

STRESS INTENSITY FACTORS OF EDGE CRACKS IN DISSIMILAR JOINT
PLATES

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To my parents and my beloved family



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ABSTRACT

Nowadays, there are many applications in which need the combination or different materials. The development of this is caused by the mechanical wear problem, a high temperature situation or other conditions in which different properties are required from different parts of the same applications. This problem brings about the need for joining dissimilar materials. However the combination process between two dissimilar materials can caused mechanical mismatch and may lead to catastrophic failure or crack. Hence, this study will focused on the stress intensity factor on edge crack between dissimilar joint plates. The study focused on the investigation of edge crack behavior of dissimilar joint plates and to find out stress intensity factor (SIF) of dissimilar joint plates under different conditions by using finite element software. In this research, the investigation simulation are conducted by using finite element analysis software, ANSYS. A program that consist a coding of joining dissimilar material with centre and offset edge crack have been developed using ANSYS software. Data of stress intensity factor, K produced by ANSYS software then transform to dimensionless stress intensity factor, F . Relationship between mechanical mismatch, α , ratio of stress, β , relative crack depth, a/w and relative offset distance, b/h to the dimensionless SIF, F are analyze and discussed.

ABSTRAK

Pada masa kini, terdapat banyak aplikasi yang memerlukan gabungan 2 atau lebih bahan-bahan yang berbeza. Perkembangan ini disebabkan oleh masalah mekanikal bahan, keadaan suhu yang tinggi atau keadaan lain di mana sifat yang berbeza diperlukan dari bahagian yang berlainan pada aplikasi yang sama. Masalah ini telah membawa kepada keperluan bagi menyambungkan dua bahan yang berbeza dalam satu aplikasi yang sama. Walau bagaimanapun proses gabungan antara dua bahan yang tidak serupa boleh menyebabkan terjadinya ketidak padanan mekanikal dan boleh menyebabkan kegagalan bencana atau retak. Oleh itu, kajian ini akan memberi tumpuan kepada faktor keamatan tekanan pada retak di antara dua gabungan bahan yang berbeza. Kajian ini memberi tumpuan kepada kelakuan retak tepi kepada gabungan bahan yang berbeza seterusnya mencari nilai faktor keamatan tekanan yang mungkin terjadi kepada gabungan bahan berbeza ini dengan kehadiran pelbagai parameter berbeza. Dalam kajian ini, simulasi ujian dibuat dengan menggunakan perisian, ANSYS. Satu program yang mengandungi koding bagi menghasilkan gabungan bahan berbeza dengan kehadiran retakan di tepi telah dibangunkan menggunakan perisian ANSYS. Data faktor keamatan tekanan, K yang dihasilkan oleh perisian ANSYS kemudian diubah kepada faktor keamatan tekanan tanpa dimensi, F . Seterusnya, hubungan antara sifat mekanikal berbeza, α , nisbah tekanan, β , nisbah kedalaman retak, a/w dan nisbah jarak retak, b/h , kepada faktor keamatan tekanan tanpa dimensi, F dianalisis dan dibincangkan.

CONTENTS

TITLE	i
DECLARATION	ii
DEDICATION	iii
ACKNOWLEDGEMENT	iv
ABSTRACT	v
ABSTRAK	vi
CONTENTS	vii
LIST OF FIGURES	x
LIST OF TABLES	xii
LIST OF SYMBOLS	xiv
CHAPTER 1 INTRODUCTION	1
1.1 Background of study	1
1.2 Problem Statement	2
1.3 Project objectives	3
1.4 Project scopes	3
1.5 Summary	

CHAPTER 1 LITERATURE REVIEW	5
2.1 Overview	5
2.2 Fracture mechanics	5
2.2.1 Fracture process	6
2.2.2 Fracture modes	7
2.2.3 Elementary fracture mechanics	8
2.3 Stress intensity factor	11
2.3.1 Stress intensity factors using numerical method	13
2.4 Finite element analysis	19
2.4.1 Basic steps in finite element analysis	21
2.4.2 ANSYS	21
CHAPTER 3 METHODOLOGY	23
3.1 Introduction	23
3.2 ANSYS simulation	23
3.2.1 Modeling assumptions	25
3.2.2 ANSYS analysis approach	28
3.3 Verification	31
3.4 Design of experiment	33
3.4.1 A centre edge crack between dissimilar material	33
3.4.2 An offset edge crack between dissimilar material	34
CHAPTER 4 RESULT & DISCUSSION	36
4.1 Overview	36
4.2 Verification	37
4.3 Centre edge crack in dissimilar material simulation	39
4.4 Offset edge crack in dissimilar material simulation	41
4.4.1 Effect of mechanical mismatch, α to the value of dimensionless SIF, F	41
4.4.2 Effect of ratio of stress, β to the dimensionless SIF, F	45

	ix
4.4.3 Effect of relative crack depth, a/w and relative offset distance, b/h to the value of dimensionless SIF, F	48
CHAPTER 5 CONCLUSION	57
5.1 Overview	57
5.2 Conclusion	57
REFERENCES	59
APPENDIXES	62



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LIST OF FIGURES

2.1	Modes of crack	8
2.2	Effect of thickness on K_C	10
2.3	Failure stress against fracture toughness graph	10
2.4	Coordinate system around an interface crack	12
2.5	Interface crack at bi material specimen	12
2.6	Geometrical configuration of bi-material bonded plate	13
2.7	The edge interface crack s in a bonded strip	13
2.8	Constant value of C1 and C2 for various combinations of material	15
2.9	Distribution of stress intensity factors	18
2.10	FEM model of meshes near crack tip	19
2.11	Step of process in numerical simulation	20
2.12	Example of ANSYS capabilities	22
3.1	Flowchart of the structural analysis by ANSYS	24
3.2	Geometrical configurations of parts	25
3.3	Edge crack on the joining line of material	26
3.4	Edge crack offset to material 1	26
3.5	Edge crack offset to material 2	27
3.6	Difference between exact analysis and FEM	27
3.7	Library of element type	28
3.8	Define material model behavior	28
3.9	Examples of nodes	29
3.10	Create key point in active coordinate system	29
3.11	Meshing element from ANSYS	30
3.12	Result of stress intensity factor	31

3.13	Simulation parts of centre edge crack between dissimilar material	34
3.14	Simulation parts of offset edge crack between dissimilar material	35
4.1	The comparison of verification data a) $\alpha = 2$, b) $\alpha = 4$ and c) $\alpha = 10$	38
4.2	Plotted graph of centre edge crack simulation (a) subjected to in plane tension, (b) subjected to bending moment, (c) subjected to both cases	40
4.3	Dimensionless SIF, F versus relative crack depth, a/w , (a) upper cracks, (b) lower cracks and (c) SIF ratio between upper and lower cracks.	44
4.4	Dimensionless SIF, F versus relative crack depth, a/w , (a) upper cracks, (b) lower cracks (c) combined cases upper and lower crack (d) SIF ratio between upper and lower cracks.	47
4.5	Dimensionless SIF, F versus relative crack depth, a/w , (a) upper cracks, (b) lower cracks (c) SIF ratio between upper and lower cracks.	50
4.6	Dimensionless SIF, F versus relative crack depth, a/w , (a) upper cracks, (b) lower cracks (c) SIF ratio between upper and lower cracks.	55
4.7	Dimensionless SIF, F versus relative crack depth, a/w , (a) upper cracks, (b) lower cracks (c) combined cases upper and lower crack (d) SIF ratio between upper and lower cracks.	56



LIST OF TABLES

2.1	Order of stress singularity for various combinations of materials	14
2.2	Values of C_1	16
2.3	Values of C_2	17
2.4	Stress intensity factors across thickness	18
3.1	Simulation condition for verification analysis of edge crack	32
3.2	Result for F of an edge crack between rectangular dissimilar material by Toshiro <i>et al</i> (2000)	32
3.3	Condition for simulation of centre edge crack between dissimilar material	33
3.4	Condition for simulation of offset edge crack between dissimilar material	35
4.1	Result of verification process	37
4.2	Result of center edge crack simulation with value of α is 1,2,3 and 4	39
4.3	Offset edge crack simulation data	41
4.4	The Dimensionless SIF value for upper material crack, F_1 of offset edge crack for $\beta = 1.0$ and α value 0.3, 1.0 and 3.0	42
4.5	The Dimensionless SIF value for lower material crack, F_2 of offset edge crack for $\beta = 1.0$ and α value 0.3, 1.0 and 3.0	43
4.6	The Dimensionless SIF value for upper material crack, F_1 of offset edge crack for different β , α and a/w value	45

4.7	The Dimensionless SIF value for lower material crack, F_2 of offset edge crack for different β , α and a/w value	46
4.8	The Dimensionless SIF value for upper material crack, F_1 of offset edge crack for $\beta = 0.5$ and $\alpha = 1$	48
4.9	The Dimensionless SIF value for lower material crack, F_2 of offset edge crack for $\beta = 0.5$ and $\alpha = 1$	49
4.10	The Dimensionless SIF value for F_1/F_2 of offset edge crack for $\beta = 0.5$ and $\alpha = 1$	49
4.11	The Dimensionless SIF value for upper material crack, F_1 of offset edge crack for $\beta = 0.5$ and $\alpha = 3$	51
4.12	The Dimensionless SIF value for lower material crack, F_2 of offset edge crack for $\beta = 0.5$ and $\alpha = 3$	52
4.13	The Dimensionless SIF value for F_1/F_2 of offset edge crack for $\beta = 0.5$ and $\alpha = 3$	52
4.14	The Dimensionless SIF value for upper material crack, F_1 of offset edge crack for $\beta = 0.5$ and $\alpha = 5$	53
4.15	The Dimensionless SIF value for lower material crack, F_2 of offset edge crack for $\beta = 0.5$ and $\alpha = 5$	53
4.16	The Dimensionless SIF value for F_1/F_2 of offset edge crack for $\beta = 0.5$ and $\alpha = 5$	54

LIST OF SYMBOLS

K	Stress intensity factor
E	Young modulus
ν	Poisson ratio
a	Length of crack
w	Width of model
b	Interval of crack
h	Height of model
K_C	Fracture toughness
σ	Tensile stress
F	Dimensionless stress intensity factor
t	Thickness
M	Moment
α	Mechanical mismatch
β	Ratio of stress
a/w	Relative crack depth
b/h	Relative offset distance

CHAPTER 1

INTRODUCTION

1.1 Background of study

Nowadays, there are many applications in which need the combination or different materials. The development of this is caused by the mechanical wear problem, a high temperature situation or other conditions in which different properties are required from different parts of the same applications. This problem brings about the need for joining dissimilar materials. However the combination process between two dissimilar materials can caused mechanical mismatch and may lead to catastrophic failure or crack.

Failure or crack is a conditions in which solid materials fail under the action of external loads. Crack has seemed like a main phenomenon in mechanics of materials. Crack can cause failure of a component especially on a joining and assembly process. Failure of materials will cause huge cost to the industries. What is more worrying is the failure or crack can lead to the accidents involving human life. Because of this, field known as fracture mechanics have been introduced to overcome this problem.

For the past 50 years, fracture mechanics have been introduced in accordance to the crack studies. Fracture mechanics methodology is based on the assumption that all engineering materials contain cracks from which failure starts. The estimation of the remaining life of machine or structural components requires knowledge of the redistribution of stresses caused by the introduction of cracks in conjunction with a crack growth condition. Cracks result in high stress elevation in

the neighborhood of the crack tip, which should receive particular attention since it is at that point that further crack growth takes place.

Cracks can be classified according to various criteria. First criteria is the origin of the crack. One need to classified either the cracks are due to shrinkage and temperature variations in restrained elements or due to load producing local tension. Other than this, crack also can be classified in accordance to its shape and pattern either it is a single crack, multiple crack or branching cracks. Third criteria is the position of the cracks. In general, there are three type of crack position which are the centre cracks, single edge crack or multiple edge cracks. Last criteria for classification of crack is the crack deformation modes which have four modes namely opening mode (mode I), sliding mode (mode II), tearing mode (mode III) and last mode which is the mixed mode.

1.2 Problem statement

The failure of cracked components is governed by the stresses in the vicinity of the crack tip. The singular stress contribution is characterized by the stress intensity factor, K . Stress intensity factors or also known as driving force for fracture is dependent on the geometry of the component and on the special loading conditions (tension, bending, thermal stresses, etc).

As the stress intensity factors is one of the main problem in studying the propagation of crack, this project focus on the study of stress intensity factors of offset edge cracks in dissimilar joint plates. Currently there is limited stress intensity factor for offset edge crack in literature especially for cracks occurred in dissimilar joint plates. Therefore this study focus on the stress intensity factors for offset edge cracks in the dissimilar joint plates under tension and bending loadings.

1.3 Project objectives

Based on the problem statement, there are two objectives for this study which are:

- i. To investigate edge crack behavior of dissimilar joint plates using finite element method
- ii. To find out stress intensity factor (SIF) of dissimilar joint plates under different conditions using ANSYS Software

1.4 Project scope

This study cover the edge crack modelling using ANSYS for finite element analysis.

The scope for study are:

- i. Each analysis involve two type of material with fixed value of modulus elasticity, E (200 GPa) for material one.
- ii. Two materials with different mechanical properties are joined with an assumption that both materials are elastic.
- iii. The cracks are located at the edge of dissimilar joint plates. Two conditions of cracked are assumed which are at the centre of the dissimilar joint plates and offset cracks.
- iv. Stress intensity factors result obtained by changing data of young modulus, E , for material 2, ratio of a/w , b/h and ratio of the pressure from tension loading to pressure from bending loading, β .
- v. The dimensionless stress intensity factors, F at the crack tips are calculated and discussed.

1.5 Summary

Rapid development in the field of manufacturing has seen many improvements have been made to improve the quality of human life. This includes the usage of several manufacturing processes which allows the joining process of two different materials in order to get better quality of the product. However, theoritically combination of materials usually will exposed to some continuos stress that allow some crack in the

joining area. This crack if not treated well will propagate and cause a very big impact to the human life.

To overcome this problem, the understanding of fracture mechanics, fundamental of fatigue and finite element method is important to ensure the successful for this research. The nature of crack tip core regions and stress intensity factors are important factors in understanding fracture mechanics. An assumption of two dimensional plane stress or plane strain delivers useful two dimensional results with reasonable accuracy. This research depends on the theory value of the stress intensity factors and verified by using ANSYS software for finite element analysis.



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CHAPTER 2

LITERATURE REVIEW

2.1 Overview

This chapter provide a comprehensive review related to the topic contain in this study. It explained on the concept of stress intensity factors in conjunction to the growth of the crack. In conclusion, this chapter explained further on the concept behind the crack initiation and crack growth related to the stress intensity factors by using numerical method which in this research by using ANSYS software.

2.2 Fracture mechanics

According to Gopichand *et al* (2012) fracture mechanics is a field of solid mechanics that deal with the mechanical behaviour of cracked bodies. Fracture is a problem that society has faced for as long as there have been man made structures.

Barsom & Rolfe (1999) in his book explained that fracture mechanics is a method of characterizing the fracture behaviour of sharply notched structural members (cracked or flawed) in terms that can be used directly by the engineer. Fracture mechanics is based on a stress analysis in the vicinity of a notch or crack. It can also be used to predict the crack approach a critical size in fatigue or by environmental influences. The fracture mechanics approach have three important variables which are :

- i. Fracture toughness of the material
- ii. Applied stress
- iii. Flaw size

Based on Anderson (2005) there are two alternatives approaches for fracture analysis. There are:

- i. Stress intensity approach

Each stress component is proportional to a single constant, K_I . This constant is called stress intensity factor. It is completely characterizes the crack tip conditions in a linear elastic material. The formula for stress intensity factor is given by,

$$K_I = F\sigma\sqrt{\pi a} \quad (2.1)$$

- ii. Energy criterion

The energy approach states that crack extension occurs when the energy available for crack growth is sufficient to overcome the resistance of the material. The materials resistance may include the surface energy, plastic work or other type of energy dissipation associated with a propagating crack.

2.2.1 Fracture process

Generally, fracture process occurs in a material in four steps as explained by Naman (2012). The steps are described below:

- i. The first step is local yielding in the vicinity of defects or material and geometric singularities. The degree of singularity has a major influence on the magnitude of the plastic zone and the stress concentration. In repeated loading, there is hardening, which raises the yield stress, σ_y . The material located near the notch tip becomes very strong, resulting in the creation of a first crack.
- ii. Second step is the formation of cracks. This step can be due to surface treatments, with the treatment or thermal loading generating residual stresses

well above the yield strength. The material may also have cracks from static or variable mechanical loading.

- iii. The third step is the real beginning of cracks. This propagation can be sudden or successive. Often there is successive propagation with the size of the crack increasing until it reaches a critical size, causing sudden propagation.
- iv. The final step is the sudden propagation. It may be accompanied by generalized large strain (necking) or can occur without significant strain for brittle fracture.

2.2.2 Fracture modes

From a macroscopic point of view, there are two main types of fracture which are plane fracture and inclined fracture. Plane fracture corresponds to a flat fracture surface that is generally perpendicular to the direction of maximum principal stress. While inclined fracture presents a crack angle in the direction transverse to the direction of propagation. It is often accompanied by large strains.

For a plate with a through thickness crack, the loading on the crack is typically described as one of three types or modes. The modes of crack are described as below:

- i. Mode I : Crack opening mode, where the displacements at the lips of the crack are perpendicular to the direction of propagation.
- ii. Mode II : In plane shear mode, where the displacements at the lips of the crack are parallel to the direction of propagation.
- iii. Mode III : Out of plane shear mode, where the displacements at the lips of the crack are parallel to the toe of the crack.

In addition, the crack may be simultaneously subjected to a combination of these loading modes, known as mixed mode loading.

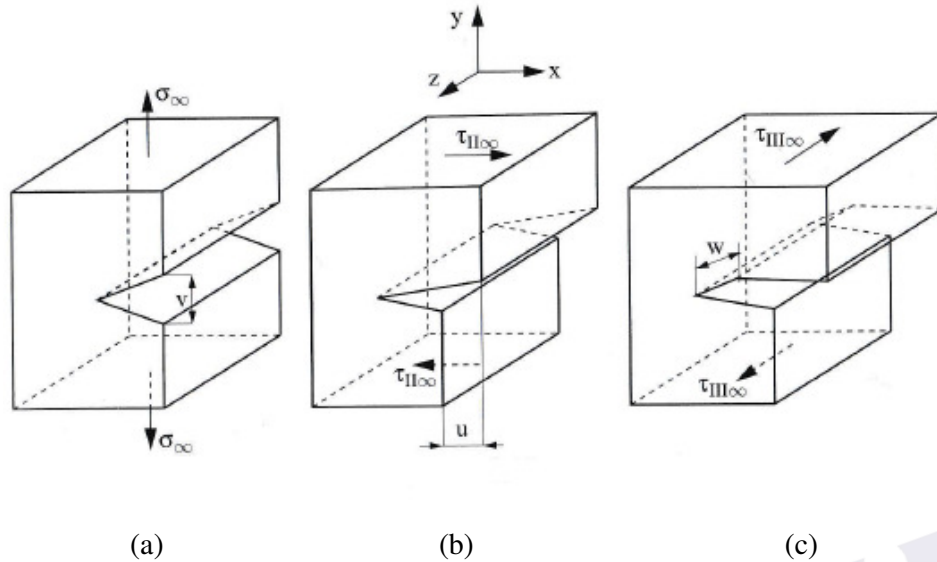


Figure 2.1 : Modes of crack (a) mode I, (b) mode II and (c) mode III
(Naman, 2012)

2.2.3 Elementary fracture mechanics

The geometries of cracks, with radius of curvatures approaching zero at the crack tip cause stress fields that approach infinity proportional to the reciprocal of the square root of the distance from the crack tip (Bykov, 1984). This occurs even at low load levels. As such, commonly used failure measures such as Von Mises are not applicable (Shukla & Dally, 2010). As stated by Xian-Kui & Joyce (2012), the stress intensity factor, K or also known as SIF was first proposed by Irwin (1957) and can be thought of as a measure of the effective local stress at the crack tip. An increasing stress intensity factor, K indicates the stress near the crack tip is increasing. With this linear elastic fracture mechanics approach of characterizing the crack tip stresses, small amounts of plasticity may be viewed as taking place within the crack tip stress field and neglected for the characterization (Paris & Sih, 1965). Stress intensity factor, K is designated by the mode of loading, such as K_I , K_{II} and K_{III} . Stress intensity factor, K is usually expressed in the following units:

1. MPa \sqrt{m} for ISO units
2. ksi \sqrt{in} for imperial units

Stress intensity factor, K can be determined using closed form solutions, finite element analysis and a number of other techniques. The solutions relate the remote loading, geometry of the specimen and the crack size to the stress intensity factor, K . Using the stress intensity factor in design requires knowledge of the critical stress intensity factor or fracture toughness, K_C .

The critical stress intensity factor or fracture toughness, K_C is a mechanical property that measures a materials resistance to fracture. Fracture toughness is used in structural integrity assessment, damage tolerance design, fitness for service evaluation, and residual strength analysis (Xian-Kui *et al*, 2012). As stated before, K_C is further expressed according to the loading mode, such as K_{IC} , K_{IIC} , K_{IIIC} for mode I, II and III respectively. When the stress intensity factor reaches the materials structure toughness an existing crack will undergo unstable crack extension (Shukla *et al*, 2010). Since K_C is material specific its value must be determined for each material of concern. Further, K_C can vary with temperature, component thickness and strain rate.

The critical stress intensity factor, K_C is strongly dependent on plate thickness (Szab & Babuska, 2011). For thin plates it is often the case that the plastic zone around a crack is on the order of the plate thickness. This allows K_C to reach a maximum value ($K_{C(max)}$). As plate thickness increase, the size of the plastic zone decreases lowering the toughness of the material to some level *below* $K_{C(max)}$. As plate thickness continue to increase, the plastic zone size becomes constant and K_C reaches an asymptotic value $K_{C(min)}$, known as plane strain fracture toughness (Anderson, 2005). This is shown in Figure 2.2.

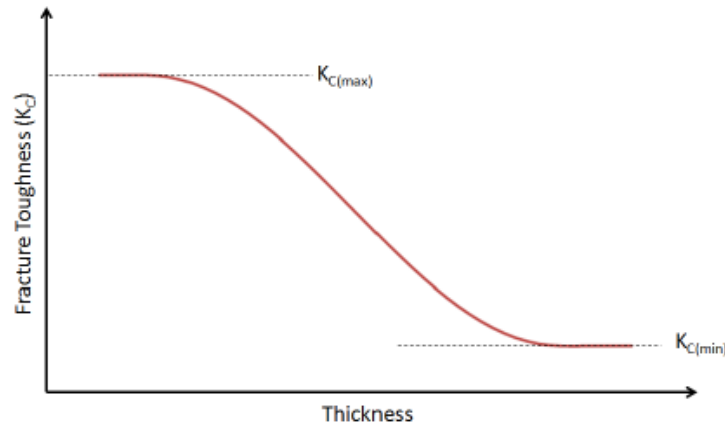


Figure 2.2 : Effect of thickness on K_C (Anderson, 2005)

Besides thickness, the fracture toughness property is analogous to the yield strength property. In tensile test, the material sustain is a stress and will remain elastic until the stress level applied exceeds the yield strength. If yield strength is used as failure criterion, the material fails after the stress level surpasses the yield strength of the material.

For low toughness materials, brittle fracture is the governing failure mechanism and critical stress varies linearly with fracture toughness K_{IC} . Figure 2.3 shows the effect of fracture toughness on the governing failure mechanism.

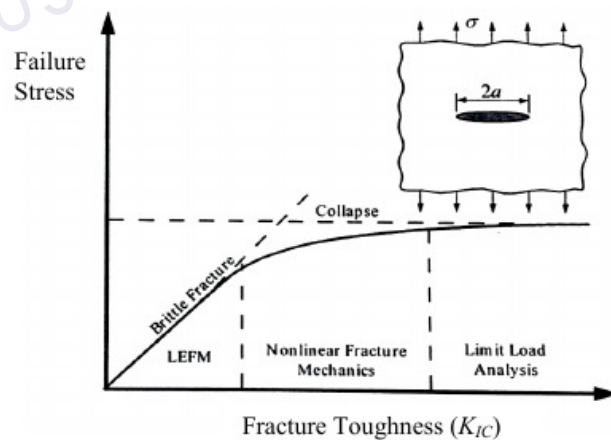


Figure 2.3 : Failure stress against fracture toughness graph (Anderson, 2005)

From the figure, it shows that failure will occur when the value of $K = K_{IC}$ where K is the driving force for fracture and K_{IC} is a measure of material resistance.

2.3 Stress Intensity Factor

Stress intensity factors is a measure of the stress field intensity near the tip of an ideal crack in a linear-elastic solid when the crack surfaces are displaced in an opening mode (Xian-Kui *et al*, 2012) Stress intensity factors can be determined for certain cases if the geometry and remote loading is known. By using a method developed by Westergaard (1930), Irwin (1957) found that the stress and displacement fields in the vicinity of crack tips subjected to the three deformation modess. However, as this research focused on the stresses applied on the plate, the formula for mode I stress intensity factors was given below.

For mode I:

$$\sigma_x = \frac{K_I}{(2\pi r)^{1/2}} \cos \frac{\Theta}{2} \left[1 - \sin \frac{\Theta}{2} \sin \frac{3\Theta}{2} \right] \quad (2.2)$$

$$\sigma_y = \frac{K_I}{(2\pi r)^{1/2}} \cos \frac{\Theta}{2} \left[1 + \sin \frac{\Theta}{2} \sin \frac{3\Theta}{2} \right] \quad (2.3)$$

$$\tau_{xy} = \frac{K_I}{(2\pi r)^{1/2}} \sin \frac{\Theta}{2} \cos \frac{\Theta}{2} \cos \frac{3\Theta}{2} \quad (2.4)$$

$$\sigma_z = \nu(\sigma_x + \sigma_y) \quad (2.5)$$

Based on Erdogan (1965), stress intensity factor of an interface crack is the distribution of stress around an interface crack tip. In the coordinate system in Figure 2.4 the stress formula along the x_1 axis near an interface crack tip is:

$$\sigma_{22} + i\sigma_{12} = \frac{K_I + iK_{II}}{\sqrt{2\pi r}} \frac{r^{i\alpha}}{1_k} \quad (2.6)$$

where K_I and K_{II} are the mode I and II stress intensity factors of an interface crack respectively.

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