

**DYNAMIC LOAD FORECASTING FOR COMMERCIAL POWER
NETWORK**

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For my beloved mother, father, brothers and sisters



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ABSTRACT

Load forecasting is an important component for power system energy management system. The electrical load is the power that an electric utility needs to supply in order to meet the demands of its customers. It is therefore very important to the utilities to have advance knowledge of their electrical load, so that they can ensure the load is met and thus minimising any interruptions to their service. It also plays a key role in reducing the generation cost, and also essential to the reliability of power systems. The electric power demand in Universiti Tun Hussein Onn Malaysia (UTHM) has increased as the power system network is getting larger with more consumption is to be expected. This loading trend is certain to continue in the near future. The aim of this project is to forecast the medium term loading of UTHM. Linear regressions and polynomial based methods as well as artificial neural networks (ANN) approach have been adapted in the load forecasting from 2006 to 2012. The results attained are validated with the real data obtained from the Tenaga Nasional Berhad (TNB) which represents the monthly load electric consumption in UTHM. By comparing the forecasted results with the real data, the most suitable method has been proposed. When the approaches are compared according to their highest prediction error, the highest error for linear regression and Polynomial equation approaches are very high compared to the ANN approach. Generally the ANN approach has produced better results.

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LIST OF SYMBOLS AND ABBREVIATIONS

T	-	Estimated time
y_T	-	Real value for moment T
λ	-	Discount factor
\tilde{y}_t	-	Forecasted value
a_t	-	Value of the intercept
b_t	-	Value of the slope
h	-	Time horizon
α, β	-	Discount factors (constants)
e_t	-	Forecast error
SS_E	-	Sum of squared forecast errors
S_{t-p+h}	-	Seasonal adjustment
$x(t)$	-	Variable
t	-	Time
$T(t)$	-	Trend variation at time t
$S(t)$	-	Seasonal variation at time t
$C(t)$	-	Cyclical variation at time t
$I(t)$	-	Irregular variation at time t
Y	-	Dependent variable
X	-	Independent variable
N	-	Number of variable
Y_{tr}	-	Average value for the dependent variable
W	-	Weighting matrix
K_{th}	-	Output node
J_{th}	-	Hidden node
STLF	-	Short-term load forecasting

MTLF	-	Medium-term load forecasting
LTLF	-	Long-term load forecasting
UTHM	-	Universiti Tun Hussein Onn Malaysia
kWh	-	Kilowatt hour
MWh	-	Megawatt hour
TNB	-	Tenaga Nasional Berhad
MAPE	-	Mean Absolute Percentage Error
ANN	-	Artificial Neural networks
PSS	-	Statistical package for the social sciences
QSAR	-	Quantitative Structure-Activity Relationship



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CHAPTER 1

INTRODUCTION

1.1 Project Background

Accurate electrical load forecasting is a crucial issue for resource planning and management of electrical power generation utilities. Load forecasting can be divided into three categories. Short-term load forecasting (STLF) that covers a period of one hour to one month. Medium-term load forecasting (MTLF) covers a period of one month up to one year. It is essential for scheduling fuel supplies and maintenance operation. Long-term load forecasting (LTLF) predicts the requirements of energy for more than one year. Capital assignment and infrastructure plans draw upon long-term forecasts.

Many tasks, in power generation industry, such as unit commitment, security assessment and enhancement of security depend on the near future load prediction including daily peak load [1]. Peak load forecasting inaccuracy has a negative impact on the economics of these utilities. For these reasons, many researchers in the last 20 years have tackled this area to devise more accurate and efficient techniques of load prediction [2].

The main resource of any power generation utility is the generation units. Efficient management of these units means running them in minimum cost to satisfy the requirements of consumers. To achieve this purpose starting-up and shutting down the generating units should be performed according to schedule. The

scheduling process is also known as unit commitment. As load demand varies from hour to hour and from day to day, and starting-up a unit needs time, it is necessary to have a demand prediction on hourly basis. This prediction should be provided one hour, one day, or one month a head [1].

If a power generation system is able to meet consumers demand at both normal and Emergency conditions the system is said to be secure. If this is not the case, the system is said to be insecure. The process of specifying whether a system is secure or insecure is called security assessment [3]. The set of actions necessary to restore the secure state of a system is called security enhancement. Both security assessment and security enhancement need load prediction. Load forecasting contributes in the decision-making concerning, among others, the processes of unit commitment and security enhancement. If the forecast is inaccurate the generation will be either above or below the required demand. If the prediction is too high, extra generation units will be put into operation without real need. If the prediction is too low shortfall will take place. Correcting the second situation either by activating standby units or purchasing electric power from neighbor countries. Thus, in both situations the generation utility will pay extra cost. Because of the important role of load forecasting in power system, researchers in the last two decades have been trying and experimenting to develop new techniques to increase the accuracy and efficiency of load forecasting models [2].



1.2 Problem Statements

Load forecasting problem is receiving great and growing attention as being an important and primary tool in power system planning and operation. Importance of load forecasting becomes more significant in developing countries with high growth rate. In recent years the electrical energy consumption is increased. A noticeable increase of electric energy consumption in UTHM University and especially after 2008 observed a significant increase in energy consumption due to developments that take place in all university facilities in terms of new section, library, and other new buildings. Since this development will be accompanied by increasing demand for energy, so it is necessary to perform the forecasting study to estimate the increase of energy demand that meet the needs of the future development plans, and helps the authority to take the right decisions regarding the investment and future plans.

1.3 Project Objectives

The objective of the project is to study the possible use of forecasting technique for UTHM power system loads, and estimating the annually load demand. It measurable objectives are as follows:

- a) To analysis the historical data collected for UTHM in past years.
- b) To propose dynamic load forecasting method for power consumption in UTHM.
- c) To simulate the power system load forecast and determine the medium term power load demand for the next years.

1.4 Project Scopes

- a) Collecting data for UTHM power loads since 2006 to 2012 and analysing it to determine the growth rate kWh.
- b) Study the load forecasting techniques and choose the suitable techniques and choose the suitable techniques used.
- c) Calculate the power loads using static methods to predict loads, to study the behaviour of future loads resulting using Excel.
- d) Simulate the power loads using dynamic method to forecast the load demand of UTHM network by using ANN, MATLAB software.
- e) Comparison between the electrical loads resulting by different approaches methods according to their highest predictions % error.



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CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Load forecasting is of the most difficult problems in distribution system planning and analysis. However, not only historical load data of a distribution system play a very important role on peak load forecasting, but also the impacts of meteorological and demographic factors must be taken into consideration. Generally, load forecasting methods are mainly classified into two categories: classical approaches and ANN based techniques. Classical approaches are based on dynamic methods and Static methods forecast future value of a variable by using a mathematical combination of the historic information. Section 2.2 describes the study of the load models of electricity supply systems. Section 2.3 discusses Static load forecasting methods, section 2.4 describes the Dynamic load forecasting methods used and section 2.5 describes the Model validation methods.

2.2 Load models

For a long time, it has been recognized that the operation and performance of electricity supply systems are strongly influenced by the characteristics of the supplied load as given in Figure 2.1. Accordingly, the selection of the load model in a particular power system study will have a significant effect on the results of the study and, therefore, corresponding design decisions. Too optimistic load models can lead to inadequate system design or reinforcement, and may result in either costly upgrades or insecure systems, more vulnerable to various types of disturbances and collapse. Too pessimistic load models can, on the other hand, lead to unnecessary capital expenditure and uneconomical operation of the supply system.

In the past, more pessimistic load models have often been favored, in order to accommodate conservative safety and design margins. However, the most pessimistic load representation is sometimes hard to determine. Supply systems are increasingly being operated near to their operational margins, due to growing demand, economic and environmental pressures to run these systems close to their maximum capacity. Representative and accurate load models are thus becoming increasingly important for correct assessment of network performance. In addition, accurate load models can facilitate better decision making in relation to financial investment.

Traditionally, load models have been developed for use in high and medium voltage studies, with low voltage distribution systems and associated loads represented as a combined single (i.e. aggregate) load model. Accordingly, these aggregate loads are relatively insensitive to variations in characteristics of individual loads. However, the move away from centralized to distributed generation of electricity requires distribution networks to be considered in more detail. This also means that distribution system load should be considered in greater detail. When representing these smaller aggregate distribution system loads, the characteristics of individual load components become more important, and should be given more attention than has been the case previously.

Load models will play an increasingly important role in system design and planning. A major reason for this is the anticipated higher penetration of distributed generation and subsequent need for more efficient management of distribution networks. that system load characteristics will strongly affect optimal location, size and financial feasibility of distributed generation. Furthermore, the types and numbers of loads commonly found in the distribution system have changed significantly in the last few decades, with most significant and most prominent increase of non-linear loads. This represents a major change in load inventory, which should be taken into account when considering load models for use in studies of modern power supply systems. Although the importance of accurate representation of system load is generally known, certain assumptions are often made about load models. This is due to the lack of or uncertainty in available load modeling data. Therefore, efforts in clarifying or presenting new load models are of general importance to all aspects of supply system design and operation [4].



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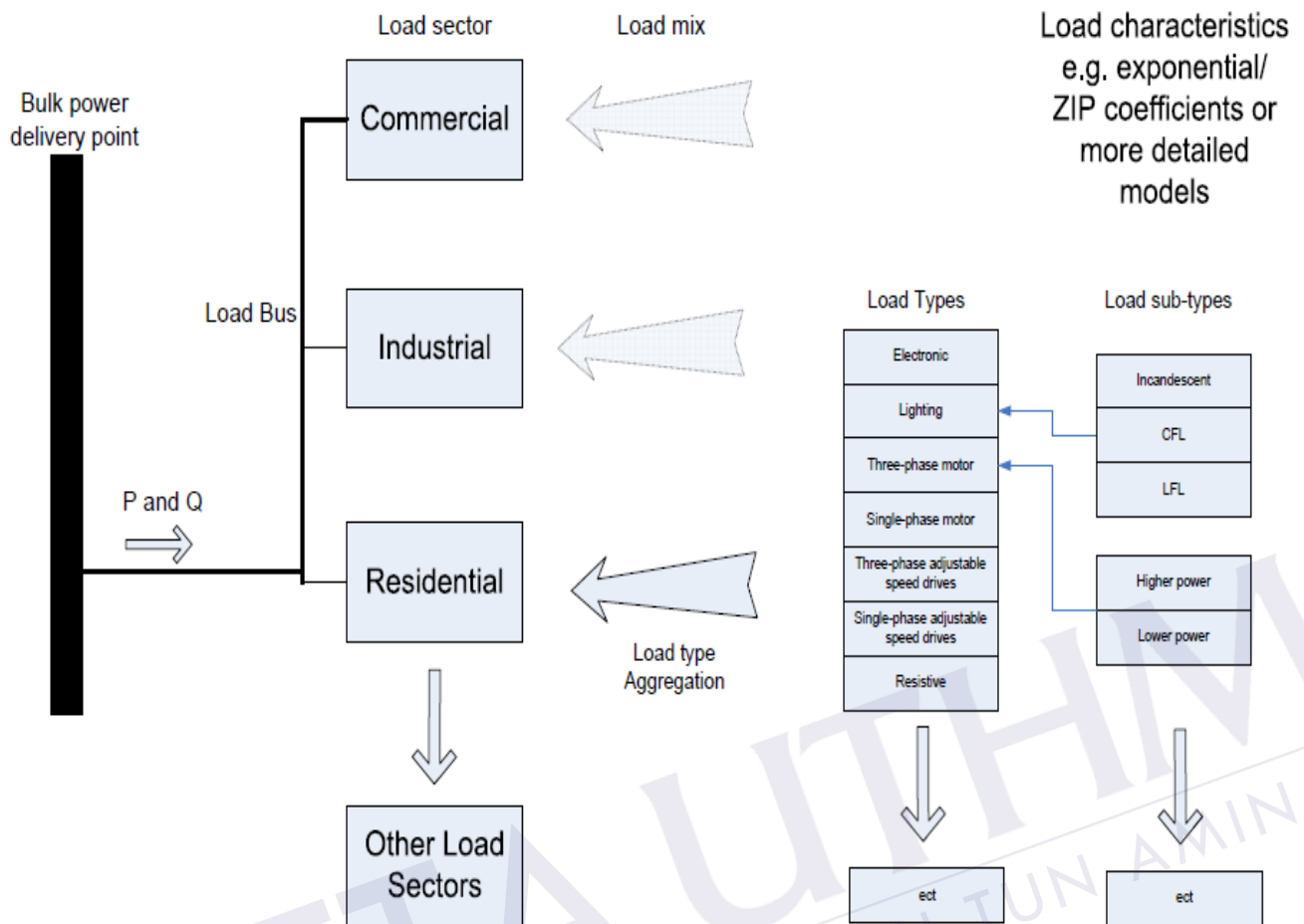


Figure: 2.1 Component-based approaches to load modelling

In this report, the term “load” is defined as either a single device or load type connected to the supply system, or an aggregation of load types connected to the supply system. A load model is mathematical or analytical representation of the changes in active and/or reactive power demand of a load, usually as a function of the changes in applied voltage and, in some cases, frequency. Load models can be generally defined as either static or dynamic. A static model is used to represent the active and reactive power demand of the load as a function of voltage and frequency at a particular instant time. A dynamic load model represents active and reactive power demand as a function of voltage, frequency and time.

Dynamic models are typically represented in the form of differential equations and used in voltage stability studies, studies of inter-area oscillations and long term stability studies. A commonly used dynamic load model is based on the

equivalent circuit of an induction motor [5], this is because induction motor load represents a significant proportion of the total system load. According to [6], the lack of dynamic motor models in power system studies is often thought to be the main cause of differences in results between field-measurements and large-scale simulations. When static and dynamic models are used together, this is known as a composite dynamic load model [7].

2.3 Static load forecasting methods

These methods forecast future load based on its historical values. The goal is to infer the pattern in the historical data series and extrapolate that pattern into the future. The load is considered as a time series embedding hourly, daily and seasonal patterns. Several techniques have been used for the analysis of these methods such as multiple linear regressions, moving average process, general exponential smoothing. Problems encountered with this approach include the inaccuracy of prediction and numerical instability. In general, statistical methods work well unless there is a drastic change in the variables that have a potential effect on the load pattern.

2.3.1 Exponential Smoothing methods

The method used for the load forecast is based on time series and takes into consideration only the history of the consumption in order to establish a pattern in the past that might be useful and similar with the present load curves. This technique uses exponentially decreasing weights as the observation get older. Recent observations are given relatively more weight in forecasting than the older observations. Exponential Smoothing is used to generate the smoothed values in order to obtain estimates power load. Exponential smoothing types currently used are [8]:

The first order exponential smoothing;

The second order exponential smoothing;

The higher order exponential smoothing;

The Holt-Winters mechanism.

2.3.1.1 The first-order exponential smoothing

It uses a recursive equation that can also be seen as the linear combination of the current observation and smoothed observation of the previous time unit. As the latter contains the data from all previous observations, the smoothed observation at moment T (estimated time) is in fact the linear combination of the current observation and the discounted sum of all previous observations [8]:

$$\tilde{y}_{T+1} = \lambda \cdot y_T + (1 - \lambda) \cdot \tilde{y}_T \quad (2.1)$$

Where:

\tilde{y}_{T+1} is the estimated value for moment $T+1$;

y_T - the real value for moment T ;

$\lambda \in [0, 1]$ - discount factor

\tilde{y}_t - forecasted value;

The discount factor represents the weight put on the previous observation while $(1 - \lambda)$ is the weight put on the smoothed value of the previous observations. The most important issue for the exponential smoothers is the choice of the discount factor, λ [8, 9].

2.3.1.2 The second-order exponential smoothing

The first-order exponential smoothing method was extended by Holt for presenting time series with trend (and random component). This approach adjusts the time series considering the trend to be linear [8]:

$$\tilde{y}_{T+h} = a_t + h \cdot b_t \quad (2.2)$$

Where

a_t - the value of the intercept;

b_t - the value of the slope;

h - time horizon.

For the one step ahead forecasting, the value of h is 1. Parameters a_t and b_t are calculated as follows:

$$a_t = \alpha \cdot y_t + (1 - \alpha) \cdot (a_{t-1} + b_{t-1}) \quad (2.3)$$

$$b_t = \beta \cdot y_t + (1 - \alpha) \cdot (a_t + a_{t-1}) + (1 - \beta) \cdot b_{t-1} \quad (2.4)$$

Where

$\alpha, \beta \in [0, 1]$ are the discount factors (constants);

a_t, b_t - values for this parameters at time t ;

a_{t-1}, b_{t-1} - values for this parameters at time $t - 1$.

The constants α, β will be chosen for the smallest sum of the squared forecast errors (the value of λ from the first-order exponential smoothing will be determined in same manner) [8, 9],

$$e_t = \tilde{y}_t - y_t \quad (2.5)$$

$$SS_E = \sum_{t=1}^T e_t^2 \quad (2.6)$$

Where

e_t - the forecast error;

\tilde{y}_t - forecasted value;

y_t - real value;

SS_E - sum of squared forecast errors.

2.3.1.3 The higher-order exponential smoothing

The first and second order exponential smoothing can be extended to the general n-th degree polynomial model presented in the equation below:

$$y_t = \beta_0 + \beta_1 \cdot t + \frac{\beta_2}{2!} \cdot t^2 + \dots + \frac{\beta_n}{n!} \cdot t^n + \varepsilon_t \quad (2.7)$$

Where ε_t are assumed to be independent with mean 0 and constant variance σ_ε^2 . In order to estimate the parameters the next equations will be used:

$$\begin{aligned} \tilde{y}_T^{(1)} &= \lambda \cdot \tilde{y}_T + (1 - \lambda) \cdot \tilde{y}_{T-1}^{(1)} \\ \tilde{y}_T^{(2)} &= \lambda \cdot \tilde{y}_T^{(1)} + (1 - \lambda) \cdot \tilde{y}_{T-1}^{(2)} \\ \tilde{y}_T^{(n)} &= \lambda \cdot \tilde{y}_T^{(n-1)} + (1 - \lambda) \cdot \tilde{y}_{T-1}^{(n)} \end{aligned} \quad (2.8)$$

Where $\tilde{y}_T^{(n)}$ is the estimated value for the n-th order exponential smoothing [8].

2.3.1.4 The Holt-Winters mechanism for seasonal time series

Some time series data exhibit cyclical or seasonal patterns that cannot be effectively modeled using the polynomial model as given in equations (2.8). Several approaches are available for the analysis of this data as given in Figure 2.2. The methodology of the Day-Ahead forecast by Holt and winters and is generally known as winters' method; in this case, a seasonal adjustment is made to the linear trend model. Two types of adjustments are used, namely the multiplicative and the additive model a multiplicative model and the estimated values are calculated as in the following equation [9]:

$$\tilde{y}_{T+h} = (a_t + h \cdot b_t) \cdot S_{t-p+h} \quad (2.9)$$

Where

REFERENCES

- [1] D. W. Bunn and E. D. Farmer, "Economic and Operation Context of Electric load prediction", in D. W. Bunn and E. D. Farmer, (ed.), Comparative models for Electrical load forecasting, pp.3-11, New York: John Wiley and Sons Ltd., 1985.
- [2] Andrew P. Douglas et al., "Risk Due Load forecast Uncertainty in Short Term Power System Planning", IEEE Transactions on Power System, Vol. 13, No. 4, PP. 1499, November 1998.
- [3] Robert Fich1, "Security Assessment and Enhancement, in Application Of Artificial Neural Network to Power System", M. A. El-Sharkawy And Degmar Neural (eds.), pp. 104-127, New Jersey: The Institute of Electrical and Electronic Engineers, Inc., 1996
- [4] S. Cray, "Steady state stability of composite systems," Electrical Engineering, vol. 52, pp. 787–792, Nov. 1933.
- [5] IEEE Task Force on Load Representation for Dynamic Performance, "Standard load models for power flow and dynamic performance simulation," IEEE Trans. Power Syst., vol. 10, no.3, pp. 1302–1313, Aug 1995.
- [6] P. Ju and D. Ma, "A composite dynamic-static model of electric power load," IEEE Trans. Control and Decision, vol. 4, no.2, pp. 20–23, 1989.

- [7] IEEE Task Force on Load Representation for Dynamic Performance, "Load representation for dynamic performance analysis (of power systems)," IEEE Trans. Power Sys t., vol. 8, no. 2, pp. 472–82, May 1993.
- [8] D. C. Montgomery, C. L. Jennings, M. Kulahci, *Introduction to Time Series Analysis and Forecasting*, Publisher John Wiley and Sons, ISBN 978-0-471-65397-4.
- [9] Introduction to Time Series Analysis
<http://www.itl.nist.gov/div898/handbook/pmc/section4/pmc4.htm>
- [10] Ganesan, Rajesh, et al. "Regression and ANOVA: an integrated approach using SAS software." IIE Transactions 36.12 (2004): 1211-1216.
- [11] Inés, Romero Navarro. "Dynamic Load Models for Power Systems- Estimation of Time-Varying Parameters During Normal Operation." TEIE 1034 (2002).
- [12] Bunnoon, Pituk. "Mid-Term Load Forecasting Based on Neural Network Algorithm: a Comparison of Models."
- [13] Park, Dong C., et al. "Electric load forecasting using an artificial neural network." *Power Systems, IEEE Transactions on* 6.2 (1991): 442-449.
- [14] Ghods, Ladan, and Mohsen Kalantar. "Different Methods of Long-Term Electric Load Demand Forecasting; A Comprehensive Review." *Iranian Journal of Electrical & Electronic Engineering* 7.4 (2011): 249.
- [15] Yasar, Y. Aslan S. Yavasca C. "LONG TERM ELECTRIC PEAK LOAD FORECASTING OF KUTAHYA USING DIFFERENT APPROACHES.
- [16] Adhikari, Ratnadip, and R. K. Agrawal. "An Introductory Study on TimeSeries Modeling and Forecasting." arXiv preprint arXiv:1302.6613 (2013).

- [17] Zhang, Guoqiang, B. Eddy Patuwo, and Michael Y Hu. "Forecasting with artificial neural networks:: The state of the art." *International journal of forecasting* 14.1 (1998): 35-62.
- [18] Oreskes, Naomi, Kristin Shrader-Frechette, and Kenneth Belitz. "Verification, validation, and confirmation of numerical models in the earth sciences." *Science* 263.5147 (1994): 641-646.
- [19] Veerasamy, Ravichandran, et al. "Validation of QSAR Models-Strategies and Importance.
- [20] Gavin, Henri. "The Levenberg-Marquardt method for nonlinear least squares curve-fitting problems." *Department of Civil and Environmental Engineering, Duke University* (2011).
- [21] M.I.A. Lourakis. *A brief description of the Levenberg-Marquardt algorithm implemented by levmar*, Technical Report, Institute of Computer Science, Foundation for Research and Technology- Hellas, 2005.
- [22] K. Madsen, N.B. Nielsen, and O. Tingleff. *Methods for nonlinear least squares problems*. Technical Report. Informatics and Mathematical Modeling, Technical University of Denmark, 2004.