, 5(4), pp. 310-317.
- Awartani, M. & Hamdan, M. H. (1999). Some admissible geometries in the study of steady plane flow of a dusty fluid through porous media. *Applied Mathematics and Computation*, 100(1), pp. 85-92.
- Bachok, N., Ishak, A. & Pop, I. (2011). Stagnation-point flow over a stretching/shrinking sheet in a nanofluid. *Nanoscale Research Letters*, 6(623), pp. 1-10.
- Bejan, A. (2004). *Convection Heat Transfer*. Third edition. Canada: John Wiley & Sons, Inc.

- Chen, C. H. (1998). Laminar mixed convection adjacent to vertical, continuously stretching sheets. *Heat and Mass Transfer*, 33(5-6), pp. 471-476.
- Choi, S. U., & Eastman, J. A. (1995). Enhancing thermal conductivity of fluids with nanoparticles (No. ANL/MSD/CP-84938; CONF-951135-29). Argonne National Lab., IL (United States).
- Crane, L. J. (1970). Flow past a stretching sheet. *Journal of Applied Mathematics and Physics (ZAMP)*, 21, pp. 645–647.
- Daniel, Y. S., Aziz, Z. A., Ismail, Z. & Salah, F. (2018). Effects of slip and convective conditions on MHD flow of nanofluid over a porous nonlinear stretching/shrinking sheet. *Australian Journal of Mechanical Engineering*, 16(3), pp. 213-229.
- Das, M., Mahatha, B. K. & Nandkeolyar, R. (2015). Mixed convection and nonlinear radiation in the stagnation point nanofluid flow towards a stretching sheet with homogenous-heterogeneous reactions effects. *Procedia Engineering*, 127, pp. 1018-1025.
- Gireesha, B. J., Chamkha, A. J., Rudraswamy, N. G. & Krishnamurthy, M. R. (2015). MHD flow and heat transfer of a nanofluid embedded with dust particles over a stretching sheet. *Journal of Nanofluids*, 4(1), pp. 66-72.
- Gireesha, B. J., Manjunatha, S. & Bagewadi, C. S. (2012). Unsteady hydromagnetics boundary layer flow and heat transfer of dusty fluid over a stretching sheet. *Afrika Matematika*, 23(2), pp. 229-241.
- Grubka, L. J. & Bobba, K. M. (1985). Heat transfer characteristics of a continuous, stretching surface with variable temperature. *Journal of Heat Transfer*, 107(1), pp. 248-250.
- Ibrahim, W. & Gamachu, D. (2020). Dusty nanofluid past a centrifugally stretching surface. *Mathematical Problems in Engineering*, 2020.
- Jamaludin, A., Nazar, R. & Pop, I. (2019). Mixed convection stagnation-point flow of a nanofluid past a permeable stretching/shrinking sheet in the presence of thermal radiation and heat source/sink. *Energies*, 12(5), pp. 1-20.
- Janke, V. R. R., Naramgari, S. & Vangala, S. (2015). MHD flow of a nanofluid embedded with dust particles due to cone with volume fraction of dust and nano particles. *Procedia Engineering*, 127, pp. 1026-1033.

- Kastner, W. (2013). *Application Overview: Wire Drawing*. Retrieved from <https://www.manufacturing.net/article/2013/03/application-overview-wire-drawing>
- Konch, J. & Hazarika, G. C. (2016). Effects of variable viscosity and thermal conductivity on MHD free convection flow of dusty fluid along a vertical stretching sheet with heat generation. *International Research Journal of Engineering and Technology*, 3(2), pp. 1029-1038.
- Kraus, A. D., Welty, J. R. & Aziz, A. (2011). *Introduction to Thermal and Fluid Engineering*. Florida: CRC Press.
- Krishnamurthy, M. R., Prasannakumara, B. C., Gireesha, B. J. & Gorla, R. S. R. (2016). Effect of chemical reaction on MHD boundary layer flow and melting heat transfer of Williamson nanofluid in porous medium. *Engineering Science and Technology, an International Journal*, 19(1), pp. 53-61.
- Lok, Y. Y., Ishak, A. & Pop, I. (2011). MHD stagnation-point flow towards a shrinking sheet. *International Journal of Numerical Methods for Heat & Fluid Flow*, 21(1), pp. 61-72.
- Mahanthesh, B., Gireesha, B. J. & Gorla, R. S. R. (2016). Nonlinear radiative heat transfer in MHD three-dimensional flow of water based nanofluid over a nonlinearly stretching sheet with convective boundary condition. *Journal of The Nigerian Mathematical Society*, 35(1), pp. 178-198.
- Manjunatha, S., Gireesha, B. J. & Bagewadi, C. S. (2012). Effect of thermal radiation on boundary layer flow and heat transfer of dusty fluid over an unsteady stretching sheet. *International Journal of Engineering, Science and Technology*, 4(4), pp. 36-48.
- Mansur, S. (2015). *Aliran Olakan Lapisan Sempadan Nanobendalir melalui Permukaan Rata dan Mengufuk dengan Pelbagai Syarat Sempadan*. Universiti Kebangsaan Malaysia: Ph.D. Thesis.
- Mansur, S., Ishak, A. & Pop, I. (2017). Stagnation-point flow towards a stretching/shrinking sheet in a nanofluid using Buongiorno's model. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*, 231(2), pp. 172-180.
- Murshed, F. K., Roychowdhury, I., Chattopadhyay, A. & Sandeep, N. (2015). Radiation effect on boundary layer flow of a nanofluid over a nonlinearly

- permeable stretching sheet. *Advances in Physics Theories and Applications*, 40, pp. 43-54.
- Nandy, S. K. & Mahapatra, T. R. (2013). Effects of slip and heat generation/absorption on MHD stagnation flow of nanofluid past a stretching/shrinking surface with convective boundary conditions. *International Journal of Heat and Mass Transfer*, 64, pp. 1091-1100.
- Naramgari, S. & Sulochana, C. (2016). Dual solutions of radiative MHD nanofluid flow over an exponentially stretching sheet with heat generation/absorption. *Applied Nanoscience*, 6(1), pp. 131-139.
- Naramgari, S., Sulochana, C. & Kumar, B. R. (2016). Unsteady MHD radiative flow and heat transfer of a dusty nanofluid over an exponentially stretching surface. *Engineering Science and Technology, an International Journal*, 19(1), pp. 227-240.
- Nazar, R., Jaradat, M., Arifin, N. & Pop, I. (2011). Stagnation-point flow past a shrinking sheet in a nanofluid. *Open Physics*, 9(5), pp. 1195-1202.
- Noghrehabadi, A., Pourrajab, R. & Ghalambaz, M. (2012). Effect of partial slip boundary condition on the flow and heat transfer of nanofluids past stretching sheet prescribed constant wall temperature. *International Journal of Thermal Sciences*, 54, pp. 253-261.
- Oztop, H. F. & Abu-Nada, E. (2008). Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *International Journal of Heat and Fluid Flow*, 29(5), pp. 1326-1336.
- Patil, P. M., Kulkarni, M. & Hiremath, P. S. (2020). Effects of surface roughness on mixed convective nanofluid flow past an exponentially stretching permeable surface. *Chinese Journal of Physics*, 64, pp. 203-218.
- Pavithra, G. M. & Gireesha, B. J. (2013). Effect of internal heat generation/absorption on dusty fluid flow over an exponentially stretching sheet with viscous dissipation. *Journal of Mathematics*, 2013.
- Prabhakara, S. & Deshpande, M. D. (2004). The no-slip boundary condition in fluid mechanics. *Resonance*, 9(5), pp. 61-71.
- Raju, C. S. K., Sandeep, N., Babu, M. J. & Sugunamma, V. (2016). Dual solutions for three-dimensional MHD flow of a nanofluid over a nonlinearly permeable stretching sheet. *Alexandria Engineering Journal*, 55(1), pp. 151-162.

- Ramya, D., Raju, R. S., Rao, J. A. & Chamkha, A. J. (2018). Effects of velocity and thermal wall slip on magnetohydrodynamics (MHD) boundary layer viscous flow and heat transfer of a nanofluid over a non-linearly-stretching sheet: a numerical study. *Propulsion and Power Research*, 7(2), pp. 182-195.
- Sahoo, B. & Do, Y. (2010). Effects of slip on sheet-driven flow and heat transfer of a third grade fluid past a stretching sheet. *International Communications in Heat and Mass Transfer*, 37(8), pp. 1064-1071.
- Sandeep, N. & Kumar, M. S. (2016). Heat and mass transfer in nanofluid flow over an inclined stretching sheet with volume fraction of dust and nanoparticles. *Journal of Applied Fluid Mechanics*, 9(5).
- Sandeep, N. & Saleem, S. (2017). MHD flow and heat transfer of a dusty nanofluid over a stretching surface in porous medium. *Jordan Journal of Civil Engineering*, 11(1), pp. 149-164.
- Sandeep, N., & Sulochana, C. (2015). Dual solutions for unsteady mixed convection flow of MHD micropolar fluid over a stretching/ shrinking sheet with non-uniform heat source/sink. *Engineering Science and Technology, an International Journal*, 18(4), pp. 738 - 745.
- Sandeep, N., Sulochana, C. & Kumar, B. R. (2017). Flow and heat transfer in MHD dusty nanofluid past a stretching/shrinking surface with non-uniform heat source/sink. *Walailak Journal of Science and Technology (WJST)*, 14(2), pp. 117-140.
- Shah, N. F. M. & Ali, A. (2017). Hydromagnetic boundary layer flow of a dusty fluid in a porous medium over a stretching sheet with slip effect. *Malaysian Journal of Fundamental and Applied Sciences*, 13(3), pp. 170-173.
- Shampine, L., Kierzenka, J. & Riechelt, M. (2000). Solving boundary value problems for ordinary differential equation in MATLAB with bvp4c. *Tutorial Notes*, 75275, pp. 1-27.
- Siddiqa, S., Begum, N., Hossain, M. A., Gorla, R. S. R. & Al-Rashed, A. A. (2018). Two-phase natural convection dusty nanofluid flow. *International Journal of Heat and Mass Transfer*, 118, pp. 66-74.
- Sulochana, C. & Sandeep, N. (2016). Flow and heat transfer behavior of MHD dusty nanofluid past a porous stretching/shrinking cylinder at different temperatures. *Journal of Applied Fluid Mechanics*, 9(2), pp. 543-553.

- Vajravelu, K. & Nayfeh, J. (1992). Hydromagnetic flow of a dusty fluid over a stretching sheet. *International Journal of Non-Linear Mechanics*, 27(6), pp. 937-945.
- Walker, J. S. (2013). *Physics*. 4<sup>th</sup> ed. Malaysia: Pearson Malaysia Sdn Bhd.
- Wang, C. Y. (1990). Liquid film on an unsteady stretching surface. *Quarterly of Applied Mathematics*, 48(4), pp. 601-610.
- Wong, S. W., Awang, A. O. & Ishak, A. (2011). Stagnation-point flow over an exponentially shrinking/stretching sheet. *Zeitschrift für Naturforschung A*, 66(12), pp. 705-711.
- Xuan, Y. & Li, Q. (2003). Investigation on convective heat transfer and flow features of nanofluids. *Journal of Heat Transfer*, 125(1), pp. 151-155.
- Yacob, N. A. & Ishak, A. (2012a). Micropolar fluid flow over a shrinking sheet. *Meccanica*, 47(2), pp. 293-299.
- Yacob, N. A. & Ishak, A. (2012b). Stagnation point flow towards a shrinking/stretching sheet in a micropolar fluid with a convective surface boundary condition. *The Canadian Journal of Chemical Engineering*, 90, pp. 621-626.
- Yacob, N. A. & Ishak, A. (2014). Flow and heat transfer of a power-law fluid over a permeable shrinking sheet. *Sains Malaysiana*. 43(3), pp. 491-496.



## VITA

The author was born on June 24, 1995, in Gombak, Selangor. She went to secondary school for lower form at Sekolah Menengah Kebangsaan (Perempuan) Kapar, Klang, Selangor and went to Sekolah Menengah Sains Kuala Selangor (KUSESS), Selangor during upper form. She took her foundation in Science at the Centre of Foundation Studies, Universiti Technology Malaysia (UiTM Campus Puncak Alam), Selangor. She pursued her degree at the Universiti Tun Hussein Onn Malaysia (UTHM), Johor, and graduated with a Bachelor's Degree in Science (Mathematical Technology) with Honour in 2018. She participated as co-author in a paper on boundary layer flow. After graduation, she continued her study for Master's Degree at UTHM with a Master of Science (by research). At the same time, she was doing a part-time job as a private tutor. In addition, the author has published an article entitled Boundary Layer Flow of Dusty Nanofluid over Stretching Sheet with Partial Slip Effects in Journal of Advanced Research in Fluid Mechanics and Thermal Sciences, Akademia Baru (indexed by SCOPUS).



PERPUSTAKAAN UNIVERSITI TUN HUSSEIN ONN MALAYSIA