CAPUTO-FABRIZIO FRACTIONAL DERIVATIVE FOR MAGNETIC BLOOD FLOW OF NEWTONIAN AND CASSON FLUID IN AN INCLINED ARTERY

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A thesis submitted in
fulfillment of the requirement for the award of the
Doctor of Philosophy in Science

Faculty of Applied Sciences and Technology
Universiti Tun Hussein Onn Malaysia

NOVEMBER 2022

This thesis is consecrated to my beloved parents;

Haji Jamil bin Ahiya' and Hajah Gayah binti Panot.

ACKNOWLEDGEMENTS

First of all, I thank Allah (S.W.T.) the most gracious, the most merciful, for the countless wisdom and blessings he bestowed upon me during the course of my PhD programme until its completion. It is indeed a great pleasure to express my profound gratitude to all and sundry who have contributed towards seeing this work achieve success.

The role played by my supervisor Professor Dr Rozaini Bin Roslan, would never be over emphasise, so, I would like to acknowledge his outstanding and efficient work and contributions by introducing me to this charming area and regular discussions sessions to give guide in some grey and difficult areas until the success of this thesis is finally achieved. Furthermore, I would like to also acknowledge the remarkable contributions by my Co-supervisor Dr Salah Uddin and for everything he did to me towards the success of this thesis. Their contributions and participation have been so helpful to this study.

I am grateful to the staff of the Department of Mathematics Universiti Tun Hussein Onn Malaysia, for being kind to me and provided me with all the necessary materials and pieces of advice. Special thanks to all my colleagues in the research group, all my friends and well-wishers for their kind motivation toward the success of this study. It is an honor for me to express my deep gratitude to my beloved father, mother, brother, sisters and friend for their moral support, encouragement and prayers throughout my studies.

ABSTRACT

The use of mathematical models to investigate blood flow activity has become an invaluable method for studying and understanding the circulatory system. In light of many clinical conditions, the blood flow issue of an inclined artery is significant from a physiological perspective. The current study analyzes blood flow with magnetic particles through inclined stenosed and multi-stenosed arteries, where impact of blood flow by Newtonian and Casson fluids was considered. The flow was driven by an oscillating pressure gradient and subjected to an external inclined magnetic field for all models. The Caputo–Fabrizio time fractional-order model without singular kernel was used to solve the nonlinear governing equations. The Laplace and finite Hankel transforms, as well as the Robotnov and Hartley's functions, were applied to obtain analytical solutions. Moreover, Mathcad software was utilized to construct blood velocity, temperature profiles, and magnetic particle velocity from different physiological parameters on blood flow through an inclined artery. The effects of various important factors, including body acceleration, thermal radiation, porosity and electric field on the transportation of magnetic particles flow of blood have been analyzed. The current findings were compared to those mentioned in previous studies, demonstrating that they are in good agreement. Numerical findings reveal that the fractional parameter order and inclination angle affect blood and magnetic particle distributions. Some significant findings show that the fractional- order derivative, electric field, porosity, Reynolds number, and Casson fluid parameter can enhance blood and magnetic velocities. Both fluid flow velocities have similar trends in fractional parameters; however, Casson fluid is slower than Newtonian fluid. Radiation and metabolic heat both play an essential role in controlling blood temperature. The temperature of the blood flow increases as the radiation and metabolic heat source values increase. Meanwhile, the Hartmann number and porosity decelerate the blood flow and magnetic particle velocity. These findings facilitate the clinical research of a variety of arterial diseases.

ABSTRAK

Penggunaan model matematik untuk mengkaji aktiviti aliran darah menjadi kaedah bernilai untuk memahami dan mempelajari sistem peredaran darah. Secara klinikal masalah aliran darah dalam arteri condong adalah penting dari sudut perspektif fisiologi. Kajian semasa mengambilkira penyelidikan aliran darah dengan zarah magnetik melalui arteri stenosis condong dan berbilang stenosis. Penyelidikan ini mengkaji kesan aliran darah oleh bendalir Newtonian dan Casson. Aliran bendalir bagi semua model didorong oleh kecerunan tekanan berayun dan juga tertakluk kepada medan luaran elektrik condong. Model peringkat pecahan masa Caputo-Fabrizio tanpa inti singular telah digunakan untuk menyelesaikan persamaan menakluk tak linear. Penyelesaian analitik diperoleh menggunakan jelmaan Laplace, jelmaan Hankel terhingga, dan fungsi Robotnov dan Hartley. Perisian Mathcad telah digunakan untuk melakar profail halaju, taburan suhu dan halaju zarah magnetik dari pelbagai parameter fizikal pada aliran darah melalui arteri condong. Analisis terhadap kesan dari pelbagai faktor penting, termasuk pecutan badan, sinaran terma, keliangan dan elektrik pada aliran darah zarah magnetik telah dijalankan. Keputusan yang didapati telah dibandingkan dengan kajian terdahulu dengan hasil yang memuaskan. Penemuan berangka mendedahkan bahawa terbitan peringkat pecahan dan sudut kecondongan mempengaruhi taburan zarah darah dan magnetik. Beberapa penemuan yang signifikan menunjukkan bahawa turutan terbitan pecahan, medan elektrik, keliangan, nombor Reynolds, dan parameter bendalir Casson meningkatkan halaju darah dan magnetik. Kedua-dua halaju aliran bendalir mempunyai arah aliran yang sama dalam parameter pecahan; bagaimanapun, cecair Casson adalah lebih perlahan daripada cecair Newtonian. Sinaran, dan haba metabolik memainkan peranan penting dalam mengawal suhu darah. Suhu aliran darah meningkat apabila nilai sumber haba radiasi dan metabolik meningkat. Sementara itu, nombor Hartmann dan keliangan memperlahankan aliran darah dan halaju zarah magnetik. Hasil kajian ini memudahkan penyelidikan secara klinikal terhadap pelbagai penyakit arteri.

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LIST OF ABBREVIATIONS

BFD	-	Biomagnetic Fluid Dynamics
CF	-	Caputo-Fabrizio derivative notation in the thesis
FDM	-	Finite difference method
GM	-	Galerkin Method
HPM	-	Homotopy Perturbation Method
LSM	-	Least Square Method
MDP	-	Magnetite dusty particle
MHD	-	Magnetohydrodynamic flow
MRI	-	Magnetic Resonance Imaging

NOMENCLATURE

Roman Letters

A_0	-	steady part of the pressure gradient
A_1	-	amplitude of the pressure fluctuation
A_g	-	amplitude of the acceleration
B_0	-	externally applied constant magnetic field
$\overrightarrow{B}$	-	magnetic flux intensity
C_p	-	specific heat capacity
$D_t^{(\alpha)}$	-	Caputo-Fabrizio fractional order derivative
d	-	region of the stenosis
$\frac{du}{dt}$	-	material time derivative
$\overrightarrow{E}$	-	electric field intensity
E_z	-	electric field in the axial direction
e_{ij}	-	(i,j)-th component of the deformation rate
f_p	-	frequency of the pulse rate
g	-	acceleration due to gravity
Gr	-	Grashof number
Ha	-	Hartmann number
$H^1(a, b)$	-	class of all integrable functions on [a, b]
$\vec{J}$	-	current density
J_0, J_1	-	Bessel functions of first kind with zero and first order
K	-	Stokes constant
K_1	-	non-dimensional electrokinetic width
k_p	-	porosity parameter
k	-	thermal conductivity

L_0	-	stenosis length
$\mathscr{L}$	-	Laplace transform
$M(\alpha)$	-	normalization function
m	-	parameter in determining the stenosis shape
m_s	-	average mass of magnetic particles
N	-	magnetic particles per unit volume
Pr	-	Prandtl number
p	-	pressure N/m^2
Q_m	-	metabolic heat source
R	-	particles concentration parameter
Re	-	Reynolds number
R_0	-	radius of the normal artery
R_z	-	radius of the artery in the stenosed and multi stenosed region
r	-	radial component
r_n	-	positive roots of the bessel function
T	-	fluid temperature
T_ω	-	wall temperature
T_a	-	absolute temperature
T_s	-	surface temperature
T_∞	-	bulk temperature
t	-	dimensionless time
U_0	-	characteristics velocity
$u(r,t)$	-	blood flow velocity
$\nabla\overrightarrow{u}$	-	viscous dissipation
u_r	-	radial component of velocity
u_0	-	characteristic velocity
u_θ	-	angular component of velocity
u_z	-	axial component of velocity
$\overrightarrow{V}$	-	velocity field
V_0	-	characteristic velocity
$v(r,t)$	-	magnetic particles velocity
x_1	-	diameter of pipe
z	-	axial component
$\bar{z}$	-	location of stenosis

Greek Letters

α	-	order of the fractional differential operator
β	-	Casson fluid parameter
β_T	-	coefficient of thermal expansion
β_p	-	Planck's constant
ζ^2	-	non-dimensional Womersley number
α_1, α_2	-	visco-elasticity and cross-viscosity
ε	-	dielectric constant
κ	-	Debye-Huckel parameter
μ	-	fluid viscosity
μ_B	-	plastic dynamic viscosity of the non-Newtonian fluid
μ_0	-	magnetic permeability
ρ	-	fluid density
σ	-	electrical conductivity
τ	-	Cauchy stress tensor
τ_c	-	Cauchy for Casson fluid
ν	-	kinematic viscosity
γ	-	degree of stenosis
	-	non-Newtonian fluid parameter
ω_p	-	heart pressure frequency
ω_g	-	frequency of blood flow
φ	-	lead angle of the body acceleration
θ	-	inclination of magnetic field
ϕ	-	inclination angle
ξ_s	-	maximum height of the stenosis
θ_m	-	heat absorption
η	-	radiation absorption coefficient
χ	-	thermal diffusivity

LIST OF PUBLICATIONS

STATUS OF PUBLICATIONS

(i) D.F. Jamil, S.Uddin , M.G.Kamardan , R.Roslan., (2020). The Effects of Magnetic Blood Flow in an Inclined Cylindrical Tube Using Caputo Fabrizio Fractional Derivatives. *CFD Letters*. 12(1) : 111-122.

(ii) D.F.Jamil , S.Uddin , M.G.Kamardan , R.Roslan . (2021). The effects of magnetic Casson blood flow in an inclined multi stenosed artery by using Caputo Fabrizio fractional derivatives. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*. 82(2):28-38.

(iii) D.F.Jamil , S.Saleem , R.Roslan , F.S.Al Mubaddel , M.R.Gorji , A.Issakhoc , S.Uddin., (2021). Analysis of non-Newtonian magnetic Casson blood flow in an inclined stenosed artery by using Caputo Fabrizio fractional derivatives. *Computer Methods and Programs in Biomedicine*, 203 (5):1-11.

(iv) D.F.Jamil , S.Uddin , R.Roslan., (2022). Electromagnetic Casson blood flow in multistenosed porous artery using Caputo–Fabrizio fractional derivatives. In: Ryan John Magno, Tuan Anh Nguyen. Nanotechnology for Hematology, Blood Transfusion, and Artificial Blood, Transport of nanoparticles in blood flow. *Elsevier*. ISBN:9780128239711.

CHAPTER 1

INTRODUCTION

1.1 Research background

Hemorheology is an application of study of blood circulation and its interaction with the vessel into which the flow takes place. Human blood cardiovascular system includes additional substances such as nutrients and oxygen to the cells and removes metabolic waste from the same cells. Blood flow modeling has been widely used in the past few decades to better understand the symptomatic signal of various diseases to improve existing or new treatments. A computational blood flow model is essential not only for clinical disease diagnosis, but as an integral component of more complex structure modeling. Throughout this opening section, some behavioral background is provided to lay the foundation for the development of mathematical modeling and theoretical analysis in subsequent chapters.

From the biomechanical point of view, blood is considered as an intelligent fluid, probably the most intelligent one in nature, as it is capable of adapting itself in a great extent in order to provide nutrients to the organs. Numerous experimental and theoretical studies to visualize arterial blood flow behavior have been implemented in the past. The importance of obtaining a better understanding throughout the mathematical modeling and computational simulations can determine the feasibility of a medical technique before the real clinical trials due to assumptions that may not be directly accessible through the experimental investigation. The study of blood flow via arteries is essential because it provides insight into the physiological processes. Blood may be considered either a Newtonian or a non- Newtonian fluid, depending on the circumstances.

All essential fluids, including liquids and gases, flow along a solid barrier and are subjected to shear stress at the boundary. The no-slip condition indicates that the fluid is moving at zero speed. Shear stress and strain rate are related to constant viscosity in Newtonian fluid laminar flow. In the case of non-Newtonian fluids, however, it is not continuous, and there is a loss of velocity, which causes shear stress to be transmitted onto the boundary. Shear stress varies with non-Newtonian fluid grade. Behaviour of blood as Newtonian or non-Newtonian fluid depends on the nature of the blood transportation process as well as on the size and shape of vessel. Blood having shear rate greater than 100 s^{-1} shows a Newtonian nature. For example, in large arteries, veins and in large cavities, blood exhibits Newtonian characteristics. However, for shear rate less than 100 s^{-1}, blood present non- Newtonian nature. In general, in capillaries, arterioles and in myocardium, non-Newtonian effects can be seen. With increasing flow velocity and shear strain rate, blood flows more smoothly and its viscosity decreases toward a constant, which has been commonly used as the viscosity of blood in a Newtonian model. However, in the low-velocity areas, the true viscosity is much higher than this constant, when non-Newtonian rheological models could simulate the blood viscosity variations in different shear strain rates.

According to the available research, blood's rheological characteristics and flow behavior in tubes of varied cross-sections are essential in diagnosing and treating many cardiovascular disorders. A non-Newtonian fluid at low shear rates and in narrow blood vessels, especially in sick conditions, when blood clots are prevalent, is well established in the medical community. Researchers have shown that the Casson model best captures the behavior of blood at low shear rates when the hematocrit, anticoagulants, and temperature are varied (Nagarani & Sarojama, 2007). In addition, Casson fluid has recently been investigated for blood flow in the human body under various physical characteristics (Siddiqui, Verna & Mishra, 2009; Qasim, 2014; Ali *et al.*, 2017; Gudekote, Manjunatha & Choudhari, 2018; Ahmed, 2018; Sarifuddin, 2020).

Biomagnetic Fluid Dynamics (BFD) revolves around the study of fluid flow under the interference of magnetic field. This field of fluid dynamics has fascinated many researchers (Tzirtzilakis, 2008; Turk, Tezer-Sezgin & Bozkaya, 2014; Bose, Sayan & Banerjee, 2015 ; and Tzirakis *et al.*, 2016) due to the abundance of applications suggested by medical science and bioengineering, including the development of magnetic tracers, the selective transport of drugs using magnetic particles as drug carriers and cancer treatment by magnetic hyperthermia. Tzirtzilakis (2005) proposed the concept of investigating the magnetic and electrical properties of blood under a single mathematical model. In formulating the governing equation of blood motion, it is necessary to take into account the electrical conductivity of human blood circulation and the effects of magnetism and the Lorentz force (Kenjeres & Opdam, 2009). Despite this, biomagnetic fluid behavior should be considered to establish its physiological standpoint in the model's development.

At present, there is a great deal of interest in examining blood flow via tubes and blood flow inside human circulatory systems. Generally, blood arteries have been classified as having zero inclination, which is considered horizontal in most research. These arteries appear to be uneven in terms of their physical position. The consideration of an artery's tendency brings gravity into the picture. Sanyal, Das & Debnath (2007) investigated blood flow characteristics using a mathematical model of an inclined circular tube with periodic body acceleration in the presence of a uniform magnetic field. Sreenadh, Pallavi & Satyanarayana (2011) studied the steady mathematical model of steady flow of Casson fluid through an inclined tube of non-uniform cross section with multiple stenoses. Siddiqui, Ullah & Awasthi (2017) developed a mathematical model to analyze the effects of body acceleration and slip velocity on Herschel Bulkley fluid pulsatile movement through an inclined stenotic artery.

The gradual thickening of the artery has long been recognized as an early phase of atherosclerosis formation, one of the most pervasive human diseases that eventually led to cardiovascular malfunction. Most research indicates that the disease's growth may be attributed to the deposition of cholesterol, fatty substances, cellular waste products, calcium, and fibrin. This deposition of substances is identified as stenosis in the arteries. There has been growing interest in studying blood rheology and blood flow through constricted arteries due to its great importance in the human cardiovascular system (Majee & Shit, 2017). Any study of an electrically conducting fluid flow via a stenosed artery with permeable walls is essential theoretically and for various medical and engineering concerns such as magnetohydrodynamics (MHD) generators, blood flow problems, and plasma research. The study of MHD is very useful in bio-engineering especially in the study of Magnetic Resonance Imaging (MRI), heart attack and cancer treatment (Das, Wang & Payne, 2013).

Fractional-order derivatives are as old as integer-order derivatives. This subject has limited to mathematics for the past three decades. However, the principles of fractional calculus have also implemented in other fields over the last few years due to their abundant practical uses. The use of fractional order derivative in mathematical modeling has found numerous applications such as those in physics, fluid mechanics, mathematical biology and electrochemistry (Hatami, Hatami & Ganji, 2014). Markis, Dargush & Constantinou (1963) obtained satisfactory agreement between experimental and theoretical results by employing a non-integer order Maxwell model rather than an integer order Maxwell model. The authors noted that the fractional model had a stronger memory impact than the integer-order model in that experiment. However, certain disadvantages were found in these operators, such as their singular kernel and the incredibly complex solutions produced. Caputo & Fabrizio (2015) introduced another derivative technique followed by theoretical and applied studies in several real-world problems. It replaces the singular kernel of the Caputo derivative with an exponential function where it has two representations for the temporal and spatial variables. The fractional calculus is an attractive issue of current study trends in many areas of fluid dynamics to explain the development of specific physical properties. Moreover, this approach has been applied in numerous research fields, including rheological characteristics and complex dynamics of various fluids. It is a comprehensive study of its behavior when the ordinary time derivative in constitutive equations is replaced with a fractional-order product. Furthermore, more general models of physical systems can be developed using fractional calculus rather than ordinary calculus. Recently, fractional calculus has been broadened in diverse directions, specifically, in fluid dynamics, tracer influent currents, visco-elasticity, electrochemistry, bio-engineering, neurons model in biological science, finance, and signal processing (Asjad *et al.* 2017).

Caputo-Fabrizio fractional derivative with a parameter memory has the advantage that the definition is not singular. Significant transforms (such as Laplace, finite Hankel, and Bessel transforms) are combined with Robotnov/Hartley functions to create a single closed-form solution for both the local and non-local cases. The most crucial benefit of modeling with fractional-order derivatives is that it is non-local, which makes it different from the local model. Non-integer order derivatives, like half-order derivatives, are used to show this (where we take integer-order derivatives like first-order derivatives, second-order derivatives, etc.).The local model represents the system's current stage, while the non-local model describes the system's history stage. According to Devendra, Singh and Kumar (2015) the non-local property of the fractional differential equations differentiates it from the other models, which predict the next stage of a system based on the historical background and don't rely on the system's current state. According to Caputo (2008) and Riesz (2016), in applied mathematics, fractional calculus is concerned with derivatives and integrals of arbitrary (real or complex) orders. It has recently acquired prominence and appeal, owing to well-established applications in science and engineering. It encompasses fluid flow issue modeling, electric networks, seismic wave propagation, rheology, oscillation, anomaly and reaction-diffusion, turbulence, polymer and chemical physics, electrochemistry, relaxation and dynamical processes, and many other physical phenomena in complex systems.

However, some authors argue that the non-singular kernel fractional derivative is suitable for all real-world processes. Certain operators have properties that render them unsuitable for numerous applications. The results of Ortigueira & Constantinou (2018) show that the Caputo-Fabrizio model performs poorly compared to the classical fractional derivatives. According to Giusti (2018), his result indicates that the Caputo-Fabrizio operator is nothing more than an infinite linear combination of ordinary repeated integrals of the function $f(t)$. Non-singular kernel derivatives have several drawbacks that should dissuade their use, as demonstrated by rigorous mathematical reasoning. Due to the absence of a corresponding convolution integral, they do not meet the fundamental rule of fractional calculus (Diethelm *et al.*, 2020). Meanwhile in the study of Baleanu (2020), he found that the findings of Ortigueira & Constantinou (2019) were not consistent. A fractional calculus

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