

DEVELOPMENT OF MATHEMATICAL MODEL FOR MONORAIL  
SUSPENSION SYSTEM UNDER DIFFERENT TRACK CONDITIONS

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## ABSTRACT

Traffic problems in major cities around the world during the last two decades have presented important needs of new transportation systems. Currently, there is an increased demand on public transportation systems, especially in mega cities. This increased transportations demand, pushed transportation authorities to plan new projects and expand existing monorail systems to accommodate the increase demand. This required engineers to develop and design larger monorail systems. New Monorail designs require more powerful bogies with new dimensions to accommodate more passengers, therefore new suspension system design is essential. In order to overcome new designs problems, better understanding of the suspension system is needed by mathematically modeling the system to predict some dynamic characteristics of a new design. This research work concentrates on the modeling and simulation of 15 degrees of freedom full-car Monorail suspension system. The model features the Monorail body, Front bogie and rear bogie geometries. Lagrange's equation was used to obtain the equations of motion of the monorail suspension system and system matrices. Numerical Central Difference method was used to obtain the system responses subject to sinusoidal Track excitations. Three Track scenarios that has different loads and different driving speeds were conducted to investigate the monorail suspension system, programmed in MATLAB. The system results are analyzed in terms of their dynamic responses. Fourier Fast transform was used to calculate the frequency ranges of dynamic responses. As a result, some very important characteristics of the Monorail suspension system were revealed, with indicators that helps understanding the effects of driving speeds and different loads, which can be used to better understand the system dynamic performance, to improve the original design specifications and detect Monorail suspension system problems.

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## LIST OF SYMBOLS AND ABBREVIATIONS

$m$	-	Mass
$G$	-	Gravity
$K$	-	Spring constant Stiffness
$C$	-	Damping constant of damper (Vertical)
$Z$	-	Vertical displacements
$Y$	-	Lateral displacements
$nv$	-	The number of cars in a monorail train
$I$	-	The mass moment of inertia.
$v$	-	The number of cars on the bridge.
$I$	-	An index indicating the suspension position of a car.
$i$	-	Indexing parameter
$j$	-	Indexing parameter
$n$	-	An index indicating the left and right sides of the car
$R_{vinjn}$	-	The relative displacement at springs and dampers
$\delta_{ij}$	-	Kronecker's delta
$\theta_{vx}$	-	Rolling
$\theta_{vy}$	-	Pitching
$\theta_{vz}$	-	Yawing
$DOF$	-	Degree of Freedom
$CAE$	-	Computer aided Engineering
$FEA$	-	Finite Element Method

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## CHAPTER 1

### INTRODUCTION

Traffic problems in major cities around the world during the last two decades have presented important needs of new transportation systems. Consequently, the challenge to adopt new transportation modes has yielded monorail systems. This type of transit vehicle systems generally referred to as "monorail" is a generic term applied to an extremely narrow gage vehicle system utilizing a single track and beam way structure for supporting the vehicle. The conventional transit vehicle, generally referred to as "dual-rail", requires two separate running tracks such as double steel rails for steel wheel vehicles or a double concrete running surface for rubber tire vehicles. Hence, a monorail system is characterized by the use of a single track on a support beam, which the vehicle is suspended from or which supports the vehicle from the bottom. Furthermore, with the increased use of modern train systems and high demand that increases day by day, so does the need for faster, more efficient and more comfortable train ridership.

Nowadays one of the important parts of any modern transportation mode monorail car is suspension system, suspension system plays a mandatory role in the performance of the monorail in terms of, riding comfort by isolating track vibrations and movements from passengers, and providing good handling to the monorail car in turns and while accelerating or stopping at stations. Since, suspension system in a monorail bogie holds such importance this research is investigating the characteristics of the monorail suspension system under multiple situations by developing a mathematical model that simulates the system performance under different scenarios.

The results of the research provides a preview of the suspension system and its dynamic characteristics, which helps in providing a better system performance.

### **1.1 Problem Statement**

Currently, there is an increased demand on transportation systems, especially in mega cities. Monorail systems are one of many modern rail transportation systems. This increased transportations demand, pushed transportation authority's to plan new projects and expand the existing monorail systems to accommodate the increase demand. This required engineers and designers to develop and design larger monorail systems. Thus, more designing process and optimization is needed to be done on the bases of the older designs. One of the difficulties that faces engineers in such cases is the design of bogie, especially the suspension system. New bogies design must have an optimal suspension system that can isolate the track disturbances to provide comfort for the passengers and also provide better handling for the train steering. In order to achieve such performance, the suspension system needs to be designed with a balanced compromise between comfort and handling, this requires lengthy process of trial and error and optimizing of the suspension system. For example in the case of increasing the number of cars requires more powerful bogies with new dimensions to accommodate the new body of the train and more passengers. On the other hand, in terms of establishing a design with such challenges, engineers should be aware of the dynamics response of the old system and how the new requirement of a new design will affect the suspension system dynamics however the issue is to overcome the negative effects such as uncomfortable ride or bad handling. This current issue, whenever a new design is needed, the lack of understanding on how the suspension system reacts to new features that are added to the design and how to mitigate negative effects in the designing process before fabricating the design and then be enrolled in lengthy trial and error process.

The technique is to find a proper mathematical representation method, to represent the monorail suspension system and then further investigate its characteristics. Then optimizing the model and utilize the methodology to generate successful designs, in which relay on a proper functioning mathematical

representation, then to evaluate and process the train suspension in a well-represented environment and declare the parameters, physics and displacements in relation with track scenarios. These problems need to be addressed within this research, where proper mathematical model representation, Parameters declaration, physical inspection in term of mathematical analysis, verification, optimization and simulation algorithm are needed to create a successful mathematical model of the monorail bogie suspension system that helps to solve this current issue.

## 1.2 Objectives

- 1) To establish a Monorail suspension system model.
- 2) To develop the mathematical model for the monorail bogie suspension system for a full-car model.
- 3) To investigate the performance of the Monorail train Model suspension system under different track conditions through computer simulation process.

## 1.3 Scope of research

In order to establish a better understanding of how the suspension system functions and obtain insights into the way in which the system operates, the physical laws, the dynamics and the characteristics needs to be defined, listed and expressed correctly in the most suitable method. Therefore the mathematical framework is required to solve the problem of this research. This research scope focuses on:

- i. Defining Monorail Body and Bogie Geometrical Parameters in terms of height, Length and width.
- ii. Defining Monorail body and bogie suspension system physical properties in terms of mass, spring stiffness and spring damping.
- iii. Defining Monorail Degrees of freedom in terms of global coordinates system in X,Y,Z, Roll, Pitch and Yaw parameters.

- iv. Formulating the system Equations of Kinetic, potential and dissipation of energy using Lagrange equations method.
- v. Formulating equations of motion for each defined degree of freedom and system matrices in terms of mass, stiffness and damping matrix.
- vi. Defining the bogie suspension system eigenvalue and static characteristics.
- vii. Applying Central Difference Method to solve the monorail suspension system equations.
- viii. Investigation of dynamic response of the monorail model under sinusoidal track excitations in terms of vertical, lateral, roll, pitch and yaw displacements.
- ix. Finding the frequency response range of Monorail suspension system displacements using Fast Fourier Transformation.
- x. The effects of track excitations on the performance of the suspension system.

This thesis is mainly focused on a dynamic modeling and simulation of a multi-Degree of Freedom (DoF) Monorail suspension system, by incorporating body and suspension geometries. This analysis is helpful to better understand the coupled motions of monorail bogies and body, but the nonlinear spring characteristics are not covered in this research, which may be further studied in the future. The developed model is simulated to obtain system responses in both the time and frequency domains. The Track excitations include simple sinusoidal input, but there is no random input adopted. Piecewise linearization of the real nonlinear shock absorbers is replaced by equivalent conventional linear invariant viscous damping. The mathematical model is derived using Lagrange's equation and MATLAB script is used as numerical solution, where Central Difference Method is used. Since, Monorail bogies are statically indeterminate structure, the stiffness matrix combined with boundary conditions is used to calculate the suspension static deflection and the static reaction force (static load on each tyre).

The modeling process used in this thesis combines some ideas from previous modeling practices [4], and adds some new features, because methods such as FEA and actual modal tests require comprehensive test rigs and measurement instruments, which is not practical for a research work at this level. In particular the aim of this research work is not intended to build a model covering all random track excitations.

## 1.4 Significance of Research

This research, can present a solution to an issue that appears in the first test runs of the new designs of monorail systems it can also help designers to mitigate the mentioned problems during the design phase before practical implementation on the actual field. In addition, this research introduces an easier and powerful tool, for future engineers and designers to optimize future designs with a flexible mathematical model that can accommodate different future designs and manufacturing process efficiently.

## 1.5 Limitations of Research

- i. This model can simulate the general dynamic response of Monorail suspension system if the track roughness excitations are small, but it can not represent some nonlinear characteristics of the air suspension subject to large deflection of the air suspension. Besides that, high frequency dynamic track excitations are not included in this model.
- ii. The load sharing between front and rear bogie axles are ignored, which causes the front and rear axles to act essentially independently.
- iii. Monorail bogie air suspension system is very sensitive to the gearbox transmission line torsional displacements, Besides that, it is very sensitive to the transmission line torsional displacements, because the rotational action of the transmission axle will generate displacements in the bogie angle, which will then amplify the torsional displacements. As this amplification largely depends on different suspension geometry settings, this effect is difficult to include in this general model without particular case studies. Therefore, this general dynamic model does not include any torsional displacements considerations.



## 1.6 Thesis outline

This thesis is organized into 5 Chapters, Chapter 1 discusses the research introduction, problem statement, objectives, scope of research, and significance of research, expected results and the thesis outline.

Further explanations on research background , general terms, concepts and insights of mathematical modeling basics are included in Chapter 2 .This chapter explains the literature review of the modeling of monorail bogie suspension system .

Chapter 3 explains the method used in this research in order to obtain mathematical model for the bogie suspension system, with further mathematical formations and listing of all the parameters used. Additionally, it illustrates the research work flow and completed models.

Chapter 4 explains and analyzes the research results, and investigates the founded data of this research. Chapter 5, provides the research's conclusions and recommendations.



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## CHAPTER 2

### LITURETURE REVIEW

#### 2.1 Research background

##### 2.1.1 Monorail Train

In this subchapter, a conceptual overview of the monorail train is presented, to further discuss the system definition, history, types, and advantages and disadvantage of its system components.

##### 2.1.2 Definition

There are many definitions of monorail systems and it's often confused with other modes of transportation such as LRT(Light Rail Transit) and MRT-Mass Rail Transit. However, the official definition of monorail according to monorail organization is:

“MO\*NO\*RAIL single rail serving as a track for passenger or freight vehicles. In most cases rail is elevated, but monorails can also run at grade, below grade or in subway tunnels. Vehicles are either suspended from or straddle a narrow guide way. Monorail vehicles are WIDER than the guide way that supports them.”[2]

A monorail is a rail-based transportation system based on a single rail, which acts as its sole support and its guideway. The term is also used variously to describe the beam of the system, or the vehicles traveling on such a beam or track. The term

originates from joining mono (one) and rail (rail), from as early as 1897, possibly from German engineer Eugen Langen who called an elevated railway system with wagons suspended the Eugen Langen One-railed Suspension Tramway (Einschieniges Hängebahnsystem Eugen Langen). The transportation system is often referred to as a railway. Colloquially the term "monorail" is often used to describe any form of elevated rail or people mover. More accurately, the term refers to the style of track, not its elevation.



Figure 2.1: Sydney Metro Monorail in Australia singular (mono) beam, with a train wider than guideway. [2]

### 2.1.3 Monorail History

Monorail has had a long history of inventions and technology development that is more than two centuries long. In this section some of the most significant monorails in history are considered to be a representation of the Monorails in History from 1825 to 1929. Table 2.1 shows some of examples of the history of monorail in chronological order:

Table 2.1: Monorail history [2].


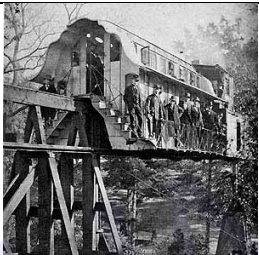

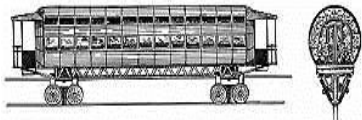
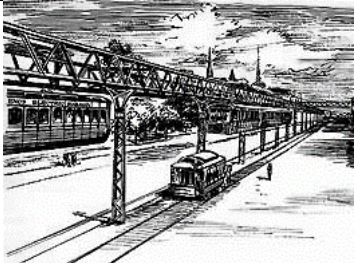

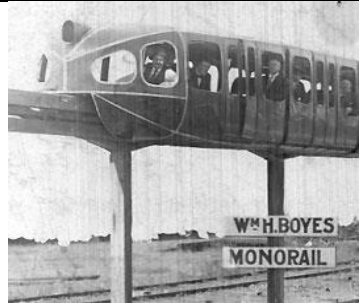
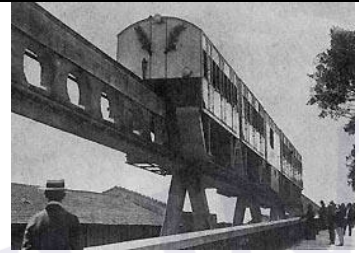

Year	Name	Description	Image
1825	Cheshunt Railway	The first passenger carrying monorail celebrated a grand opening June 25th, 1825. It had a one-horse power engine. Based on a 1821 patent by Henry Robinson Palmer, the Cheshunt Railway was actually built to carry bricks, but made monorail history by carrying passengers at its opening.	
1876	Philadelphia Centennial	General Le-Roy Stone's steam driven monorail was first demonstrated at the United States Centennial Exposition in 1876. The ornately designed double-decker vehicle had two main wheels, the rear one driven by a rotary steam engine.	
1878	Bradford & Foster Brook Monorail	A modified version of General Stone's Centennial monorail was put into use on a 6.4 kilometer line between Bradford and Gilmore, Pennsylvania. It was built to transport oil drilling equipment and personnel to Derrick City. The line was abandoned.	
1886	Meigs Monorail	Captain J.V. Meig's monorail made it as far as having a test track, but the design was so far ahead of its time that it never caught on.	
1886	Enos Electric Railway	The Enos Electric Railway, the first suspended monorail, was tested and demonstrated on the grounds of the Daft Electric Company in Greenville, New Jersey in 1886. It was built of light, open steelwork rather than massive wooden beams that most monorails to this point had used. The Greenville demonstration attracted considerable publicity in the press, but no major system was ever built..	
1901	Wuppertal Schwebebahn	Civil Engineer Eugen Langen of Cologne, Germany has left his mark on the history of monorails in a big way. His Schwebebahn (suspension railway) has operated successfully along the Wupper river for almost 100 years. It has survived two world wars and continues to operate profitably and safely today.	

Table 2.1: Monorail history (cont.)

Year	Name	Description	Image
1911	William H. Boyes Monorail	This test track was built and demonstrated in 1911 in the tideflats of Seattle, Washington. The rails were made of wood and track cost was estimated to be around \$3,000 per mile. The Seattle Times commented at the time that "the time may come when these wooden monorail lines, like high fences, will go straggling across country, carrying their burden of cars that will develop a speed of about 20 miles per hour." Like so many inventions, lack of financial backing prevented further development.	
1914	Genoa Monorail	Built for the 1914 "Esposizione Internazionale di Igiene, Marina e Colonie" exposition, this straddle-type monorail looks like a close cousin of many of today's based monorails. The "Telfer" Monorail had coaches the size of railway cars and was conceived as a mass transit system demonstrator. The line linked the exhibition site with a central square of the city. The train was built by the Italian manufacturer Carminati & Toselli and consisted of 4 coaches for passengers, with an electric locomotive located in the middle. The monorail only operated for a couple of years and was then dismantled.	
1929	The Bennie Railplane	one unique demonstration line was built by Scottish engineer George Bennie. The short test track was built over a railroad line near Glasgow, Scotland. Two electrically-powered propellers delivered 240 horsepower in a short burst for acceleration to the cruise speed of 160 kph. There were plans for a high-speed link between London and Paris, with a seaplane to carry passengers across the English Channel, but the grave economic difficulties of the 1930's doomed the Railplane from the start.	

#### 2.1.4 Monorail Types

Monorails are classified into straddle and suspended-type systems. Since the straddle-type travels by straddling the track, its center of gravity is situated above the track. The suspended-type, on the other hand, is configured suspending from the track, with its center of gravity under the track.



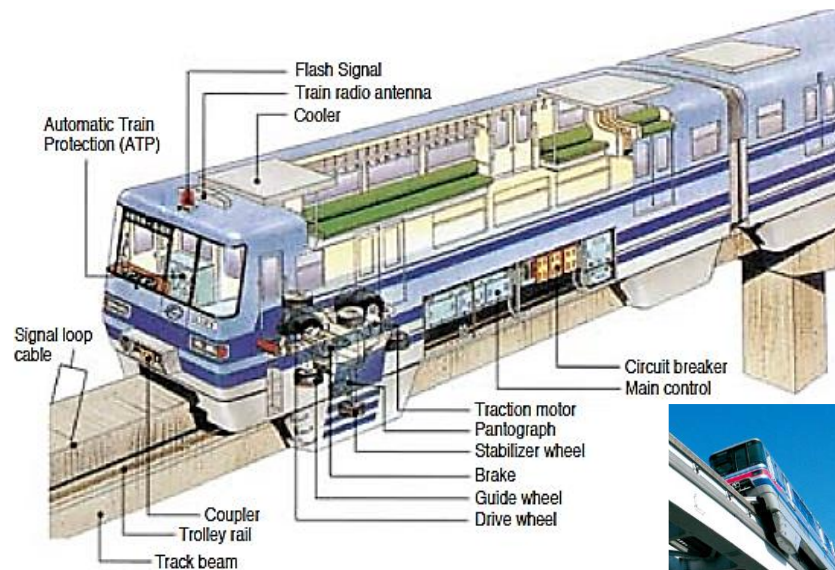


Figure 2.2: Straddle Monorail systems [3].

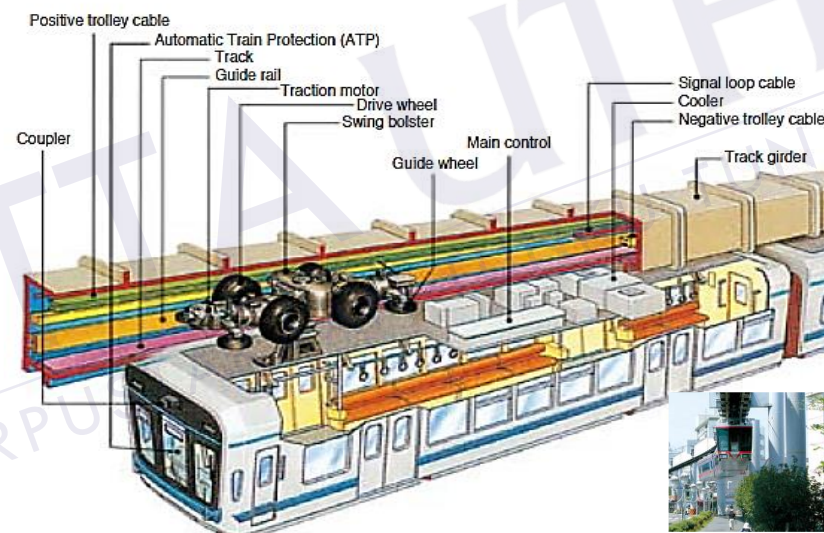


Figure 2.3 Suspended Monorail systems [3].



### 2.1.5 Monorail Verses other rail transportation systems.

In this part a comparison between Monorail system and other modes of transportation is made in terms of, Aesthetics, Construction, , Efficiency and Safety .To support the argument that monorail could serve better in many transit realms. Even though, monorail is not perfect for every situation, the following arguments present a strong case for the Monorail systems. These arguments are tabulated in the Table 2.2.

Table 2.2 Monorail Vs. Other rail systems.

Comparison Arguments	HEAVY RAIL/ SUBWAY	LIGHT RAIL/ TRAMS	MONORAIL
Aesthetics	<ul style="list-style-type: none"> <li>• When heavy rail is elevated, the guideway casts a wide shadow and blocks out much more of the sky.</li> </ul>	<ul style="list-style-type: none"> <li>• Light Rail requires a spider web of overhead wires with support posts.</li> <li>• When light rail is elevated, it's even more obtrusive with its wide, dark street-producing guideway.</li> </ul>	<ul style="list-style-type: none"> <li>• The monorail guideway can be constructed to be an enhancement</li> <li>• The beam is not very wide.</li> <li>• - Small shadow, and sky-view friendly</li> </ul>
Construction	<ul style="list-style-type: none"> <li>• Construction Time is very long.</li> <li>• Disturbs Mega cities Transportation paths.</li> <li>• Underground tunneling is risky and effects structures foundations.</li> </ul>	<ul style="list-style-type: none"> <li>• Customers can't access their establishments during the long period of construction.</li> <li>• Entire streets and underground utilities must be rebuilt to put in light rail.</li> </ul>	<ul style="list-style-type: none"> <li>• Simple construction process, mainly consists of mounting pre-built support beams that are manufactured off site. .</li> <li>• Monorail beam way can be installed far faster than the alternatives.</li> <li>• No other fixed rail can be installed as quickly and as disruption-free</li> </ul>
Efficiency	<ul style="list-style-type: none"> <li>• Steel wheels on steel rail grind and wear. Therefore, both wheels and rail require far more care than monorail tires.</li> <li>• Hard to profit from due to high maintenance costs.</li> </ul>	<ul style="list-style-type: none"> <li>• In case of trams running in the street, the schedule can be influenced by conditions during peak traffic times.</li> <li>• Also, steel wheels and rail require high maintenance cost, thus less profitability.</li> </ul>	<ul style="list-style-type: none"> <li>• Monorail run on typical tires and typically, each load tire gets over 100,000 miles of travel before being replaced.</li> <li>• Monorails regularly operate at high reliability that makes them more profitable.</li> </ul>

Table 2.2 Monorail Vs. Other rail systems.(Cont.)[34],[35]

Comparison Arguments	HEAVY RAIL/ SUBWAY SYSTEMS	LIGHT RAIL/ TRAMS	MONORAIL
Safety	<ul style="list-style-type: none"> <li>• Heavy rail is under the risk of derailments in case of poor wheel flange maintenance or unsafe track conditions.</li> <li>• Collusion risks in road crossovers.</li> </ul>	<ul style="list-style-type: none"> <li>• Derailments risk.</li> <li>• Collusion risks in road crossovers.</li> </ul>	<ul style="list-style-type: none"> <li>• Track is isolated form other transportation modes, since its elevated.</li> <li>• Train optimized design , minimizes the risk of derailment</li> </ul>
Bogie Shapes	<ul style="list-style-type: none"> <li>• H shaped Bogie is used :</li> </ul> 		<ul style="list-style-type: none"> <li>• U shaped Bogie is used :</li> </ul> 

### 2.1.6 Monorail Bogie:

Monorails have a unique bogie design, where Straddle-type monorail train has two bogies on the front and rear axles, respectively. Each bogie has driving wheels, steering and stabilizing wheels that firmly grasp the track girder to increase running stability as illustrated in the Figure 2.4.



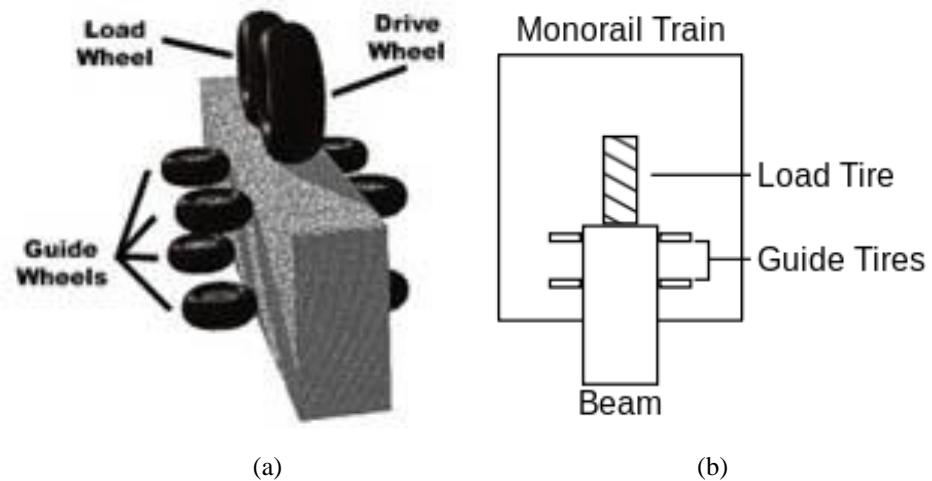


Figure 2.4 (a) Monorail bogie components.[4] (b) Monorail train bogie position[4].

The drive wheels are called the load tires. The four main load tires per train car are found at the front and rear of each monorail section. These are the tires that the monorail rides on at the top of the beam ways. The tire specifications are similar to wide truck tires. These are seen in truck or cement mixer.

The second type, the 21.5" in diameter guide tires, are the smaller tires, which ride along the sides of the beam ways and keep the train centered on the beam. In addition, there are two steering wheel tires under each cab car to help steer the suspension up to  $3^\circ$  in either direction.

Steering is accomplished in the intermediate cars by the relative geometry of the two adjacent cars. Thus, all load tires are steerable, and the tires are always tangent to the curve. There are 8 load and 24 guide tires for a total of 32 tires per train for the current SCOMI GEN2 train, where GEN3 has 4 cars per train which doubles the number of load up to 16 tiers and guide tires up to 48 tiers for a total of 64 tiers as seen in Figure 2.4.

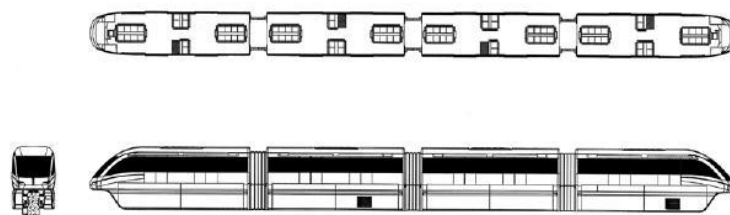


Figure 2.5: Scomi Monorail GEN 3 side, top and front view. [5]

All tires are nitrogen-filled to aide in extinguishing fires if the axle becomes too hot. Additionally they have run-flat capability and will last 100,000 miles in normal operation. The maximum rated speed for these tires is 65 mph and the load capacity is 12,800 pounds each at maximum inflation.

The main suspension of monorail cars above the axles is done with air bags (also called air springs). The air bags are inflated and deflated by an automatic leveling valve, which compensates for varying load conditions. There are also vertical hydraulic shock absorbers to add to the somewhat smooth ride, but the suspension system will be further disused within Chapter 3 in terms of technical and mathematical specification.

#### **2.1.7: Brief History of Vehicle Dynamics Development**

Vehicle dynamics is a relatively newly established discipline with a history less than 100 years. It derives from awareness of various ride problems experienced in early vehicles. In the early 1930's, engineers such as Lanchester, Olley and Broulheit began to analyze suspension kinematics [36], cornering kinematics and tire dynamics during their research on development of independent suspensions. Up to present two major research directions are formed in the category of vehicle dynamics: Ride Dynamics and Handling Dynamics.

Vehicle dynamics was in its first "golden age" in the 1950's, during which period the linear dynamics theory was established [36]. In 1993, Segel [37] made a speech to an I.Mech. E conference, giving the audience an overview of the infantile development of the vehicle dynamics. He divided the achievement of the early vehicle dynamic research into 3 stages:

- Stage 1 (up to the early of the 1930's)
  - Examination of vehicle dynamic performance based on experience.
  - Practical experience of front wheel hunting.
  - Awareness of the importance of the ride performance.

- Stage 2 (From the early of 1930's to 1952)
  - Understanding of the simple tire dynamics and definition of the slip angle.
  - Definition of “Understeering” and “Oversteering”.
  - Understanding of the steady-state cornering characteristics.
  - Establishment of the simple 2 DoF handling model.
  - Test work in relation to ride performance and presenting the concept of “flat ride”.
  - Introduction of front independent suspension.
- Stage 3 (After 1952)
  - Deeper understanding of the tire characteristics after testing and modeling.
  - Establishment of the 3 DoF handling model.
  - Extension of handling, stability and cornering response analysis.
  - Initial prediction of the ride performance using random vibration theory.

After 1950's, vehicle dynamics developed even more rapidly. During this period it was further explored in several important areas. First, test methodologies were more complete and test standards were established. Researcher's understanding of nonlinear response characteristics improved allowing improved nonlinear modelling. Second, with the development of computer aided engineering (CAE) technology, the availability of some general simulation software and the development of Multi-body System Dynamics (MBS) techniques and various numerical methods, it is now possible to simulate a complicated vehicle model with large numbers of DoF's in relatively short time and with high accuracy.

Meanwhile, active control technology began to be adopted on practical vehicles. Active suspension, active anti-roll bar, active steering and active engine mounts were developed in the past 20 years. Many designs have since then been installed on practical vehicles. Integration of modern control theory into traditional vehicle technology has been a popular research area.

In the past 60 years, although the theory of the vehicle dynamics has achieved great success in improving passenger vehicle dynamic performance, it is still deficient. Notably vehicle manufacturers currently use both subjective and objective evaluation techniques to assess vehicle dynamic performance, and pure CAE technology is

supplemented and even integrated by test results which is known as “Hybrid Modeling”.

In light of this overview, many transportation systems such as cars, trucks, airplanes had their fair share of research work, in terms of vehicle dynamics studies. The focus on these system is due to their uses and popularity. However, Monorail trains differ in terms of uses and popularity. Their characteristics can be studied using similar approaches, due to the fact that the system incorporates some of large trucks parts for example the load wheels. However the number of work and studies done on these systems is very low compared to other transportation systems and it's implemented by few specialized companies.

## **2.2 General Terms and Concepts**

In this section, related general terms and concepts are listed and discussed in relation with the research are suspension systems, mathematical models and important dynamic properties.

### **2.2.1 Suspension System**

#### **2.2.1.1 Definition**

According to Wikipedia the definition of suspension system is “*Suspension is the term given to the system of springs, shock absorbers and linkages that connects a vehicle to its wheels and allows relative motion between the two. .*”<sup>[6]</sup>

#### **2.2.1.2 Purpose**

The suspension system serves two main purposes. Firstly, contributing to the vehicle's road holding, handling and braking for good active safety and driving pleasure.

Secondly, keeping vehicle occupants comfortable and reasonably well isolated from road noise, bumps, and vibrations.

Moreover, the complete suspension system isolates the vehicle body from road shocks and vibrations which would otherwise be transferred to the passengers and load. It must also keep the tires in contact with the track, regardless of track surface. A basic suspension system consists of springs, axles, shock absorbers, arms, rods, and ball joints. When a wheel strikes a bump, there is a reaction force, and energy is transferred to the spring which makes it oscillate. Oscillations left uncontrolled can cause loss of traction between the wheel and the road surface. Shock absorbers dampen spring oscillations by forcing oil through small holes. The oil heats up, as it absorbs the energy of the motion. This heat is then transferred through the body of the shock absorber to the air.

### **2.2.1.3 Types of suspension system**

The suspension system can be categorized into passive, semi-active and active suspension system according to external power input to the system. A passive suspension system is a conventional suspension system consists of a spring and shock absorber damper without control. The semi-active suspension system has the same elements but semi-active suspension system utilized controlled dampers under closed loop control and it is using varying damping force as a control force. Active suspension system differs from semi-active suspension as its control force is produced by separate hydraulic or pneumatic actuator unit. Besides these three types of suspension systems, a skyhook type damper suspension has been considered in the early design of the active suspension system. In the skyhook damper suspension system, an imaginary damper is placed between the vehicle body and the sky. The imaginary damper provided a force on the vehicle body proportional to the Monorail body absolute velocity. As a result, the Monorail body movements could be reduced without improving the tire deflections. However, the design concept was not feasible to be realized. Therefore, the actuator has to be placed between the Monorail body and the wheel.

## **2.3 Mathematical Models of different suspension systems**

The fundamental properties of various systems, are theoretically investigated on the basis of mathematical models that are subjected to realistic inputs chosen to represent different mathematical quantities. In this section, some of these mathematical models are discussed and illustrated.

### **2.3.1 Linear Passive suspension mathematical model**

Passive suspension system can be found in controlling the dynamics of vertical motion of a vehicle. There is no energy supplied by the suspension element to the system. Even though it doesn't apply energy to the system, but it controls the relative motion of the body to the wheel by using different types of damping or energy dissipating elements. Passive suspension has significant limitation in structural applications. The characteristic are determined by the designer according to the design goals and the intended application. The disadvantage of passive suspension system is it has fixed characteristic, for example if the designer design the suspension heavily damped it will only give good vehicle handling, but at the same time the suspension system will transfer road input (disturbance) to the vehicle body. The result of this action is if the vehicle travel at the low speed on a rough road or at the high speed in a straight line, it will be perceived as a harsh road. Then, if the suspension is designed lightly damped, it will give more comfortable ride. Unfortunately this design will reduce the stability of the vehicle in making turns and lane changing. Figure 2.6 shows traditional passive suspension components system that consists of spring and damper.

### **2.3.2 Linear Semi-active suspension mathematical model**

Semi-active suspension system was first proposed in 1970's. It's provides a rapid change in rate of springs damping coefficients. It does not provide any energy into

suspension system but the damper is replaced by controllable damper. The controller's determine the level of damping based on control strategy and automatically adjust the damper to the desired levels. This type of suspension system used external power to operate. Sensors and actuator are added to detect the road profile for control input. The most commonly semi-active suspension system is called skyhook damper. Schematic diagram for semi-active suspension is shown in Figure 2.7.

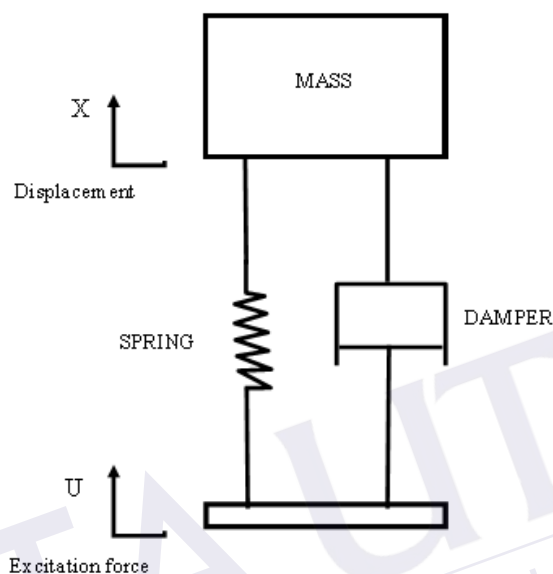


Figure 2.6: Linear Passive Suspension Components.[42]

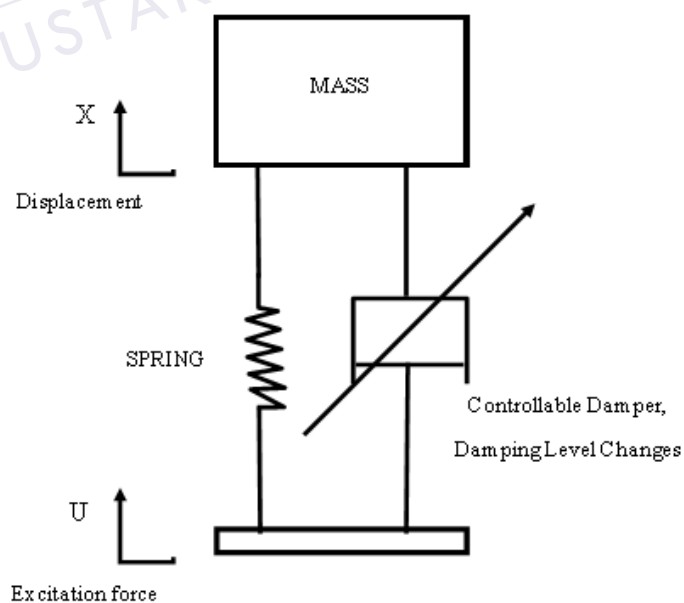


Figure 2.7 Linear Semi-active Suspension Components[42]

### 2.3.4 Dynamic properties of Monorail Train

#### 2.3.4.1 Degree of Freedom (DOF)

Degrees of freedom, in a mechanics context, are specific, defined modes in which a mechanical device or system can move. The number of degrees of freedom is equal to the total number of independent displacements or aspects of motion. A machine may operate in two or three dimensions but have more than three degrees of freedom. The term is widely used to define the motion capabilities of robots.

The number of *degrees of freedom* (DOF) of a mechanical system is defined as the minimum number of *generalized coordinates* necessary to define the configuration of the system. For a set of generalized coordinates to be minimum in number, the coordinates must be independent of each other. That is, they must form an independent set of coordinates. Figure. 2.8 shows examples of one and two degree-of-freedom planar systems.

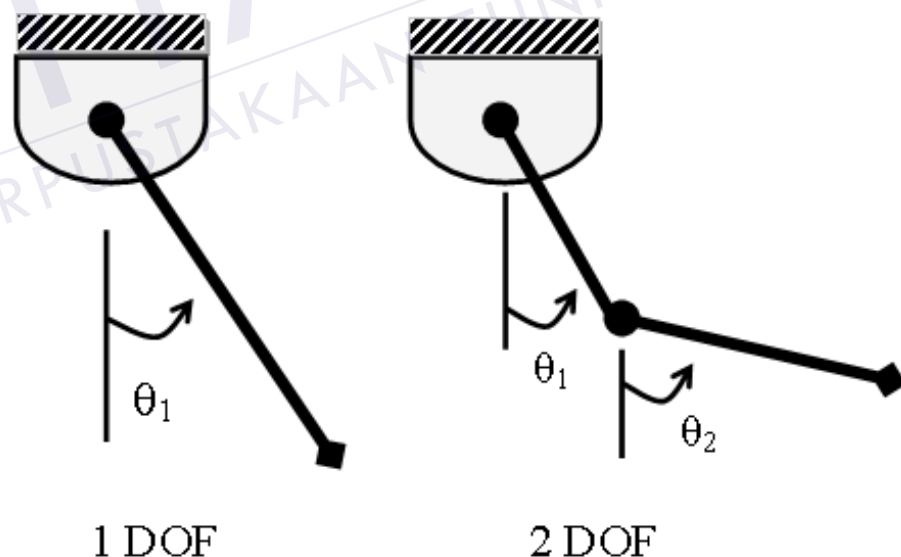


Figure 2.8 Example of one and two degree of freedom systems.[42]



### 2.3.4.2 Pitch Roll and Yaw

Pitch, Roll and Yaw are terms used to describe the orientation of an object on (xyz) axis's system, where Roll is the rotation about local x axis, Pitch is the rotation about local y axis and Yaw is the rotation about local z axis, as described in Figure 2.9 as follows:-

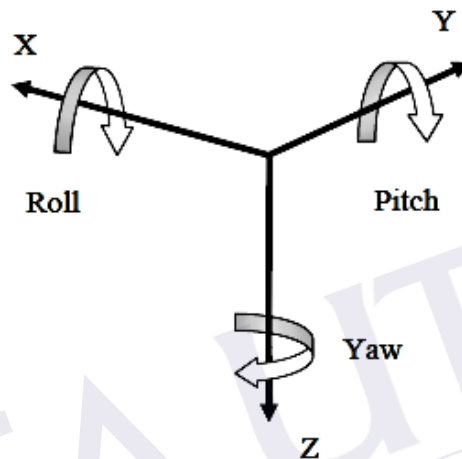


Figure 2.9 Pitch m Roll and Yaw on XYZ axis. [7]

### 2.3.4.3 Sway, Bounce Swing and Winding

**Sway**, is defined as a rhythmical movement from side to side. **Bouncing**, is the act of jumping or moving up and down jerkily, typically on something springing. Moreover, **Swing**, is known as the movement back and forth. Finally, **Winding**, is a twisting movement on a spiral course as seen on Figure 2.10.

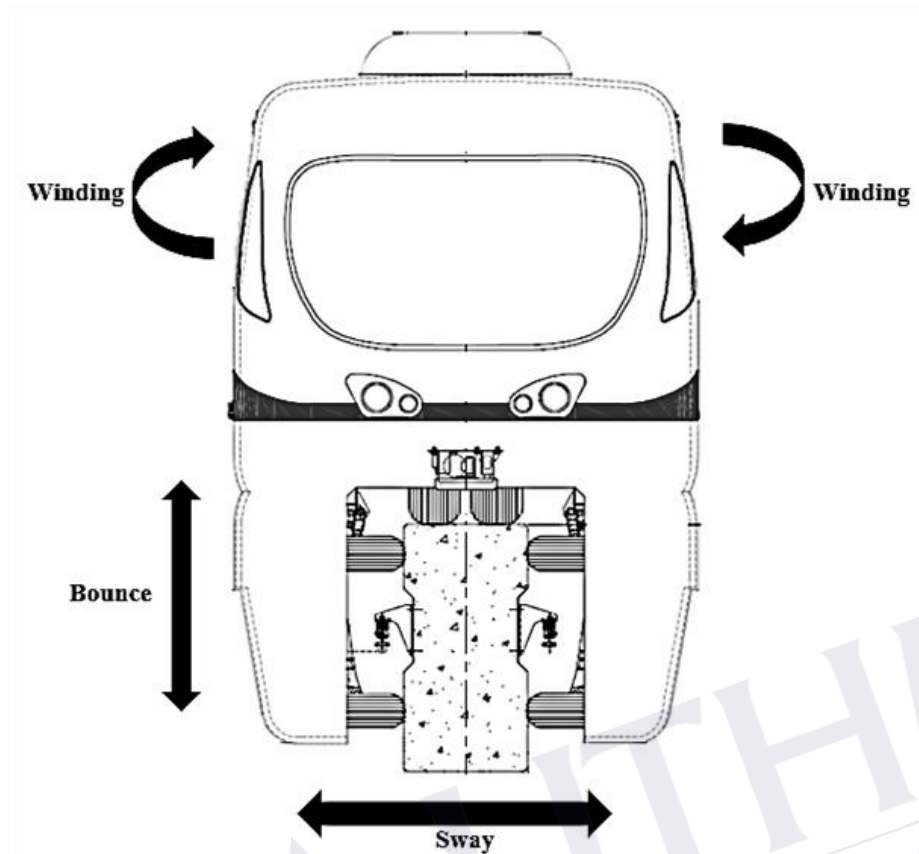


Figure 2.10 Bounce, Sway and winding motions of Monorail Train.[42]

#### 2.3.4.4 Sprung mass and Unsprung mass

The weight of the train, transmission, various mechanical and electrical components, passenger cabin, passengers and various other components whose weight is supported by the suspension of a train in total is called sprung mass. The weight of the wheels, tires, brakes are considered to be the unsprung mass of a vehicle which is defined as the mass between the track and the suspension. Thus, sprung mass is the load sitting on top of the springs and unsprung mass is the weight connected to the bottom of the suspension as illustrated on Figure 2.11.

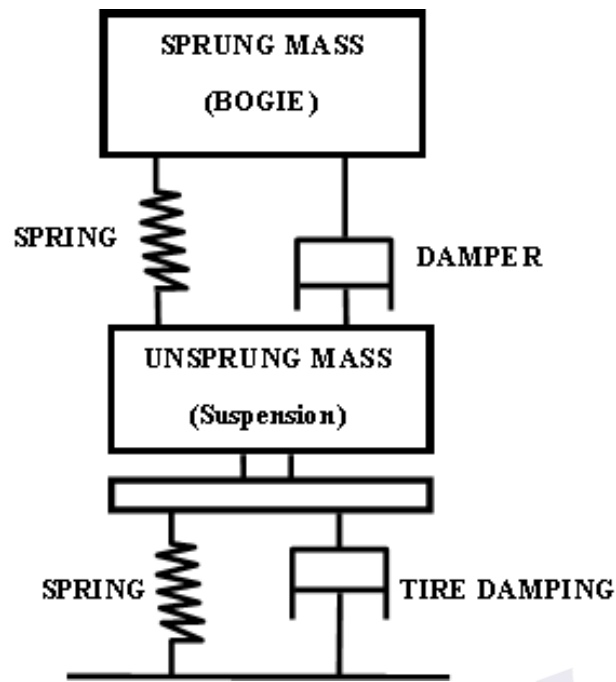


Figure 2.11 Sprung and Unsprung masses of Monorail Train. [42]

## 2.4 Previous research work

There are variety of research that handle mere aspects of this proposed work, but not many have handled the molding of a monorail system. Table 2.3, describes related previous studies that handles the modeling of monorail systems.

Table 2.3: Previous studies

Research No	Title
1.	"Investigation of train dynamics in passing through curves using a full model "
2.	" Preview Control of an Active vehicle Suspension System Based on Four Degree of Freedom Half Car Model ",
3.	" 9 DOF railway vehicle modeling and control for the integrated tilting bolster with active lateral secondary suspension "
4.	" Ride Analysis of Three Wheeled Vehicle Using MATLAB/Simulink"
5.	" Modeling and simulation of railway bogie structural vibrations"
6.	" Effects of speed, load and damping on the dynamic response of railway bridges and vehicles "

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