DEVELOPMENT OF AN ELECTROMAGNETIC NUMERICAL SOLVER BASED ON THE FINITE DIFFERENCE TIME DOMAIN (FDTD) TECHNIQUE FOR RESEARCH AND TEACHING PURPOSES

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For my beloved mother and father

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ABSTRACT

A 1D-FDTD code was developed to support plane wave excitation in 3D-FDTD domain and the code was developed using C++ programming language. First-order Mur absorbing boundary condition (ABC) is applied to keep outgoing electric and magnetic fields from being reflected into the problem space. In this thesis, the performance of 1D-FDTD scheme is then evaluated on several medium including free space, lossless dielectric medium, lossy dielectric medium and good conductors. Sine-Gaussian technique is used to excite field signal in the 1D-FDTD simulation domain and the simulation have been carried out to analyze the performance of the scheme. From the results, the 1D-FDTD scheme shows good expected results on all applied conditions. The integration of 1D-FDTD scheme into the 3D-FDTD solver is realized through the implementation of Total Field Scattered Field (TFSF) technique. The technique is used to excite plane wave into the 3D-FDTD domain and will be used for future wave propagation studies. All simulation results presented in this work were analyzed using OriginPro software.

ABSTRAK

Kod *Finite Difference Time Domain (FDTD)* pada satu dimensi telah dibangunkan untuk menghasilkan gelombang satah di dalam domain *FDTD* tiga dimensi di mana gelombang satah itu bergerak pada arah normal dengan paksi Y. Bahasa pengaturcaraan yang digunakan dalam membangunkan kod ini ialah C++. Seterusnya, kaedah sempadanan serapan jenis Mur digunakan sebagai penyerap sempadan kerana ia berkebolehan untuk memastikan medan elektrik dan medan magnet daripada terpantul semula ke ruang simulasi. Di dalam tesis ini, prestasi skim *FDTD* satu dimensi telah dinilai di dalam pelbagai keadaan. Antaranya ialah ruang kosong, ruang dielektrik tanpa kehilangan, ruang dielektrik dengan kehilangan dan akhir sekali ialah ruang konduktor yang baik. Teknik *Sine-Gaussian* telah digunakan sebagai pencetus isyarat di dalam ruang simulasi *FDTD* satu dimensi untuk menganalisa prestasinya. Daripada keputusan simulasi, skim *FDTD* satu dimensi menunjukkan keputusan yang baik seperti yang telah dijangkakan. Selepas itu, skim *FDTD* satu dimensi telah digabungkan ke dalam *FDTD* tiga dimensi yang sedia ada melalui pelaksanaan teknik *Total Field Scattered Field (TFSF)*. Teknik itu adalah untuk mencetuskan gelombang satah ke dalam ruang domain *FDTD* tiga dimensi bagi kajian pergerakan gelombang satah. Semua keputusan simulasi dalam kajian ini dianalisis menggunakan perisian *OriginPro*.

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E	Electric field intensity
H	Magnetic field intensity
B	Magnetic flux density
D	Electric flux density
σ^*	Magnetic resistivity
σ	Electric conductivity
μ	Magnetic permeability
ε	Electric permittivity
TFSF	Total Field Scattered Field
1D	One Dimension
2D	Two Dimension
3D	Three Dimension
UTHM	Universiti Tun Hussein Onn Malaysia
PS	Projek Sarjana
PEC	Perfect Electric Conductor
FDTD	Finite Difference Time Domain

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CHAPTER 1

INTRODUCTION

1.1 Project Background

Over the past years, a number of numerical and analytical approaches to Maxwell's time-dependent curl equations were broadly used with the increases in computer memory capacity and relentless advances in computational imitating efficiency. Consequently, the demand for efficient field modelling tools in electromagnetic scattering problems is ceaselessly expanding. In general, computational electromagnetic techniques have been applied to vast areas including the study of the radiation, scattering and penetration of electromagnetic wave with 3-D objects, in problems related to telecommunication, electromagnetic compatibility (EMC), microwave devices, waveguide structures, medical diagnosis and many others [4].

Apparently, the finite-difference time domain (FDTD) method has become one of the most popular methods to calculate full-wave solutions to complex electromagnetic problems. This is because of the increase in computing power and decreasing cost of workstations. Computer memory capacities are increasing rapidly. While this trend positively influences all numerical techniques, it is of particular advantage to FDTD methods, which are founded on discretizing space over a volume, and therefore inherently require a large random access memory. The FDTD method is also very attractive since it is accurate, robust, easily includes simple and highly dispersive media, can easily model complex geometries, and can give broadband solutions from a single simulation using a fast Fourier transform (FFT) on post simulated data. The sources of error in FDTD calculations are well understood, and can be bounded to permit accurate models for a very large variety of electromagnetic wave interaction problems [3].

Furthermore, the primary advantage of FDTD over other full-wave models is that, FDTD being a time domain method allows broadband analysis of the antenna by Gaussian pulse excitation. The antenna characteristics over a wide frequency range can be obtained by taking the Fourier transform of the FDTD simulation results obtained when a wideband Gaussian pulse is used as an excitation and most of widely used electromagnetic simulation software's use frequency domain methods like FEM [1].

1.2 Problem Statement

In many problems of electromagnetics the quantity of interest is the field scattered by a localized object in an otherwise uniform background, under harmonic plane wave illumination. Since FDTD is a near-field solver and only represents a finite portion of the space, it is impossible to create inside it a truly plane wave of infinite extent traveling in an arbitrary direction. Therefore, the development of the plane wave source condition allowed the earliest engineering applications of FDTD computational electromagnetics modeling. These applications were in the defense and bio electromagnetics areas, and involved the interaction of complex-shaped, inhomogeneous material structures with impinging pulsed or continuous-wave electromagnetic fields. Initially, many problems of these types located the material structure of interest far from the radiating antenna, where the incident illumination could be approximated by plane wave.

The plane wave is a constant-frequency wave whose wave fronts (surfaces of constant phase) are infinite parallel planes of constant peak-to-peak amplitude normal to the phase velocity vector. It is not possible in practice to have a true plane wave; only a plane wave of infinite extent will propagate as a plane wave. However, many waves are approximately plane waves in a localized region of space. For example, a localized source such as an antenna produces a field that is approximately a plane wave far from the antenna in its far field region. Similarly, if the length scales are much longer than the wave’s wavelength, as is often the case for light in the field of optics, one can treat the waves as light rays which correspond locally to plane waves.

1.3 Project Objectives

The objectives of the project are:

(i) To develop a stand-alone 1D-FDTD scheme for electromagnetic wave study.

(ii) To evaluate the performance of 1D-FDTD scheme on various materials and medium.

(iii) To Implement TFSF decomposition into the existing 3D-FDTD scheme for plane wave excitation in 3D space.

1.4 Scope of Project

The scopes of the project are:

(i) The 1D-FDTD scheme and the 3D-TFSF decomposition codes were developed using C++ programming language.

(ii) Using OriginPro software for data post-processing purposes.

(iii) Mur absorbing boundary condition was applied to truncate the 1D-FDTD simulation space to infinity.

(iv) The performance of the 1D-FDTD scheme was evaluated on several medium and material conditions including free space, dielectric (lossy and lossless) and PEC.

(v) Using the sine-gaussian technique to excite field signal in the 1D-FDTD simulation domain.

(vi) The plane wave is only for normal incident propagation in 3D space.

1.5 Outlines of the Thesis

Chapter 1 covers the background of the project. The problem of the project were stated follows by the objectives and the scopes of the project. More details overview of FDTD is discuss in Chapter 2.

Chapter 2 unfolds the theoretical concept of FDTD principles including the derivation of the magnetic and electric field update equations, parameters that control the stability and accuracy. The advantages of using FDTD, excitation source, absorbing boundary condition (ABC) and total field scattered field (TFSF) were discuss in general.

Chapter 3 comprises the methodology or approaches in order to achieve the objectives of the project. It shows work flow used in completion of this project. The derivation of electric field and magnetic field for Total/Scattered Field formulation in 3D were shown.

Chapter 4 discuss the results and analysis data which are addressed through the simulation of the 1D-FDTD evaluation in a various medium and plane wave implementation to the currently available FDTD code. Graphs of simulation output and comparison of results in time steps are explained in this chapter.

Chapter 5 summarize all the simulation results and the main conclusion will be concluded including the recommendations for future work are dealt with in this chapter.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Over the past few years, finite-difference time-domain (FDTD) method [1] have become increasingly prevalent in the computational electromagnetic problems due to its simplicity, efficiency, robustness and versatility scheme for highly complex configuration in the computational domain. Generally, FDTD technique is the most well-known numerical method for the solution of problems in electromagnetic simulation ranging from RF to optical frequencies. It is considered to be one of the most powerful numerical techniques for solving partial differential equations of any kind. In addition, it can be utilized to solve the spatial as well as the temporal distributions of electric and magnetic fields in various media.

In principle, FDTD is a method that divides the solution domain into finite discrete points and then replaces the partial differential equation with a set of difference equations. It has successfully been applied to many problems of propagation, radiation and scattering of electromagnetic waves such as antenna, radar, wireless communication system, high speed electronic, photonic, radiography, x-ray crystallography, bio-electromagnetic and geophysical imaging. A good measure of its success lies in the fact that many of papers on the subject have been published in journals and international symposium, apart from the books and tutorials devoted to it. Moreover, much specific and general purpose commercial software is available on the market which further extends its appeal globally. The books used as the main references in this project is written by Taflove and Hagness [1].

2.2 Rise of FDTD Methods

The FDTD method, introduced by Yee in 1966 [1], was the first technique in this class, and has remained the subject of continuous development. Since 1990, when engineers in the general electromagnetics community became aware of the modeling capabilities afforded by FDTD and related techniques, the interest in this area has expanded well beyond defense technology.

There are seven primary reasons for the expansions of interest in FDTD and related computational solution approaches for Maxwell's equations:

(i) FDTD uses no linear algebra:

Being a fully explicit computation, FDTD avoids the difficulties with linear algebra that limit the size of frequency-domain integral-equation and finite-element electromagnetics models to generally fewer than 10^6 electromagnetic field unknowns. FDTD models with as many as 10^9 field unknowns have been run; there is no intrinsic upper bound to this number [4].

(ii) FDTD is accurate and robust:

The sources of error in FDTD calculations are well understood, and can be bounded to permit accurate models for a very large variety of electromagnetic wave interaction problems [4].

(iii) FDTD treats impulsive behavior naturally:

Being a time-domain technique, FDTD directly calculates the impulse response of an electromagnetic system. Therefore, a single FDTD simulation can provide either ultra wideband temporal waveforms or the sinusoidal steady-state response at any frequency within the excitation spectrum [4].

(iv) FDTD treats nonlinear behavior naturally:

Being a time-domain technique, FDTD directly calculates the nonlinear response of an electromagnetic system.

(v) FDTD is a systematic approach:

With FDTD, specifying a new structure to be modeled is reduced to a problem of mesh generation rather than the potentially complex reformulation of an integral equation. For example, FDTD requires no calculation of structure-dependent Green functions [4].

(vi) Computer memory capacities are increasing rapidly:

While this trend positively influences all numerical techniques, it is of particular advantage to FDTD methods, which are founded on discretizing space over a volume, and therefore inherently require a large random access memory.

(vii) Computer visualization capabilities are increasing rapidly:

While this trend positively influences all numerical techniques, it is of particular advantage to FDTD methods, which generate time-marched arrays of field quantities suitable for use in color videos to illustrate the field dynamics.

Generally, the algorithm used by Yee was described by the electric field component which was spatially and temporally offset from the magnetic field component to acquire the update equations. These equations were used in a leapfrog manner to propagate the electric and magnetic fields ahead in time. The equations provide the present fields in terms of the past fields all over the computational domain. After Yee's publication, the approach was widely used with different endeavor [2-6]. The boundaries of the computational domain in FDTD need to be carefully treated when simulating problems in open regions. Spurious reflections will generally occur from the termination of the grid.

The problem can be solved by means of the well-known method called the absorbing boundary condition (ABC). It is generally meant to absorb any outgoing propagating waves without ideally producing spurious reflections. The ABCs was first proposed in 1971 by Merewether [7] to solve the open region difficulties. The development chronicle to magnify the practicability study of the technique was continued in the literature by [8-12] which were based of nonmaterial type. In contrast, Berenger presented a new idea in 1994 called the perfectly matched layer (PML) ABC which was based on material category [13]. The state of the art of Berenger's PML contributes to notably better precision when compared to the other ABCs in the written works [14-15] for broad assortment of applications.

The main handicap of FDTD lies in the truth that only consistent grids can be used. Accordingly, the geometry resemblance in FDTD is restricted to staircase-shaped boundaries which lead to a large number of computer memory requirements and the CPU time particularly when dealing with curvature geometries with fine features [16]. The total number of cells in the computational domain grows significantly due to a global fine mesh. Another FDTD weakness is the presence of error due to numerical dispersion [17-18].

2.3 Finite Difference Time Domain Method in 1D

The finite-difference time-domain (FDTD) method is arguably the simplest, both conceptually and in terms of implementation, of the full-wave techniques used to solve problems in electromagnetics. It can accurately tackle a wide range of problems. However, as with all numerical methods, it does have its share of artifacts and the accuracy is contingent upon the implementation. The FDTD method can solve complicated problems, but it is generally computationally expensive. Solutions may require a large amount of memory and computation time. The FDTD method loosely fits into the category of "resonance region" techniques, in example, ones in which the characteristic dimensions of the domain of interest are somewhere on the order of a wavelength in size. If an object is very small compared to a wavelength, quasi-static approximations generally provide more efficient solutions. Alternatively, if the wavelength is exceedingly small compared to the physical features of interest, ray-based methods or other techniques may provide a much more efficient way to solve the problem.

The FDTD method employs finite differences as approximations to both the spatial and temporal derivatives that appear in Maxwell's equations (specifically Ampere's and Faraday's laws). Consider the Taylor series expansions of the function $f(x)$ expanded about the point x_0 with an offset of $\pm\delta/2$:

$$f\left(x_0+\frac{\delta}{2}\right)=f(x_0)+\frac{\delta}{2}f'(x_0)+\frac{1}{2!}\left(\frac{\delta}{2}\right)^2 f''(x_0)+\frac{1}{3!}\left(\frac{\delta}{2}\right)^3 f'''(x_0)+\cdots \quad (2.1)$$

$$f\left(x_0-\frac{\delta}{2}\right)=f(x_0)-\frac{\delta}{2}f'(x_0)+\frac{1}{2!}\left(\frac{\delta}{2}\right)^2 f''(x_0)-\frac{1}{3!}\left(\frac{\delta}{2}\right)^3 f'''(x_0)+\cdots \quad (2.2)$$

where the primes indicate differentiation. Subtracting the second equation from the first yields:

$$f\left(x_0+\frac{\delta}{2}\right)-f\left(x_0-\frac{\delta}{2}\right)=\delta f'(x_0)+\frac{2}{3!}\left(\frac{\delta}{2}\right)^3 f'''(x_0)+\cdots \quad (2.3)$$

Dividing by δ produces:

$$\frac{f\left(x_0+\frac{\delta}{2}\right)-f\left(x_0-\frac{\delta}{2}\right)}{\delta}=f'(x_0)+\frac{1}{3!}\frac{\delta^2}{2^2}f'''(x_0)+\cdots \tag{2.4}$$

Thus the term on the left is equal to the derivative of the function at the point x_0 plus a term which depends on δ^2 plus an infinite number of other terms which are not shown. For the terms which are not shown, the next would depend on δ^4 and all subsequent terms would depend on even higher powers of δ. Rearranging slightly, this relationship is often stated as:

$$\left.\frac{df(x)}{dx}\right|_{x=x_0}=\frac{f\left(x_0+\frac{\delta}{2}\right)-f\left(x_0-\frac{\delta}{2}\right)}{\delta}=O(\delta^2). \tag{2.5}$$

The "O" term represents all the terms that are not explicitly shown and the value in parentheses, in example., δ^2, indicates the lowest order of δ in these hidden terms. If δ is sufficiently small, a reasonable approximation to the derivative may be obtained by simply neglecting all the terms represented by the "O" term. Thus, the central-difference approximation is given by:

$$\left.\frac{df(x)}{dx}\right|_{x=x_0}=\frac{f\left(x_0+\frac{\delta}{2}\right)-f\left(x_0-\frac{\delta}{2}\right)}{\delta}. \tag{2.6}$$

Note that the central difference provides an approximation of the derivative of the function at x_0, but the function is not actually sampled there. Instead, the function is sampled at the neighboring points $x_0+\delta/2$ and $x_0-\delta/2$. Since the lowest power of δ being ignored is second order, the central difference is said to have second-order accuracy or second-order behavior. This implies that if δ is reduced by a factor of 10, the error in the approximation should be reduced by a factor of 100 (at least approximately). In the limit as δ goes to zero, the approximation becomes exact.

One can construct higher-order central differences. In order to get higher-order behavior, more terms, in example, more sample points, must be used. The use of higher-order central differences in FDTD schemes is certainly possible, but there are some complications which arise because of the increased "stencil" of the difference

REFERENCES

1. Taflove, A. and Hagness, S. C., *Computational Electrodynamics: The Finite – Difference Time – Domain method*, 3rd Edition, Artech House, Inc., 2005.
2. Sullivan, D. M. (2013). *Electromagnetic simulation using the FDTD method*. John Wiley & Sons.
3. Schneider, J. B. (2010). Understanding the finite-difference time-domain method. *Scholl of electrical engineering and computer science Washington State University.–URL: http://www. eecs. wsu. edu/~ schneidj/ufdtd/(request data: 29.11. 2012).*
4. Yee, K. S., *Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media*. IEEE Transactions on Antennas and Propagation 14 (3): 302–307, 1996.
5. Taflove, A. *Application of the finite-difference time-domain method to sinusoidal steady state electromagnetic penetration problems*. IEEE Transactions on Electromagnetic Compatibility 22 (3): 191–202, 1980.
6. Bi, Z., Wu, K., Wu, C., and Litva, J., *Accurate Characterization of Planar Printed Antennas using Finite – Difference Time – Domain method*, IEEE Transactions on Antennas and Propagation, vol. 40, May 1992.
7. Loui, H. 1D-FDTD using MATLAB.
8. Xiao, Y., Maywar, D. N., and Agrawal, G. P. (2014). Reflection and transmission of electromagnetic waves at a temporal boundary. *Optics letters*, *39*(3), 574-577.
9. Sandeep, S., *Broadband analysis of microstrip patch antenna using 3D FDTD – UPML*, University of Colorado at Boulder, USA, ECEN 5134 - Term paper, 2008.
10. Schneider, J. B. (2004). Plane waves in FDTD simulations and a nearly perfect total-field/scattered-field boundary. *Antennas and Propagation, IEEE Transactions on*, *52*(12), 3280-3287.

11. Hu, F. Q. (2004). Absorbing boundary conditions. *International Journal of Computational Fluid Dynamics*, *18*(6), 513-522.
12. Zweki, M., Abd-Alhameed, R. A., Mangoud, M. A., Excell, P. S., and Vaul, J. A., *Broadband Analysis of Finite Microstrip Patch Antenna Structure using FDTD ",* 11th International Conference on Antennas and Propagation, UMIST, Manchester, UK, April 2001.
13. Sheen, D. M., Ali, S. M., Abouzahra, M. D., and Kong, J. A., *Application of the Three–Dimensional Finite–Difference Time–Domain Method to the Analysis of Planar Microstrip Circuits*, Microwave Theory and Techniques, IEEE Transactions on, 38(7), 849-857, July 1990.
14. Taflove, A. and Brodwin, M. E., *Numerical solution of steady-state electromagnetic scattering problems using the time-dependent Maxwell's equations*. Microwave Theory and Techniques, IEEE Transactions on, *23*(8), 623-630, 1975.
15. Taflove, A. and Brodwin, M. E. *Computation of the electromagnetic fields and induced temperatures within a model of the microwave-irradiated human eye.* IEEE Transactions on Microwave Theory and Techniques 23 (11): 888–896, 1975.
16. Holland, P. R., *Threde: A free-field EMP coupling and scattering code*. IEEE Transactions on Nuclear Science 24 (6): 2416–2421, 1977.
17. Berenger, J. P., *A perfectly matched layer for the absorption of electromagnetic waves.* Journal of computational physics, 114(2), 185-200., Oct.1994.
18. Mur, G., *Absorbing boundary conditions for the finite-difference approximation of the time-domain electromagnetic-field equations.* Electromagnetic Compatibility, IEEE Transactions on, (4), 377-382., Nov. 1981.
19. Fang, J. (1994). Absorbing boundary conditions applied to model wave propagation in microwave integrated-circuits. *Microwave Theory and Techniques, IEEE Transactions on*, *42*(8), 1506-1513.
20. Sandeep, S., *Electromagnetic simulation of radiometer calibration targets using Finite Difference Time Domain (FDTD) method*, University of Colorado at Boulder, USA, ECEN 5264 - Term paper, 2009.

21. Merewether, D. E., Fisher, R., and Smith, F. W. (1980). On implementing a numeric Huygen's source scheme in a finite difference program to illuminate scattering bodies. *Nuclear Science, IEEE Transactions on*, *27*(6), 1829-1833.

22. Umashankar, K. R., and Taflove, A., *A novel method to analyze electromagnetic scattering of complex objects*, IEEE Trans. Electromagnet. Compat., Vol. 24, 1982, pp. 397 – 405.

23. Ramli, K. N. (2011). *Modeling and analysis of complex electromagnetic problems using FDTD subgridding in hybrid computational methods* (Doctoral dissertation, University of Bradford).

24. Dahlan, S. H. (2012). *Contribution to the body of revolution finite-difference time domain (BoR-FDTD) method-implementations of hybrid and multi-resolution approaches for fast simulation of electrically large axis-symmetrical antenna structures* (Doctoral dissertation, Universite De Rennes).

25. Gedney, S. D. (2011). Introduction to the Finite-difference Time-domain (FDTD) Method for Electromagnetics. *Synthesis Lectures on Computational Electromagnetics*, *6*(1), 1-250.

26. Katz, D. S., Thiele, E. T., and Taflove, A. (1994). Validation and extension to three dimensions of the Berenger PML absorbing boundary condition for FDTD meshes. *Microwave and Guided Wave Letters, IEEE*, *4*(8), 268-270.

27. Ziolkowski, R. W., and Heyman, E. (2001). Wave propagation in media having negative permittivity and permeability. *Physical review E*, *64*(5), 056625.

28. Kosmas, P. (2007, October). Review of an FDTD TF/SF Formulation Based on Auxiliary 1-D grids. In *Computational Electromagnetics in Time-Domain, 2007. CEM-TD 2007. Workshop on* (pp. 1-4). IEEE.

29. Barros, F. J. B., Santos, R. O., Araújo, J. S., and Sobrinho, C. L. S. S. (2003, October). FDTD code for determination of the radiation pattern of an aperture antenna using the magnetic vector potential and parallel processing. In *Applied Electromagnetics and Communications, 2003. ICECom 2003. 17th International Conference on* (pp. 441-444). IEEE.

30. Zhao, A. P., & Raisanen, A. V. (1996). Application of a simple and efficient source excitation technique to the FDTD analysis of waveguide and microstrip circuits. *Microwave Theory and Techniques, IEEE Transactions on*, *44*(9), 1535-1539.

31. Sun, W., Loeb, N. G., & Fu, Q. (2002). Finite-difference time-domain solution of light scattering and absorption by particles in an absorbing medium. *Applied optics*, *41*(27), 5728-5743.

32. Engquist, B., and Majda, A. (1977). Absorbing boundary conditions for numerical simulation of waves. *Proceedings of the National Academy of Sciences*, *74*(5), 1765-1766.

33. Mur, G. (1998). Total-field absorbing boundary conditions for the time-domain electromagnetic field equations. *Electromagnetic Compatibility, IEEE Transactions on*, *40*(2), 100-102.

34. Hui, W., Zhi-Xiang, H., Xian-Liang, W., and Lei, W. (2011, July). Perfect plane wave injection into 3D FDTD (2, 4) scheme. In *Cross Strait Quad-Regional Radio Science and Wireless Technology Conference (CSQRWC), 2011* (Vol. 1, pp. 40-43). IEEE.

35. Gurel, L. E. V. E. N. T., OGuz, U. G. U. R., and Arikan, O. R. H. A. N. (1997, July). Accurate plane-wave excitation in the FDTD method. In *Antennas and Propagation Society International Symposium, 1997. IEEE., 1997 Digest* (Vol. 4, pp. 2164-2167). IEEE.

36. Rappaport, T. S. (1996). *Wireless communications: principles and practice* (Vol. 2). New Jersey: prentice hall PTR.

37. Free-Space, B. P. C. (2005). Michael Marcus and Bruno Pattan.

38. Andrews, L. C., and Phillips, R. L. (2005). *Laser beam propagation through random media* (Vol. 10, No. 3.626196). Bellingham: SPIE press.

39. Jackson, J. D., and Jackson, J. D. (1962). *Classical electrodynamics* (Vol. 3). New York etc.: Wiley.

40. Wang, W. C. (1986). Electromagnetic wave theory.

41. Ishimaru, A. (1991). *Electromagnetic wave propagation, radiation, and scattering*. Prentice-Hall.

42. Smith, D. R., & Schurig, D. (2003). Electromagnetic wave propagation in media with indefinite permittivity and permeability tensors. *Physical Review Letters*, *90*(7), 077405.

43. Chew, W. C. (1995). *Waves and fields in inhomogeneous media* (Vol. 522). New York: IEEE press.

44. Ulaby, F. T. (2005). *Electromagnetics for engineers*. Pearson/Prentice Hall.

45. Sadiku, M. N. (2000). *Numerical techniques in electromagnetics*. CRC press.