ANALYTICAL APPROACH TO UNIDIRECTIONAL FLOW OF NON-NEWTONIAN FLUIDS OF DIFFERENTIAL TYPE

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This thesis is declared to all those devoted their life in one way or the other on theoretical investigation in fluids mechanics

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ABSTRACT

This thesis is regarding the development of mathematical models and analytical techniques for non-Newtonian fluids of differential types on a vertical plate, horizontal channel, vertical channel, capillary tube and horizontal cylinder. For a vertical plate, a mathematical model of the unsteady flow of second-grade fluid generated by an oscillating wall with transpiration, and the problem of magnetohydrodynamic (MHD) flow of third-grade fluid in a porous medium, have been developed. General solutions for the second-grade fluid are derived using Laplace transform, perturbation and variable separation techniques, while for the third-grade fluid are derived using symmetry reduction and new modified homotopy perturbation method (HPM). For a horizontal channel, a new analytical algorithm to solve transient flow of third-grade fluid generated by an oscillating upper wall has been proposed. A new approach of the optimal homotopy asymptotic method (OHAM) have been proposed to solve steady mixed convection flows of fourth-grade fluid in a vertical channel. The accuracy of the approximate solution is achieved through the residual function. For a capillary tube, two flow problems of the second-grade fluid were developed. Firstly, oscillating flow and heat transfer driven by a sinusoidal pressure waveform, and secondly, free convection flow driven due to the reactive nature of the viscoelastic fluid. The solutions for the first problem were derived using Bessel transform technique while for the second problem by using a new modified homotopy perturbation transform method. For a horizontal cylinder, an unsteady third-grade fluid in a wire coating process inside a cylindrical die is developed. A special case of the problem is obtained for magnetohydrodynamic flow with heat transfer for second-grade fluid. Both of these two problems are solved using a new modified homotopy perturbation transform method. Data, graph and solutions obtained are shown and were found in good agreement with previous studies.

ABSTRAK

Tesis ini adalah mengenai pembangunan model matematik dan teknik analisis untuk cecair bukan Newtonian daripada pengkamiran jenis di atas pinggan menegak, mendatar saluran, saluran menegak, tiub kapilari dan silinder mendatar. Untuk plat menegak, satu model matematik bagi aliran tak mantap bendalir kedua-gred dihasilkan oleh dinding berayun dengan transpirasi, dan masalah magnetohydrodynamic (MHD) Aliran bendalir ketiga gred di medium berliang, telah dibangunkan. Penyelesaian am bagi cecair kedua-gred yang diperolehi dengan menggunakan jelmaan Laplace, usikan dan pemisahan pembolehubah teknik, manakala bagi cecair ketiga-gred yang diperolehi dengan menggunakan pengurangan simetri dan baru diubah suai kaedah usikan homotopi (HPM). Untuk mendatar saluran, algoritma analisis baru untuk menyelesaikan aliran fana cecair gred ketiga yang dihasilkan oleh dinding atas berayun telah dicadangkan. Pendekatan baru kaedah asimptot homotopi optimum (OHAM) telah dicadangkan untuk menyelesaikan mantap aliran olakan campuran cecair keempat gred dalam saluran menegak. Ketepatan penyelesaian hampir dicapai melalui fungsi baki. Untuk tiub kapilari, dua masalah aliran bendalir kedua-gred telah dibangunkan. Pertama, didorong aliran dan haba berayun pemindahan oleh tekanan gelombang sinus, dan kedua, perolakan percuma mengalir didorong kerana sifat reaktif cecair likat kenyal itu. Penyelesaian untuk masalah pertama telah diperolehi dengan menggunakan Bessel mengubah teknik manakala bagi masalah kedua dengan menggunakan baru diubahsuai pengusikan homotopi mengubah kaedah. Untuk mendatar silinder, bendalir ketiga gred tak mantap dalam proses salutan wayar di dalam sebuah silinder die dibangunkan. Satu kes khas masalah diperolehi untuk aliran magnetohydrodynamic dengan haba memindahkan cecair untuk kedua-gred. Kedua-dua masalah adalah diselesaikan menggunakan homotopi baru diubahsuai pengusikan mengubah kaedah. Data, graf dan penyelesaian yang diperolehi ditunjukkan dan tidak terdapat dalam perjanjian yang baik dengan kajian sebelum ini.

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LIST OF ABBREVIATIONS

HPM	-	Homotopy perturbation method
OHAM	-	Optimal homotopy asymptotic method
ADM	-	Adomian decomposition method
HPTM	-	Homotopy perturbation transform method
MHD	-	Magnetohydrodynamic flow
HAM	-	Homotopy analysis method

NOMENCLATURE

Roman Letters

a	-	Lower limit of range integration
a_n	-	Constant defined by equation (8.24)
$\mathbf{a}_z$	-	Unit vector along $z-$axis
A	-	Constant pressure gradient
A_e	-	Rate constant
$\mathbf{A}_i\,(i = 1, 2, 3, 4)$	-	Revlin-Ericksen tensors
$A(r)$	-	Function defined by equation (8.24)
$A_1(r)$	-	Function defined by equation (8.87)
b	-	Upper limit of range integration
b_n	-	Constant defined by equation (8.25)
B_0	-	Applied magnetic field
B_1	-	Amplitude of the oscillatory pressure gradient
$B_1(r)$	-	Function defined by equation (8.88)
B_s	-	Amplitude of the steady pressure gradient
$\mathcal{B}$	-	Boundary differential operator
$B(r)$	-	Function defined by equation (8.25)
$\mathbb{B}(y,t)$	-	Source term arising form integration
Br	-	Brinkman number
c	-	Constant wave speed
c_n	-	Constant defined by equation (8.26)
c_p	-	Specific heat capacity at constant pressure
$C(r,t)$	-	Function defined by equation (8.26)
$C_1(r,t)$	-	Function defined by equation (8.89)
$C_2(r,t)$	-	Function defined by equation (8.90)
$C_i\,(i = 1, 2, ..., m)$	-	Auxiliary constants
C_F	-	Skin friction coefficient

C_0	-	Initial concentration of the reactant species
$\frac{d}{dt}$	-	Material time derivative
$D(r,t)$	-	Function defined by equation (8.27)
$D_2(r,t)$	-	Function defined by equation (8.92)
$\mathcal{D}$	-	Second order differential operator
$E_2(r,t)$	-	Function defined by equation (8.92)
$E(r,t)$	-	Function defined by equation (8.29)
E_c	-	Eckert number
E_A	-	Activation energy
${}_1F_1$	-	Confluent hypergeometric function (Kummer function)
g	-	Acceleration due to gravity
g_1, g_2	-	Known analytical function
g_3, g_4	-	Source terms
G_1	-	Differential operator for velocity field
G_2	-	Differential operator for temperature field
Gr	-	Grashof number
$h\left(q\right), h_1\left(q,y\right), h_2\left(q,y\right)$	-	Auxiliary functions
$h(r,t)$	-	Function defined by equation (8.84)
h	-	Reference length between two point
H_n	-	He polynomials
i	-	Imaginary units
$\mathbf{I}$	-	Identity tensor
J_n	-	Bessel function of fist kind and order n
$\mathbf{J} \times \mathbf{B}$	-	Magnetic body force
k	-	Thermal conductivity
k_0	-	Constant wall velocity
K	-	Permeability of the porous medium
l	-	Length of die
L	-	Linear operator
$\mathbf{L}$	-	$\nabla \mathbf{V}$
M	-	Magnetic parameter
N	-	Nonlinear operator
Nu	-	Nusselt number

p	-	Pressure gradient
p^*	-	Modified pressure gradient
P	-	Darcian parameter
Pr	-	Prandtl number
q	-	Embedding parameter
Q	-	Forchheimer parameter
Q_r	-	Heat reaction parameter
r	-	Transverse distance to flow direction
R_1	-	Radius of wire
R_2	-	Radius of die
R	-	Universal gas constant
$\mathbf{R}$	-	Darcy's resistance due to porous medium
$\mathcal{R}$	-	Linear differential operator of less order than $\mathcal{D}$
R_m	-	Residual error function
Re	-	Reynolds number
$S_1(r,t)$	-	Function defined by equation (8.85)
$S_2(r,t)$	-	Function defined by equation (8.86)
$S(x,s)$	-	Kernel of the transform
t	-	Time
$\mathbf{T}$	-	Cauchy stress tensor
u_0	-	Initial approximation for velocity
u_s	-	Steady velocity
u_t	-	Transient velocity
u	-	Velocity field
U_0	-	Characteristic velocity
U_1	-	Dimensional fluid velocity of wire
U_2	-	Dimensional fluid velocity of die
$\mathbf{V}$	-	Velocity vector
V_0	-	Amplitude of wall oscillations
V_w	-	Dimensional transpiration velocity
W	-	Solution of Kummer's equation

W_1	-	First particular solution of Kummer equation
W_2	-	Second particular solution of Kummer equation
X_1	-	Time translation
X_2	-	Space translation
X_3	-	Third prolongation operator
$X_1 - cX_2$	-	Wave-front type travelling solutions
X, Y	-	Banach spaces
x, y	-	Perpendicular distances
z	-	axial position

Greek Letters

$\alpha_1, \alpha_2, \alpha$	-	Viscoelastic parameters of second-grade fluid
$\beta_1, \beta_2, \beta_3, \beta$	-	Viscoelastic parameters of third-grade fluid
β_c	-	Separation constant
β_T	-	Volumetric coefficient of thermal expansion
γ_0	-	Amplitude of the pressure
$\gamma_1, \gamma_2 ..., \gamma_6, \gamma_7, \gamma$	-	Viscoelastic parameters of fourth-grade fluid
γ_c	-	Euler-Mascheroni constant
$\delta(t)$	-	Dirac delta function
ϵ	-	Activation energy parameter
ε	-	Perturbation parameter
ζ	-	Radii ratio
θ_0	-	Bulk fluid temperature
θ	-	Temperature field
θ_s	-	Steady temperature
θ_t	-	Transient temperature
θ_w	-	Wall temperature
λ	-	Non-dimensional reactant consumption parameter
μ	-	Dynamic viscosity
ν	-	Kinematic viscosity
ξ	-	Non-dimensional transpiration velocity

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