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Mathematical Modeling of Ischaemic Central Retinal Vein Occlusion Using Finite Volume Method

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Abstract. This study analyzes blood flow due to stenosis of blood vessels in the eye nerve in eye stroke. Eye stroke occurs due to the rupture of blood vessels in the retinal nerve of the eye caused by one of them stenosis of blood vessels, which results in loss of eye vision. Mathematical modeling as an applied mathematics science is one alternative to solve this problem. Mathematical modeling aims to obtain a formula that describes the state of blood flow according to actual conditions. The mathematical model is built using the finite volume method and will then be solved numerically using the SIMPLE algorithm. Based on the results of this simulation, we will know what level of narrowing (45-90% stenosed) of the renal vein is prone to causing renovascular hypertension in the kidney. The study results show that a 90% thickness can be dangerous because at this level of narrowing, the blood flow rate exceeds normal limits, and the pressure on the vein walls also increases. In contrast, the type of flow that appears becomes laminar.

INTRODUCTION

One area of science, mathematics, has a significant impact on the development of other fields, such as health and medicine. It is hoped that the simulation solution can help provide input while solving existing concrete problems because simulations are used to solve real problems in one branch of mathematics, applied mathematics. One sickness that happens a ton recently is eye stroke, which is an illness of deficiency of one's vision because of a few variables, for instance, because of blockage of the retinal nerve veins. When a retinal vein blockage (occlusion) results in vision loss, an eye stroke occurs.

Central retinal vein occlusion (CRVO) is a common ocular condition that, if left untreated, can lead to the development of neovascular glaucoma, which can cause painful blindness. [1]. One of the serious late complications of central retinal vein occlusion is neovascular glaucoma. Neovascular glaucoma was significantly less common than previously thought in eyes without treatment for ischemic-type central retinal vein occlusion. In some patients, the amount of treatment that can be done is limited by the pupil's inability to dilate because of glaucoma, cataract, or massive retinal edema that have already developed. [1]. RVO can appear in a variety of ways, such as many small bleeds in the eye or even complete vision loss. It can also cause problems such as lack of blood flow, more bleeding, swelling, and severe vision loss. Treatment and what we can expect for a person's vision varies depending on how severe their RVO is. [2]. TIn the developed world, the prevalence of retinal vein occlusions is 5.20 per 1000, and the prevalence of CRVO is 0.8 per 1000. [3]. CRVO can be divided into two groups: perfusion (non-ischemic) and ischemic. Non-ischemic CRVO usually causes small changes in vision, whereas ischemic CRVO has a worse impact on vision. [4]. Some things that can increase a person's likelihood of CRVO getting worse are being over age 50, having high blood pressure, glaucoma, diabetes, and high cholesterol. [4][5][6]. Thus, CRVO is generally considered a disease of the elderly. In younger patients, hypercoagulable state has been found to be significantly associated with CRVO; however, diabetes mellitus and hypertension were not significant [7].

This sickness is related with foundational conditions, for example, hypertension, diabetes mellitus, and obstructive rest apnea [8]. In view of the consequences of examination from Jingyi Mama, it tends to be realized that this eye stroke illness can cause vision misfortune in patients, particularly when there is an unexpected change in vision [9].

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In medical departments, the term "stenosis," which means "stricture," is frequently used to describe a constriction in particular areas. [10][11]. A stenosed artery is a narrowing of the blood vessel caused by the buildup of atherosclerotic plaque in the vessel's lumen. [12]. One of the factors that contributes to blood vessel rupture when flow pressure exceeds normal pressure is stenosis. Comparative exploration has been directed by Roy [13] connected with displaying blood stream in vein stenosis demonstrates the way that 90% blockage in a course can be very risky, causing the stream example to become creating laminar stream. Andayesh [14] demonstrated that other parameters had no significant impact on hemodynamics and that the percentage of stenosis, and the diameter of the vein were the dominant geometric parameters.
The mathematical model obtained from this problem is:
Continuity equation

$$\frac{\partial u_i}{\partial x} = 0 \tag{1}$$

Momentum equation

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x} = -\frac{1}{\rho}\nabla P + \mu\frac{\partial u_i}{\partial x} \tag{2}$$

with:
ρ is density, μ is viscosity, u is velocity. Based on Pousielle's Law, it is stated that “the law that the speed of a fluid moving through a narrow is straightforwardly relative to the strain of the fluid and the fourth force of the sweep of the slender and is conversely corresponding to the thickness of the fluid and the length of the fine", can be written:
with ΔP is pressure gradient, is viscosity of fluid L is length of tube, Q is flow, r is the radius of the tube.

According to Pralhad and Schultz [15] the boundary conditions for the radius at stenosis of arterial blood vessels are expressed as follows:

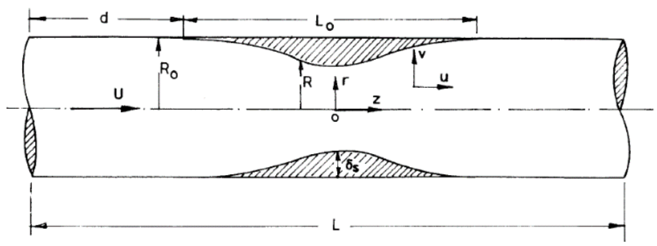


FIGURE 1. Flow geometry of stenoses blood vessels.

$$R(z) = R_0 - \frac{\delta s}{2}\left(1 + \cos\frac{2\pi}{Lo}\left(z - d - \frac{Lo}{2}\right)\right) \quad ; d \le z \le d + Lo \tag{3}$$
$$R(z) = R_0 \quad ; others \tag{4}$$

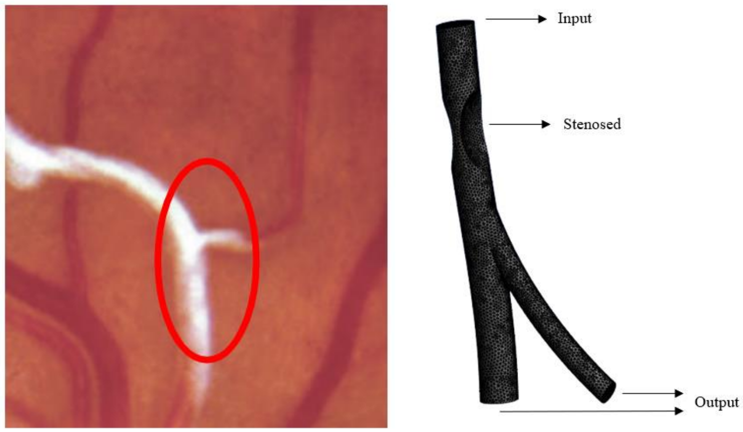
With:

R(z)	= radius on stenosis
R(0)	= the vein radius
δs	= the maximum height of the stenosis
L	= the arterial length
Lo	= the total length of the stenosis
z	= flow direction
d	= stenosis location

In this study, a simulation and analysis of the flow of narrowing of arteries due to eye stroke will be carried out based on clot thickness and the initial speed and pressure of blood flow using finite volume method. Based on similar research conducted by Wong [16], it can be known that patients with hypertension should have an ophthalmological assessment to detect hypertensive retinopathy or other retinal vascular complications. Individuals with moderate hypertensive retinopathy are at increased risk of cardiovascular disease such as peripheral artery and ocular strokes.

METHODS

This research is a type of simulation research that is a replication of system behavior [16]. A simulation is a model that contains a set of variables that display the main characteristics of a real-life system. A numerical approach (CFD) was performed to analyze blood flow in Ischemic Central Retinal Vein Occlusion. Numerical simulation has been extensively used to predict flow behavior, particularly Computational Fluid Dynamics (CFD) and Fluid-Structure Interaction (FSI) [17]. The first step is to build a mathematical model of the problem under study, namely the flow model due to stenosis of blood vessels in the retinal nerve of the eye. Furthermore, the model obtained will be completed using the finite volume method and the results will be simulated using the MATLAB application. The next step will be made 3D geometric shapes of the stenosis of the arteries and simulated using the ANSYS FLUENT application.



(a) (b)

FIGURE 2. (a) MRA image of the patient's ischemic central retinal vein occlusion (b) 3D mask of ischemic central retinal vein occlusion.

Ansys Fluent is a type of CFD (Computational Fluid Dynamics) program written in C language that has efficient and more flexible data structures. Ansys Fluent is Finite Volume Method (FVM) based software. Ansys Fluent uses a control-volume-based approach to convert a general scalar transport equation to an algebraic equation that can be solved numerically. This consists of integrating the transport equation for each control volume, resulting in a discrete equation expressing the conservation law on a control-volume basis [18]. To solve the fluid flow equation, we use the SIMPLE (Semi-Implicit Method for Pressure Linked Equation) algorithm. This algorithm is a method that uses the relationship between velocity and pressure to obtain a mass conservation value and a pressure field value [19]. The last step of this study is to analyze the simulation results both through the MATLAB and FLUENT applications for further conclusions.

RESULT AND DISCUSSION

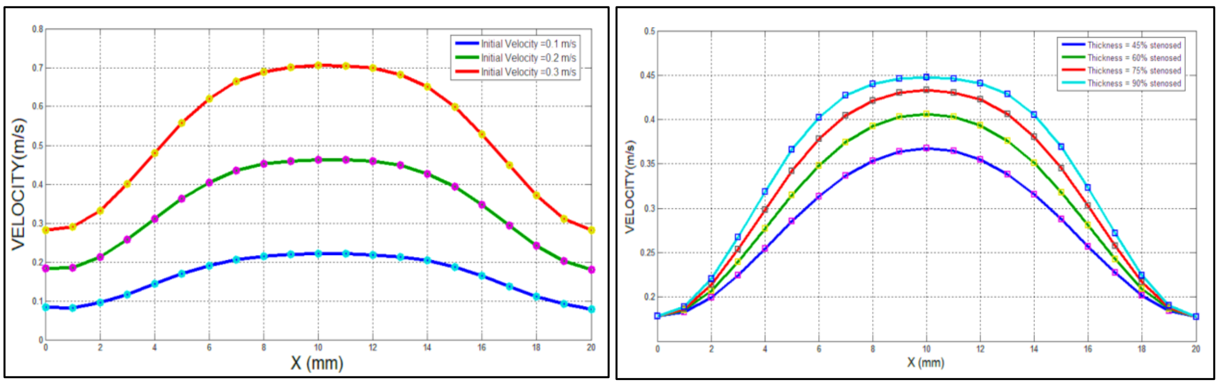
Based on the data search, data were obtained from variables that affect blood flow rate in stenosis of vein due to eye stroke [20]. The obtained model is then completed using the finite volume method using SIMPLE algorithm technique. In the first case, the effect of the clot thickness of 0.108mm, 0.144mm, 0.180mm and 0.216mm will be simulated with an initial flow velocity of 0.3 m/s using the MATLAB application.

TABLE 1. Table captions should be placed above the tables.

Parameter	Value
Retinal Blood Vessel diameter CRVE (Central Retinal Venular Equivalents)	233.5 – 235.2 μm
Blood density	1.054 – 1.060 kg/m^3
Viscosity	0.003 Kg/ms
Hypertensive blood pressure	Systolic: > 140 mmHg Diastolic: > 90 mmHg

Velocity vein	0.49 – 0.19 m/s
Pressure	17331 Pa
Thermal Conductivity	0.44 W/mK

The first simulation using this MATLAB application is to determine the flow velocity profile around stenosis based on the initial velocity of the flow with a fixed stenosis length. In this simulation, the initial velocity of flow (a) 0.1m/s (b) 0.2m/s (c) 0.3m/s with the thickness of the stenosis is 0.216mm. The result can be known that the flow speed at the starting point corresponds to the initial speed of each flow, after that the closer to the center of stenosis, the initial speed of the flow will increase, then the speed will decrease after passing through the center of stenosis (Fig. 3a). In this second simulation, it was examined how the effect of stenosis thickness on flow velocity, namely (a) 0.108mm, (b) 0.144mm, (c) 0.180mm, and (d) 0.216mm with the same initial flow velocity of 0.2m/s. The results showed that the greater the thickness of the clot will result in the greater the speed of blood flow around the blood vessels due to the stenosis (Fig. 3b).



(a) (b)

FIGURE 3. The effect of initial velocity on flow rate. (a) based on the initial velocity (b) based on thickness of stenosis

TABLE 2. Velocity profile based on initial velocity.

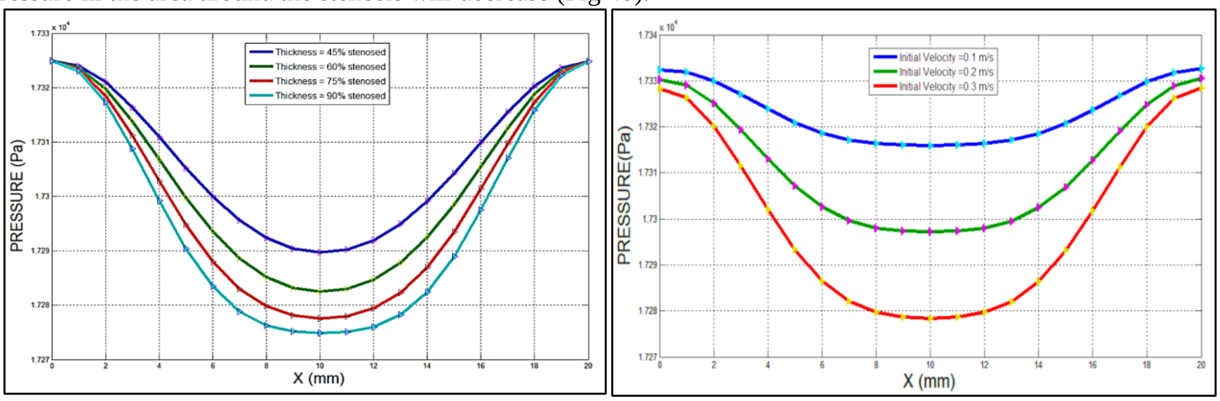
Initial velocity	Thickness of the stenosis	Max velocity
0.1m/s	0.216mm	0.22080m/s
0.2m/s	0.216mm	0.46308m/s
0.3m/s	0.216mm	0.70545m/s

TABLE 3. Velocity profile based on thickness of stenosis.

Thickness of the stenosis	%	Initial velocity	Max velocity
0.108mm	45%	0.2m/s	0.36760m/s
0.144mm	60%	0.2m/s	0.40641m/s
0.180mm	75%	0.2m/s	0.43296m/s
0.216mm	90%	0.2m/s	0.44727m/s

The velocity at the stenosis center for the initial flow speed of 0.2m/s is 0.2208m/s, while for the speed of 0.2m/s is 0.46308m/s and for the initial flow speed of 0.3m/s is 0.70545m/s which exceeds the normal limit of blood flow rate of 0.4m/s. Based on the results of this simulation, the maximum limit of the initial flow speed is 0.3m/s with the same stenosis thickness of 0.216mm. The magnitude of the velocity value at the center of the stenosis reaches 0.36760m/s for a thickness of 0.108mm (45% of radius), 0.40641m/s for a thickness of 0.144mm (60% of radius), 0.43296m/s for a thickness of 0.180mm (75% of radius) and 0.44727m/s for a thickness of 0.4427mm (90%). With the results of this simulation, the maximum thickness limit is 90% because it has exceeded the normal limit of blood flow speed, which is 0.4m/s. The results of this study are consistent with similar research conducted by Roy, M. [13] that in the presence of high blood pressure and a greater narrowing ratio resulting in greater flow velocity will have more damaging effects on artery wall stenosis than normal blood pressure.

Next simulation using the MATLAB application is the pressure profile due to the stenosis of blood vessels in the eye. The first simulation is to determine the effect of clot thickness on pressure on blood vessel walls, while in the second simulation is to determine the effect of the initial velocity of flow with a fixed thickness on pressure on blood vessel walls. The results showed that the pressure on the blood vessels decreased in the stenosis area but increased again after the stenosis area. The greater the thickness of the stenosis results in pressure on the blood vessel wall in the area around the clot but increases after the stenosis area (Fig 4a). The simulation results for the effect of the initial velocity of the flow with a fixed stenosis thickness appear that the greater the incoming flow velocity results in the pressure in the area around the stenosis will decrease (Fig 4b).



(a) (b)

FIGURE 4. Pressure outlet profile (a) based on thickness of the stenosis (b) based on initial velocity.

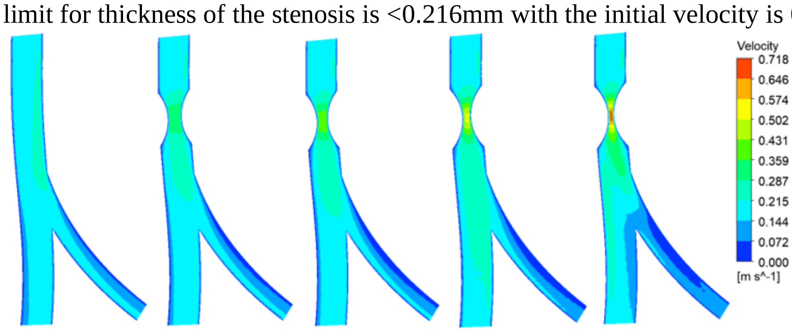
TABLE 4. Pressure based on initial velocity.

Initial velocity	Thickness of the stenosis	Pressure
0.1m/s	0.216mm	17.316Pa
0.2m/s	0.216mm	17.297Pa
0.3m/s	0.216mm	17.278Pa

TABLE 5. Pressure based on thickness of stenosis.

Thickness of the stenosis	%	Initial velocity	Pressure
0.108mm	45%	0.3m/s	17.290Pa
0.144mm	60%	0.3m/s	17.282Pa
0.180mm	75%	0.3m/s	17.278Pa
0.216mm	90%	0.3m/s	17.275Pa

The next simulation uses Ansys. 3D shapes are created with different thicknesses of the stenosis (a) 0.108mm (b) 0.144m (c) 0.180mm and (d) 0.216mm. The result can be seen that for the thickness of the size to get bigger, the flow speed will increase greater. In simulation (a), the flow speed is still quite normal in the stenosis area with an average speed of 0.3-0.4 m/s. For simulation (b) the flow speed is getting bigger with an average speed of 0.4-0.5 m/s, while for the last simulation (c) the flow speed has exceeded the normal limit of 0.5-0.6 m/s. Based on the results of this simulation, the normal limit for thickness of the stenosis is <0.216mm with the initial velocity is 0.3m/s.



(a) (b) (c) (d) (e)

FIGURE 5. Velocity streamlines due to thickness difference (a) undeformed model (b) 45% stenosed (c) 60% stenosed (d) 75% stenosed (e) 90% stenosed

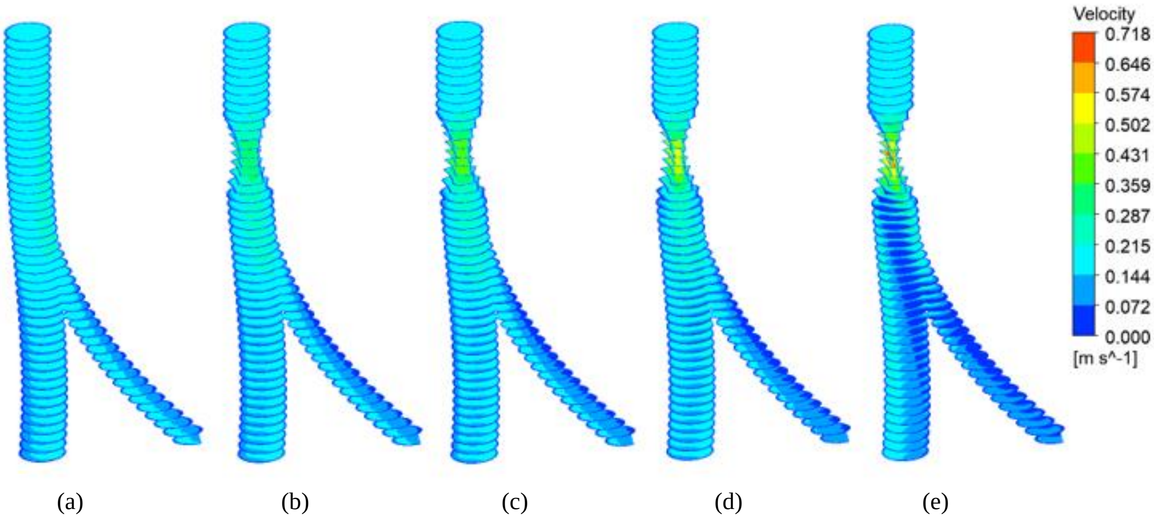


FIGURE 6. Velocity magnitudes in the domain are shown on different of thickness (a) undeformed model (b) 45% stenosed (c) 60% stenosed (d) 75% stenosed (e) 90% stenosed

Further simulation to determine the effect of clot thickness on pressure around the channel wall and analyze areas prone to channel stenosis. This simulation uses different thicknesses of the stenosis, namely (a) undeformed (b) 0.108mm (c) 0.144mm (d) 0.180mm (e) 0.216mm. The result can be seen that for the thickness of the size getting bigger, the flow pressure in the stenosis area goes down, but after the stenosis area will get bigger (red color). Large pressure on the walls of the cartilage channels results in rupture of blood vessels. Based on the simulation results, the area that is prone to impact is after the stenosis. This corresponds to research conducted by Guyton [21] that the pressure on the wall increases after stenosis.

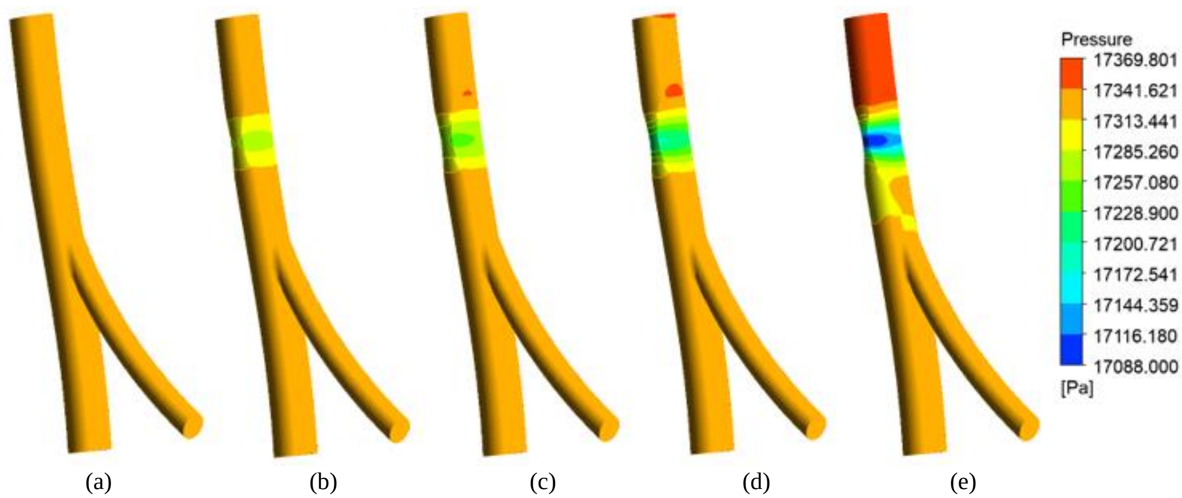


FIGURE 7. Pressure profile due to thickness difference (a) undeformed model (b) 45% stenosed (c) 60% stenosed (d) 75% stenosed (e) 90% stenosed

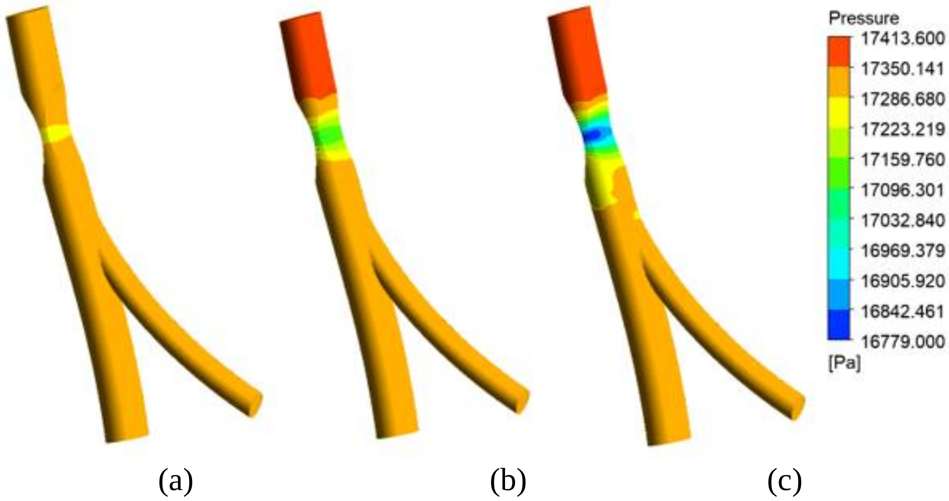


FIGURE 8. Pressure profile due to initial velocity difference (a) 0.1m/s (b) 0.2m/s (c) 0.3m/s

The third simulation using ANSYS is to find out how the initial velocity of flow is (a) 0.1m/s (b) 0.2m/s (c) 0.3m/s with a fixed thickness of the stenosis on the pressure on the vessel wall. The results can be seen that in the first simulation using an initial flow velocity of 0.1m/s, the pressure on the highest wall is after the stenosis with 17350 Pa, while the lowest pressure is on the wall of the stenosis. Likewise, for other simulations, the area with the highest pressure is in the area after stenosis, which is 17370 Pa for the initial flow velocity of 0.2m/s and 17413 Pa for the initial flow velocity of 0.3m/s. Based on the results of the simulation, it can be seen that the greater the initial velocity of flow, the more vulnerable vessel walls are, especially in the area after the stenosis.

CONCLUSION AND SUGGESTIONS

In this paper, we compare simulation results on how clot thickness and initial velocity affect the flow rate due to stenosis in central retinal vein occlusion resulting in blood vessel rupture. Simulations were performed with Ansys and MATLAB-based software using FVM, respectively, to solve the fluid flow problem. Based on the results, it was found that Ansys and MATLAB provided similar qualitative and quantitative results for all analyzed variables. The greater values of thickness and initial velocity of flow will result in increased blood flow velocity at the stenosis area. The pressure is inversely proportional to the previous results, namely the greater the value of thickness and initial velocity of flow, the pressure will also decrease in the stenosis area. Based on the simulation results, the model built is effective enough to measure the amount of flow velocity and pressure in the case of central retinal vein occlusion.

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