A NEW EMPIRICAL MODEL TO PREDICT STRESS INTENSITY FACTOR FOR DOUBLE INTERACTING SURFACE CRACKS LOCATED IN HOLLOW CYLINDER

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A thesis submitted in
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I hereby declare that the work in this thesis is my own except for quotations and summaries which have been duly acknowledged.

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DEDICATION

Special dedication to my beloved father and mother for their boundless efforts and support
To my brothers and sisters
To my wife Juriya, and my angels Maria, Zahraa, and Farah
To my English teacher Mehar
To my entire friends
Thanks for everything

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Firstly, all praise and thanks to Allah for His Divine Guidance and help in completing this research project.

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Finally, to others who have contributed either directly or indirectly towards the successful completion of this research. May Allah blessing give, and happiness is upon you always. Amin

ABSTRACT

Fracture in cylinders is one of the most popular types of failure. Owing to the impact of production processes, nondestructive testing, and severe operational conditions, etc., cracks exist. The cracks could be detected in single or multiple form, where multiple cracks considered among the significant concerns that cylinders expected to experience. This is because in the existence of multiple neighboring cracks, crack interaction can take place between cracks and accelerates fracture process and lead to a catastrophic failure. Consequently, this study focuses on the problem of double interacting surface cracks located on external and internal surfaces of a hollow cylinder and oriented into parallel and non-coplanar parallel cracks configuration. Stress Intensity Factor (SIFs) has been chosen as the driving force to define the crack interaction. The SIFs have been analyzed for a wide variety of crack geometry, and cylinder type as well as separation distances utilizing finite element software Ansys under different types of mechanical loadings. Based on the analysis results, an empirical mathematical model was produced to predict the SIFs for double parallel cracks using the SIFs for a single crack, for thick and thin cylinders, separately. The empirical model was verified in terms of performance evaluation metrics, which exhibited prediction error less than 5%. Also, it is shown that crack interaction influence for parallel cracks demonstrated by shielding interaction influence only, while both shielding, and amplification impacts produced for non-coplanar cracks. The crack separation distance (horizontal and angular) between the cracks displayed substantial influence on interaction since it exhibited the ability to convert the interaction behavior from shielding to amplification impact (for angular). The presented results in this research serve the literature database since SIFs for a wide variety of cracks geometry have been introduced under different types of loading. Besides, the proposed mathematical model could be used easily and confidently as it displayed a high rate of accuracy.

ABSTRAK

Keretakan pada silinder adalah salah satu jenis kegagalan yang paling popular. Oleh kerana kesan proses pengeluaran, ujian tidak merosakkan, dan keadaan operasi yang teruk, dan lain-lain, terdapat keretakan. Retakan tersebut dapat dikesan dalam bentuk tunggal atau berganda, di mana beberapa retakan dianggap antara kebimbangan besar yang diharapkan dapat dialami oleh silinder. Ini kerana dengan adanya banyak retakan tetangga, interaksi retak dapat berlaku antara retakan dan mempercepat proses patah tulang dan mengakibatkan kegagalan bencana. Oleh yang demikian, kajian ini memfokuskan pada masalah keretakan permukaan berinteraksi berganda yang terletak di permukaan luaran dan dalaman silinder berongga dan berorientasikan kepada konfigurasi retak selari dan bukan koplanar. Stress Intensity Factor (SIF) telah dipilih sebagai pendorong untuk menentukan interaksi retak. SIF telah dianalisis untuk pelbagai jenis geometri retak, dan jenis silinder serta jarak pemisahan menggunakan perisian elemen hingga Ansys di bawah pelbagai jenis beban mekanikal. Berdasarkan hasil analisis, model matematik empirikal dihasilkan untuk meramalkan SIF untuk retak selari berganda menggunakan SIF untuk satu retakan, untuk silinder tebal dan nipis, secara berasingan. Model empirikal disahkan dari segi metrik penilaian prestasi, yang menunjukkan ralat ramalan kurang dari 5%. Juga, ditunjukkan bahawa pengaruh interaksi retak untuk retak paralel ditunjukkan dengan pengaruh interaksi perisai hanya, sementara kedua-dua pelindung, dan kesan penguatan dihasilkan untuk retakan bukan koplanar. Jarak pemisahan retak (mendatar dan sudut) antara retakan menunjukkan pengaruh yang besar terhadap interaksi kerana menunjukkan keupayaan untuk mengubah tingkah laku interaksi dari perisai ke kesan amplifikasi (untuk sudut). Hasil yang disajikan dalam penyelidikan ini melayani pangkalan data literatur kerana SIF untuk berbagai geometri retak telah diperkenalkan di bawah berbagai jenis pemuatan. Selain itu, model matematik yang dicadangkan dapat digunakan dengan mudah dan yakin kerana ia menunjukkan tahap ketepatan yang tinggi.

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LIST OF SYMBOLS AND ABBREVIATIONS

a	-	Crack depth (mm)
a/c	-	Crack aspect ratio
a/t	-	Relative crack depth
c	-	Half crack length (mm)
D_i	-	Internal diameter (mm)
D_o	-	Outer diameter (mm)
E	-	Young's modulus
$F_{Ben-EXT}$ $F_{Ben-INT}$	-	Normalized SIFs for external and internal cracks under bending
$F_{EQV-EXT}$ $F_{EQV-INT}$	-	Normalized equivalent SIFs for external and internal cracks under mixed mode
F_{t-EXT} F_{t-INT}	-	Normalized SIFs for external and internal cracks under tension
$F_{Tor\text{-}II-EXT}$ $F_{Tor\text{-}II-INT}$	-	Normalized mode II SIFs for external and internal cracks under torsion
$F_{Tor\text{-}III-EXT}$ $F_{Tor\text{-}III-INT}$	-	Normalized mode III SIFs for external and internal cracks under torsion
K	-	Stress intensity factor
L	-	Length (mm)
MAE	-	Mean Absolute Error
MAPE	-	Mean Absolute Percentage Error
Q	-	Shape factor for an elliptical crack
R^2	-	Coefficient of determination
s	-	Horizontal separation distance (mm)
t	-	Wall-thickness (mm)
α	-	Overlapping angle
π	-	Pai

Ψ	-	Interaction factor
Θ	-	Parametric angle of the crack

CHAPTER 1

INTRODUCTION

1.1 Background

Generally, hollow cylinders are mechanical structures that are broadly used in the industry, such as offshore, pressure vessels, pipelines, and aerospace structures. Owing to their long-dated service, aging and deterioration of these structures are unavoidable. The structural integrity of these structures same like any other component, considered an important issue in terms of economic view, safety, and environmental damages. Therefore, it is essential to keep these structures safe as possible.

Basically, two types of failure could be found in cylinders, the first caused by deterioration, where the decrease of wall-thickness lead to loss of strength, and the second type, stress concentration at the tips of cracks or in general defects in the pipes, produces loss of toughness [1]. An examination of the failures of cylinders in service exposes that most failures are of fracture type, which is caused by the propagation of a defect or crack and following cylinder collapse [2].

Moreover, exploitation for a long time of such elements facilitates their failure, which can be accelerated by the influence of outside environmental circumstances such as corrosion or erosion [3]. In addition, since the impact of the production process, nondestructive testing, and severe operational conditions, etc., cracks exist during the service of pipes [4]. These cracks could be found in the form of single or multiple cracks, where the multiple cracking is one of the most common problems in aging aircraft, pressure vessel, and piping structures [5]. Such type of cracking often happens in localized patches or colonies due to various kinds of material failures, such as in Stress Corrosion Cracking (SCC) [6], fatigue [7], and corrosion fatigue [8]. Furthermore, more than one-third of the pipeline's failure is associated with the cracking as reported by [9].

Usually, the field used to study cracked structures is known as Fracture Mechanics (FM). In the FM field, two concepts could be utilized to study a cracked structure; they are the Linear Elastic Fracture Mechanics (LEFM), and the Elastic-Plastic Fracture Mechanics (EPFM). LEFM is used when the plastic deformation in the examined problem is limited to a small region, while EPFM applicable to the materials show time-independent, nonlinear behavior (plastic deformations). In the LEFM concept two main parameters, the energy release rate (G) and the Stress Intensity Factors (SIFs, K), where both of them were proposed by Irwin in [10],[11]. On the other hand, Crack Tip Opening Displacement (CTOD) proposed by Wells in [12], and the *J*-integral proposed by Rice in [13], considered the most important parameters in EPFM. By using LEFM and EPFM parameters, abundant research has been conducted to study the cracked structures, either for cases of single or multiple cracks. Various crack configurations were examined in terms of cracked body parameters as well as crack geometry. Where all these studies aimed to cover the potential crack configurations, that could occur practically, and give detailed descriptions for each case in order to prevent failure and increase the structural integrity.

The present work focuses on double surface cracks problem located either on the external or internal surface of a hollow cylinder, where the crack interaction could take place between the cracks, which may accelerate the fracture process and lead to a catastrophic failure. Also, the concept of LEFM has been chosen for this study for many reasons, such as to utilize the SIFs criterion as the driving force for the cracks interaction, and in order to avoid the plastic deformations that could occur. Moreover, the presence of plastic deformations lead to underestimate the stresses in the regions surrounding the crack, consequently present conservative and unreliable result. Therefore, the interaction effect explored under various types of loading as well as various geometric crack parameters. It should be noted, currently, a wide variety of approaches performed to compute the SIFs, for example, the integral transform method, the Westergaard method, the singular integral equation method, conformal mapping, Laurent series expansion, boundary collocation method, the Green's function method, the continuous distribution dislocation method, the boundary element method, the body force method, the displacement discontinuity method, and the Finite Element Method (FEM) [14]. Based on [15], it has shown that FEM is an extremely effective tool in SIFs determination; thus, FEM has been selected as the utilized method to obtain SIFs for the present work.

1.2 Problem Statement

Hollow cylinders are considered among the essential elements in the industry due to its excessive usage. Therefore, due to the general use of hollow cylinders, the structural integrity of these structures is deemed an essential task from many viewpoints, such as economic, safety, and environmental damages. Two types of failure could be found in cylinders, the first caused by deterioration, and the second type, caused by crack propagation. An examination of the in-service cylinder's failure implies that the majority of the failures found to be fracture type.

Cracks could exist during the service of hollow cylinders, due to the impact of the production processes, nondestructive testing, and severe operational conditions, etc. These cracks could be found in the form of single or multiple cracks, where the multiple cracking is one of the most common problems in aging aircraft, pressure vessel, and piping structures. Multiple surface cracks problem considered one of the most critical issues that hollow cylinders expected to experience, especially when it is recognized that about 33% of cylinder's collapse is related to cracking, as stated in [9]. Many researchers tried to solve various crack configurations in terms of cracked body parameters and crack geometry. Where all these studies aimed to cover the potential crack configurations that could occur practically and give detailed descriptions for each case in order to prevent premature failure and increase the structural integrity. Therefore, the cracks configuration (orientation) within the structure has a significant influence on the crack interaction.

The cruciality of multiple surface cracks problem could be demonstrated by the eventuality of cracks interaction occurrence. Generally, the cracks interaction impact on the value of the SIFs can be in the form of amplification or shielding. Furthermore, due to the amplification effect caused by the cracks interaction, the value of SIFs at the crack tips may extend to a value further than the material yielding, consequently accelerating the fracture process and lead to a catastrophic failure.

Multiple surface cracks interaction in terms of longitudinal alignment were examined in hollow cylinders under various types of loading in [16]–[20]. Where the FEM has been utilized to obtain the Stress Intensity Factors (SIFs) along the crack fronts, where SIFs criterion was adopted to calculated the interaction factor between the cracks, which is defined as the ratio of SIFs for the case of two cracks to the single crack case.

On the other hand, in terms of circumferential cracks in hollow cylinder, the interaction problem between circumferential and embedded cracks was examined in [21], [22]. The interaction behavior has been examined by performing 3-D elastic-plastic analysis via the Crack-Tip Opening Displacement criterion (CTOD). Also, the influence of the crack interaction on the Limit Load (LL) was determined by conducting finite element analysis to two similar circumferential cracks located on the internal and external surface of a hollow cylinder in [23].

It has been stated that hollow cylinders frequently contain cracks in a circumferential orientation at weld joints, which indicates the vital role that the circumferential cracks play in the safety analysis of hollow cylinders [24]. Additionally, it has been shown that the case of two interacting cracks is more critical than the case of more than two cracks.

Furthermore, the available literature dealt with the cracks interaction problem from different outlooks, such as the crack alignment (axial or circumferential), cracks configuration, crack geometry, loading type, and the utilized criterion as well as the employed method. However, none of the existing studies dealt with the interaction of multiple parallel coplanar and noncoplanar circumferential surface cracks located on hollow cylinders. Because of the problem of cracks interaction strongly depends on the orientation of the cracks within the structure, thus it is crucial to investigate and solve the above-mentioned cracks configurations. Also, solving such type of crack configurations can serves the literature database in future as well as helps to prevent the unexpected failure in hollow cylinders. Since hollow cylinders considered one of the common utilized element in the industry, preventing hollow cylinders' failure can protect the industry in terms of safety and reduce the expenses in terms of cost.

1.3 Research Objectives

In the present study, the interaction between double surface cracks located on the hollow cylinder is examined by the finite element method for different crack configurations and crack geometrical parameters. Where the Stress Intensity Factor (SIF) has been chosen to be the driving force for the crack interaction. Thus, several research objectives were proposed as follows:

(i) To investigate and analyze surface cracks interaction in terms of the SIFs as well as interaction factor for double circumferential surface cracks located on external and internal surfaces of hollow cylinders.

(ii) To formulate an empirical mathematical model to predict SIFs for double parallel circumferential interacted cracks.

(iii) To validate the proposed model in terms of performance evaluation metrics.

1.4 Research Scopes and limitations

The interaction of double circumferential surface cracks located on hollow cylinders were examined in this study. Two types of cylinders were considered depending on the cylinder wall-thickness; they are thick and thin cylinders. Both thick and thin cylinders assumed to be straight and homogeneous, with wall-thickness to internal radius ratio (t/R_i)= 0.25 and 0.1 for thick and thin cylinders, respectively. While the total length of the cylinder for both taken as 750mm, and outer diameter to internal diameter ratio (D_o/D_i)= 1.25 and 1.10 for thick and thin, individually. The cylinders modeled with material properties of Young's modulus 200 GPa, and Poisson's ratio of 0.3.

Two types of cracks configurations were inspected; they are parallel and noncoplanar parallel surface cracks. Both parallel and noncoplanar parallel orientations were tested on the external and internal surfaces of both thick and thin cylinders. The crack geometrical parameters, crack aspect ratio (ratio of crack depth to half crack length) assumed to vary from 0.4 to 1.2, and the relative crack depth (ratio of crack depth to cylinder wall-thickness) is assumed to be 0.2, 0.5, and 0.8, for both cylinders. Besides, in parallel cracks, the horizontal separation distance (s) between the cracks ranges from 3mm, 6mm, 12mm, and 24mm for both cylinders. All the above-mentioned parameters have been selected based on the available literature, which include a wide variety of crack geometrical parameters.

On the other hand, the noncoplanar parallel crack configuration, the interference between the cracks is defined by inclination angle α, where α= 10°, 20°, and 30° for the thick cylinder, and α= 5°, 10°, and 15° for the thin cylinder. It should be noted that the discrepancy between the examined inclination angles for thick and thin cylinders was due to the difference in the utilized crack geometry. Moreover, the consumed crack geometry for the thick cylinder was higher than those of thin cylinder, which means if the same inclination angles were used for thick cylinder applied to the thin cylinder, no interaction effect attained in the thin cylinder.

For the loading, four types of loading were examined; they are tension, bending, torsion, and a combination of them. All the loads applied remotely through a remote point as it is commonly used, where this point is connected to one end of the cylinder and the other end is fixed with (0) degree of freedom. All loading ratios kept in the linear elastic regions to minimize the plastic deformations regions surrounding the crack front which could lead to underestimate the SIFs.

The analysis performed via finite element software Ansys, where the semi-elliptical crack option was used to insert the cracks to the model. The examined cases considered in the whole study was for single and two cracks only, where only circumferential crack orientation was investigated.

It should be remarked that the empirical mathematical model proposed in this study valid for the case of double parallel cracks under tension, bending, and mixed-mode loading. Also, each equation can be used for external and internal surface cracks either for thick or thin cylinders depending on the horizontal separation distance, 3mm to 24mm.

1.5 Thesis Organization

In this section, a brief description for each chapter of this thesis is presented, where five chapters were composed to illustrate the sequence of the thesis layout as following:

Chapter 1. Introduction. In this chapter, an overview of the research background is presented as well as the problem statement, objectives, and scopes and limitations of the study.

Chapter 2. Literature review. This chapter describes the fracture mechanics field along with linear elastic and elastic-plastic fracture mechanics concepts. Besides reviews to the available methods utilized to determine stress intensity factors for single and multiple cracks on different bodies.

Chapter 3. Methodology. The details of the analysis work were stated in this chapter. The cylinders and cracks geometry were defined beside the applied software settings. Also, the required mathematical equations used for the normalization is presented in this chapter.

Chapter 4. Results and discussion. This chapter describes and discusses the results obtained from the analysis.

Chapter 5. Conclusion and future work. The overall conclusion of the current work is presented in this chapter based on the obtained results and their analysis. Recommendations were presented as future work in this chapter.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter presents a literature review of the past and recent studies on the multiple crack interaction in the fracture mechanics field. Multiple crack problem has gained the researcher's attention since they have a significant effect on the fatigue life of the component. Therefore, to introduce a complete literature review regarding multiple crack problems, and to be more comfortable with the used parameters in this study, different subjects have been presented. This study considered two types of cylinders; therefore, types of cylinders and their applications in practical introduced first. Then, surface cracks are reviewed, explaining the common types, the anatomy of the crack, and the essential factors controlling a surface crack. Next, the tool that has to be applied to deal with these cracks is the fracture mechanics; hence the initiation of this field along with the available concepts and parameters used in this field is reviewed in order to evaluate the mechanical problems. Stress Intensity Factor (SIFs) or K, is one of the important criterions used in fracture mechanics; therefore, methods of determining SIFs reviewed extensively, since SIFs is the driving force for the crack interaction in this study. After that, dealing with multiple crack problems become much more manageable. The next stage is to present the available research and studies dealt with single and multiple surface cracks related to crack interaction problems.

2.2 Thick and thin cylinders

Cylindrical structures, either solid or hollow (thick and thin) considered among the widely used components in the industry. For example, shafts, control rods, pressure vessels, the piping in offshore drilling operations, tubing and drill pipes used in oil and gas fields, the pipelines that carry oil and natural gas, and tubes used in heat exchanger [25]. Typically, cylinders are classified into two categories, corresponding to the equations which govern their stress state. If the thickness of the wall is small with respect to the internal diameter, within limits explained next, it assumed that the stresses are equally distributed across it, simplifying the treatment to a considerable extent [26].

It should be stipulated that there is no difference between the two types, either thin or thick cylinders from the perspective of continuum mechanics. The discrepancy is entirely conventional. At this point, the more widespread global convention, whereby the ratio of wall thickness, t, to the inner diameter, d_i, for thin cylinders is according to the following [25]:

$$\frac{t}{d_i} \leq \frac{1}{20} \tag{2.1}$$

Typically, Equation (2.1) is expressed by an equivalent form by introducing a nondimensional diameter ratio δ, where $\delta = d_e/d_i$, or its mutual $\beta = 1/\delta$, keeping in mind that the subscripts (e and i) denoting to the external and internal diameters of the cylinder. Therefore, we have:

$$\delta \leq 1.10 \text{ or } \beta \geq 0.91 (\sim 0.90) \tag{2.2}$$

Thus, depending on the equations as mentioned above, it could be specified if the examined cylinder falls in the thin cylinder category or not. Whereas, thick cylinders by convention deemed to be cylinders which satisfy any one of the following equivalent requirements [27]:

$$\frac{t}{d_i} > \frac{1}{20}\ ;\ \delta > 1.10\ ;\ \beta < 0.9 \tag{2.3}$$

Hence, as mentioned earlier, for the thin cylinder, it is assumed that the stresses are equally distributed across it, which lead to simplify the treatment to a substantial extent. In the case of a thick cylinder, it is not conceivable to assume that stresses are uniformly oriented through it. In this situation, the approach to the problem is more complicated, as it includes simultaneously employing the equilibrium equations and the compatibility equations, both must be fulfilled, as well as the boundary conditions.

It should be known that the international European standards involving pressure vessel design analysis and response analysis do not permanently agree with the value of ratio δ to be used as the separation between thin and thick cylinders. Sometimes, these differences are rooted in empirical rather than theoretical considerations. Consequently, the more straightforward relations deriving from the thin assumption are extended well beyond these conventional limits to include a sizable portion of the values of ratio δ (the lower values) that, at least from the viewpoint of the above convention, should be considered as being in the category of thick cylinders. It is also worth noting that a few standards introduce an intermediate-range midway between thin and thick cylinders [25].

Generally, due to the heavy use of cylindrical structures in the industry in the last decades [28], the structural integrity of this kind is deemed extremely important. One of the primary reasons that could lead to failure of such type is the presence of the surface cracks. The failures are triggered by the propagation of a crack or surface defect that produces local plastic deformation, due to mechanical influence or external interference or corrosion pitting, Which may be in the shape of a tooth or crack, or variations of it [29]. Overall, the next section discusses surface cracks deeply.

2.3 Surface cracks

Owing to the massive use of the cylindrical structures nowadays, particularly hollow cylinders, the safety of these structures plays an important role. This could be considered from the perspective of cost or either in terms of human safety or environmental damage. Therefore, the structural integrity of this kind of structure took extensive interest and effort from the involved people (scientists and organizations). Moreover, exploitation for a long time of such elements facilitates their failure, which can be accelerated by the influence of outside environmental circumstances such as corrosion or erosion [3]. In addition, since the impact of the production process, nondestructive testing and severe operational conditions, etc., cracks exist during the service of pipes [4]. Furthermore, more than one-third of the pipeline's failure is associated with the cracking as reported by [9].

Stress Corrosion Cracking (SCC), considered one of the highly acute failures causes in pipes, where the cracks initiate and propagate in a corrosive environment at the presence of mechanical loading [30]–[32]. Generally, many factors affect SCC in hollow cylinders; they are: environmental, mechanical, and metallurgical [33], where all of these factors are required for SCC, and each can be dominant beneath specific conditions.

Surface cracks have been studied extensively in the literature due to its foremost task in leading to failure. In fact, the presence of surface cracks accelerates the fracture process, especially for the case of unstable crack growth. Thus, the application of structural integrity assessment techniques to evaluate the safety state of the cracked pipe is required [34]. Hence, a deep understanding to the nature of the surface cracks, exploring the common types of these cracks as well as the essential factors that control the shape of the cracks is necessary; the following section discusses the structure and configuration of the crack.

2.3.1 Anatomy of crack

In general, surface cracks, regardless of the reason for the initiation, start with irregular shapes and then take a semielliptical form after a few cyclic loadings [35]. Therefore, the majority of the researchers dealt with the surface cracks as a semi-elliptical shape. Hence, whenever it is mentioned in this study a crack or a surface crack, it means that a semi-elliptical crack depending on what has been stated in the literature.

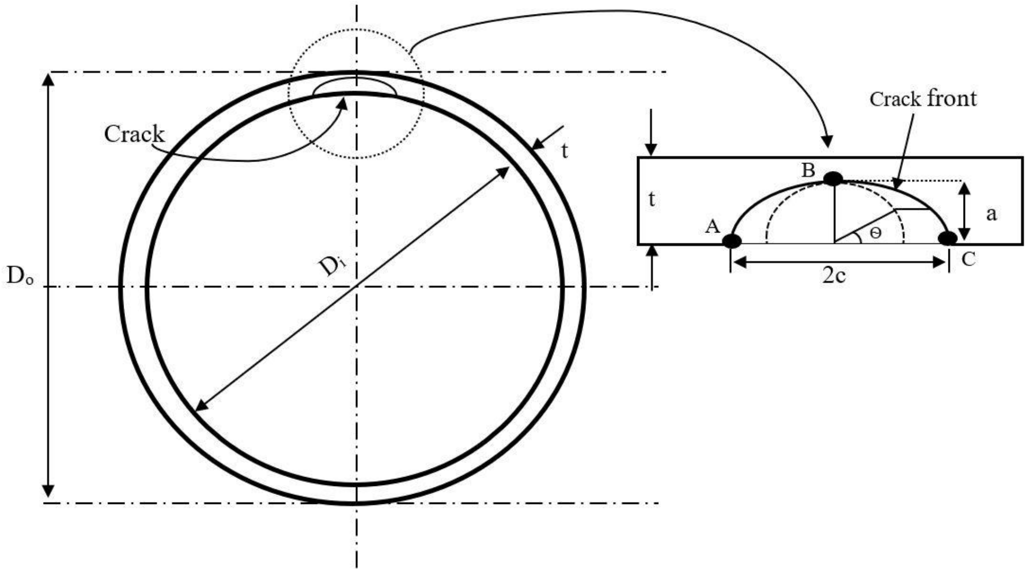


Figure 2.1: Anatomy of a semi-elliptical crack

Since this study focus on the interaction of multiple surface cracks located on hollow cylinders, for the purpose of explanation, a single semi-elliptical crack positioned on the internal surface of a hollow cylinder is shown in Figure 2.1. It can be noted that 2c, is the length of the crack, while a, denoting to the depth of the crack through the wall thickness of the cylinder, t. The crack front has two crack tips (A and C), which are the intersection points between the crack front and the cylinder surface. On the other hand, point (B) represents the deepest point on the crack front through the wall thickness. It is possible to define any point located on the crack front by using a non-dimensional coordinate named the normalized coordinates, $2\Theta/\pi$, where Θ, is the parametric angle of the crack.

Now, to describe a crack located on a surface of a hollow cylinder, two fundamental parameters are used based on the crack geometry itself as well as the cylinder wall thickness. Where, a, c, and t formulated in the non-dimensional form to present a/c, the crack aspect ratio, and a/t, the relative depth of the crack. For a/c, it is the ratio that describes the shape of the crack, which could be straight or slender or transverse across the thickness. Meanwhile, a/t illustrates the depth of the crack with respect to the thickness of the cylinder.

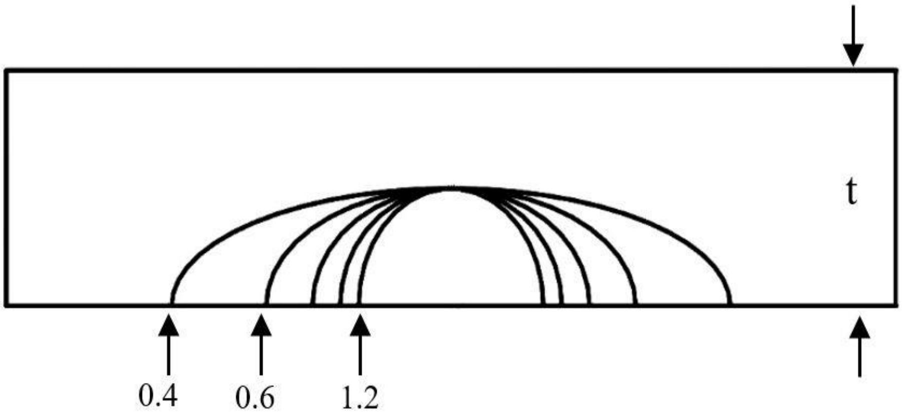


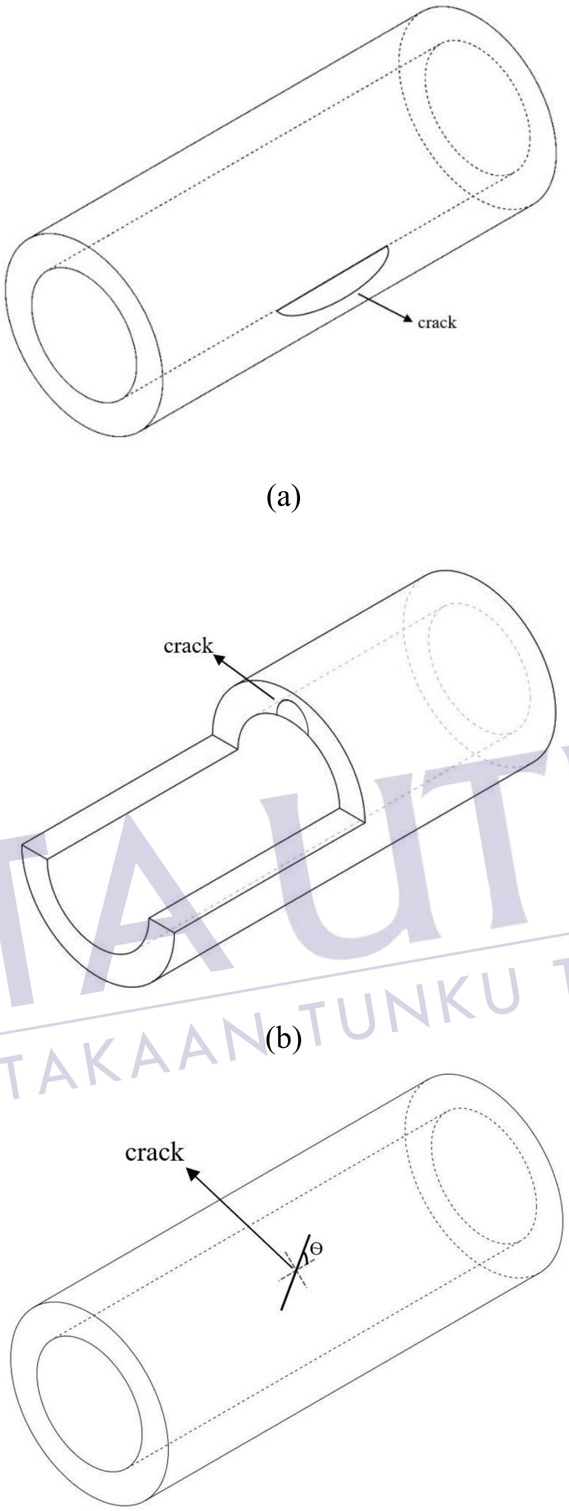
Figure 2.2: Variation of the aspect ratio (a/c) through the thickness

Figure 2.2 shows different values of crack aspect ratio through-thickness for the same value of the relative depth of the crack ratio. As depicted, the shape of the crack changes concerning a/c, where small aspect ratios present a slender profile, while in case of high value turns to transverse form. It should be noted that, for a/c=1, the crack here takes a circular shape.

2.3.2 Types of surface cracks

The aging and degradation of pipes or hollow cylinders are unavoidable because of their long-term usage [1]. Thus, several pipes in-service failures have been described worldwide [36]–[38]. Essentially, there are two failure manners for pipes; the first is based on the strength loss owing to the decrease of the pipe’s wall thickness produced by deterioration. While the second type is based on the toughness loss triggered by stress concentration at the crack tips or other defects in pipes.

As it is known, the effect of the failure of a pipe could be catastrophic and devastating. An analysis of the failure of in-service pipes exhibits that failures generally are in the fracture type, which occurs due to the propagation of a crack or defect and consequent collapse of the pipe [2]. Semi-elliptical surface crack is one of the most common forms of the defect that exists in hollow cylinders as well as in many other engineering pipes [39], [40].



(a)

(b)

(c)

Figure 2.3: Types of cracks (a) axial (b) circumferential (c) inclined.

These cracks could be divided according to their occurrence orientation on the hollow cylinder into three types. They are [41]:

(i) Axial or longitudinal.

(ii) Circumferential or radial.

(iii) Inclined.

Figure 2.3 shows the three types of surface cracks located on a hollow cylinder. It should be noted, all these crack types could be located on the external or internal surface of the cylinder, but for the sake of introducing these cracks, they were presented for the case of internal crack only.

Moreover, the axial cracks or the longitudinal cracks oriented on the longitudinal axis of the cylinder through the thickness. While, for the case of circumferential cracks or radial racks, oriented in a radial way with respect to the cylinder. Finally, when the crack oriented in such a way, not on any of the radial or axial is considered to be an inclined crack.

Now, the presence of any of the types, as mentioned above in a pipe or cylinder, generates stress intensity bordering the crack region, which leads to failure of the pipe. Therefore, to estimate the failure of the pipes with semi-elliptical surface cracks, it is essential to calculate the stress intensity factor accurately. To do so, the fracture mechanics criterions must be applied for this kind of problem, the next section introduces the main concepts of the fracture mechanics field alongside the available criterions in each concept.

2.4 Fracture mechanics field

The applied mechanics' framework for studying the behavior of cracked bodies under load is known as Fracture Mechanics (FM). A brief discussion on the initiation of the FM field, as well as the essential concepts in this field is discussed in the next sections.

2.4.1 Fracture mechanics birth

The mechanics of fracture evolved from being viewed as a scientific curiosity to an engineering discipline, mainly because of what occurred to the Liberty ships during World War II [42]. Moreover, the U.S. applied a new quick ship fabrication procedure, where the new vessels having an all-welded hull, which is the opposite of the traditional construction method, the riveted construction. This was a resounding revolution at that time, but one day in 1943, one of these vessels collapsed completely while sailing between Siberia and Alaska. Following fractures occurred in the other ships; out of 2700 ships built for the period of the war, about 400 sustained fractures, among of which 90 were deemed severe. The investigations deduced that the failure of these ships produced by a combination of three reasons [43]:

- The used welds contained crack-like defects.
- A local stress concentration detected on the deck at square hatched corners since most of the collapsed initiated at these corners.
- The steel used in manufacturing has poor toughness as assessed by impact tests.

In fact, the steel had always been sufficient for the traditional type of ships, which is the riveted; this is because of fracture unable to transmit through the panels that were joined by rivets. However, the welded ship is basically a single part of metal, no barriers significantly resisting the crack propagation; therefore, sometimes capable to transverse the whole hull. Once the sources of failure were detected, the remaining ships where modified, the hatch corners were strengthened by rounded reinforcement, while steel plates with high toughness were added to the deck at strategic positions.

Besides, a group of researchers at the Naval Research Laboratory in Washington, DC. studied the problem of fracture in detail, the field which is well-known now as fracture mechanics born in this lab during the war time and the following years [43]. In the next section, the development of fracture mechanics studies is summarized. Besides, the development of each concept is also reviewed.

2.4.2 Brief history of fracture mechanics

Several centuries ago, Leonardo da Vinci performed experiments that produced some indications related to the root cause of fracture [44]. When he evaluated the strength of iron wires, he found an inverse proportion between the strength and the length of the wire. Based on these results, it can be inferred that defects in the wire material influenced the strength. Simply, the probability of crack presence in long wire higher than those of short wire; however, the results were only qualitative [43].

In 1920, Griffith made a quantitative connection between the flaw size and fracture stress [45]. Where the propagation of brittle cracks in a glass was considered. Griffith formulated the well-known concept that an existing crack will propagate if thereby the total energy is lowered, and he assumed that there is a simple energy balance, consisting of a decrease in elastic strain energy within the stressed body as the crack extends, counteracted by the energy needed to create the new crack surfaces [46]. His theory allows the estimation of the theoretical strength of brittle solids and gives the correct relationship between fracture strength and defect size.

Irwin, the father of fracture mechanics [47], owing to his substantial experimental and theoretical contributions to the fracture mechanics field. Irwin indicated that the Griffith-type energy balance must be between (i) the stored strain energy and (ii) the surface energy plus the work done in plastic deformation. Irwin defined the 'energy release rate' or 'crack driving force', G, as the total energy that is released during cracking per unit increase in crack size [10]. He also recognized that for relatively ductile materials, the energy required to form new crack surfaces is generally insignificant compared to the work done in plastic deformation.

After that, Irwin added another major contribution by showing that the energy approach is equivalent to a stress intensity (K) approach [11]. He indicated that the stresses and displacements near the crack tip could be described by a single constant that was related to the energy release rate. This characterizing parameter is well-known now, "Stress Intensity Factor" (K). Demonstration of the equivalence of G and K offered the foundation for the advancement of the discipline of Linear Elastic Fracture Mechanics (LEFM) [46]. LEFM is often known as Griffith–Irwin theory because of the most important roles they played in its development [48]. When the fundamentals of LEFM were fairly-well founded, investigators turned their interest to the crack tip plasticity.

LEFM could be considered valid until a significant plastic deformation precedes failure. During 1960 – 1961, several analyses were performed by many researchers to correct for yielding at the crack tip [12], [49]–[51]. Where, in [49], the correction of the plastic zone was a relatively simple extension to LEFM, while [50] and [51] are based on a narrow strip of yielded material at the crack-tip, each developed slightly additional detailed models. Also, the "Crack Tip Opening Displacement" (CTOD) was introduced in [12] by Wells when he noticed that, during plastic deformations, the crack faces moving apart, which has led to this development.

Later on, in 1968, Rice developed another parameter to characterize the nonlinear material behavior ahead of crack [13]. He introduced the well-known elastic-plastic fracture parameter "*J* integral". He states that it is possible to express the nonlinear energy release rate as a line integral calculated along an arbitrary contour around the crack. Depending on the above-mentioned parameters, which significantly extend the description of fracture behavior beyond the elastic regime, which introduces the concept of "Elastic-Plastic Fracture Mechanics" (EPFM).

For more details about fracture mechanics history and development [52] and [53]. In the next section, reviewing the LEFM concept, besides the most important parameters in this concept, reviewed also.

2.4.3 Linear elastic fracture mechanics LEFM

The basis of LEFM has been laid by the establishment of each of the "energy release rate", (G) and the "Stress Intensity Factor", (K), respectively. In LEFM, the cracked structure is regarded as a linearly elastic medium, while the nonlinear impacts are assumed to be small and could be ignored. Therefore, the next following sub-sections summarizing the basics of G and K due to its importance.

2.4.3.1 The Energy Release Rate

As it is known, the "Energy Release Rate" (G), which has been proposed by Irwin in [10], was based on (or modification to) Griffith energy balance [45]. Therefore, firstly Griffith's energy balance is introduced, then the G approach is discussed.

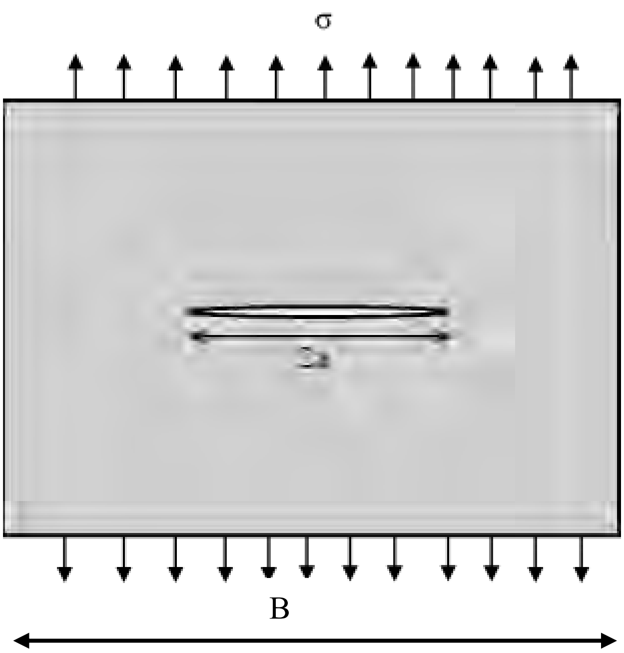


Figure 2.4: A loaded infinite width plate with a through crack

Now, considering the plate shown in Figure 2.4, the plate is subjected to uniform tensile stress, σ, containing a crack of 2a length. Assume that the width B of the plate >> 2a and plane stress conditions dominate.

For this crack, in order to extend in size, adequate potential energy must be existing in the plate to overcome the surface energy of the material. The Griffith energy balance for an incremental increase in the crack area dA, under equilibrium conditions, can be expressed by the following [43]:

$$\frac{\mathrm{dN}}{\mathrm{dA}} = \frac{\mathrm{d\Pi}}{\mathrm{dA}} + \frac{\mathrm{dW_s}}{\mathrm{dA}} = 0 \tag{2.4a}$$

or

$$-\frac{d\Pi}{dA} = \frac{\mathrm{d}W_s}{dA} \tag{2.4b}$$

Where, N is the total energy, Π, representing the potential energy supplied by the internal strain energy and external forces, W_s is the required work to create new surfaces. Griffith used the stress analysis of [54] for the cracked plate shown in Figure 2.4 to show that:

$$\Pi = \Pi_o - \frac{\pi\sigma^2 a^2 B}{E} \tag{2.5}$$

Where Π_o is the potential energy of an uncracked plate and B, is the plate thickness. Because of the formation requires the creation of two surfaces, W_s, is given by:

$$W_s = 4aB\gamma_s \tag{2.6}$$

Where γ_s is the surface energy of the material. Thus:

$$-\frac{d\Pi}{dA} = \frac{\pi\sigma^2 a}{E} \tag{2.7a}$$

and

$$\frac{dW_s}{dA} = 2\gamma_s \tag{2.7b}$$

Now, Equations (2.7a and 2.7b) and solving for fracture stress, σ_f, gives:

$$\sigma_f = \left(\frac{2E\gamma_s}{\pi a}\right)^{1/2} \tag{2.8}$$

Equation (2.8) indicates that the extension of a crack in ideally brittle materials ruled by the product of the remotely applied stress and the square root of the crack length and by material properties, as E and γ_s are material properties. Consequently, Equation (2.8) implies that crack extension in such materials happens when σ_f attains a certain critical value [46].

Since Griffith's approach is presented, it is possible to introduce the G approach, because as mentioned earlier, G, was a modification to Griffith's approach. In fact, they are essentially equivalent, except that the more convenient form to solve engineering problems is Irwin's approach [43], which included the effect of plastic deformation. Moreover, Equation (2.8) applicable only for ideally brittle materials, where a good agreement obtained between this formula and the experimental fracture strength of glass, however, Griffith's approach rigorously underestimates the metals fracture strength.

The first modification was done by [55], [56] Irwin, and Orowan individually, where the new modification gave the Griffith approach the capability to be applied to the materials that have plastic flow. Therefore, the new revised formula is given by:

$$\sigma_f = \left(\frac{2E(\gamma_s + \gamma_p)}{\pi a}\right)^{1/2} \tag{2.9}$$

Where γ_p represents the plastic work with respect to a unit area of the created surface, and typically much higher than γ_s. Despite the originality of Equation (2.9) for metals, it could generalize the Griffith expression to be applied for any type of energy dissipation as follows:

$$\sigma_f = \left(\frac{2Ew_f}{\pi a}\right)^{1/2} \tag{2.10}$$

Where w_f is the fracture energy, which might involve the effects of plastic, viscoelastic or viscoplastic depending on the material. In [10] Irwin has defined the energy release rate, G, to describe the available energy for an increase of crack extension:

$$G = -\frac{d\Pi}{dA} \tag{2.11}$$

It should be noted that the term rate, which is used in this context, does not describe a derivative related to time; precisely it is the changing rate in the potential energy with respect to the crack area. Because of G found from the derivative of potential, it could be as well called “crack extension force” or “crack driving force”. Corresponding to Equation (2.7a) for a wide plate, in-plane stress condition with a crack having a length of 2a Figure 2.4, the energy release rate could be given by:

$$G = \frac{\pi\sigma^2 a}{E} \tag{2.12}$$

As it is known earlier, in order to produce a crack to extension, G must reach a critical value G_c; therefore, beyond this value, the crack extends, thus:

$$G_c = \frac{dW_s}{dA} = 2w_f \tag{2.13}$$

Here, G_c represents a material property, which is a gauge of the fracture toughness of the material. The potential energy of an elastic body, Π, can be defined in term of the stored strain energy in the body U, and the work is done by the influence of the external forces F, as follows:

$$\Pi = U - F \tag{2.14}$$

2.4.3.2 Stress intensity approach

For a particular cracked configuration exposed to exterior forces, it is potential to obtain a closed-form representation for the stresses in the body, presuming isotropic linear elastic material disposal. Hence, [11] and [57]–[59] were among the earliest to considered such solutions. From the theory of linear elastic, the stresses in the neighborhood of a crack tip yield the form [46]:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + \ldots \quad (2.15)$$

Where σ_{ij} is the stress tensor, (r and ϴ) represent the cylindrical polar coordinates (distance and angle) of arbitrary point with respect to the crack tip as shown in Figure 2.5. While K is a quantity that defines the magnitude of the elastic stress field. It is called the "Stress Intensity Factor", f_{ij} the dimensionless function of ϴ.

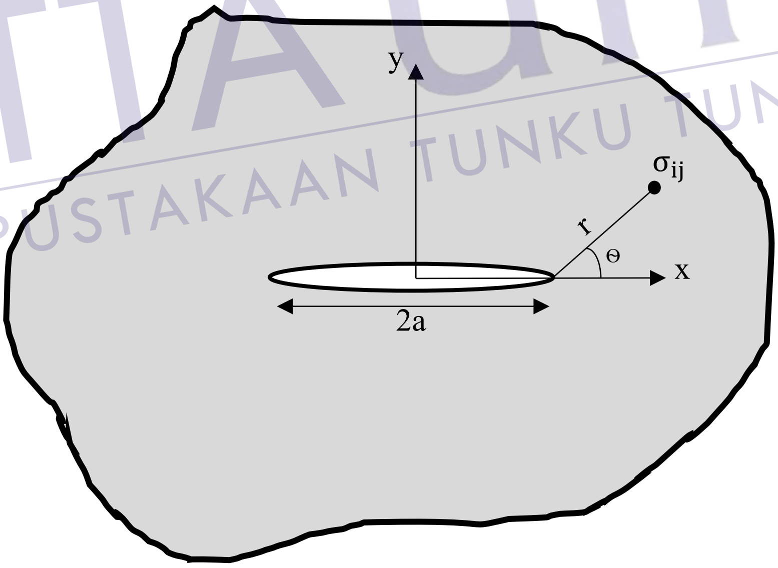


Figure 2.5: Stress at point near to crack tip

The elastic stress distribution in the neighborhood of a crack tip, Equation (2.15), reveals that as r tends to zero the stresses become infinite, i.e. there is a stress singularity at the crack tip. Since structural materials deform plastically exceeding the yield stress, there will in reality be a plastic zone close to the crack tip. Thus, the elastic solution is not entirely applicable.

The dimensional analysis illustrates that K should be linearly linked to stress and directly associated with the square root of a characteristic length. Equation (2.8) from Griffith's analysis shows that this distinguishing length is the crack length, and it turns out that the general form of the stress intensity factor could be given by:

$$K = \sigma\sqrt{\pi a}\,.\,f(a/W) \tag{2.16}$$

Where f(a/W) is a nondimensional parameter which depends on the geometry of the specimen and the crack, and σ is the applied stress (remotely). Moreover, For an infinite plate with a central crack of length 2a, f(a/W) = 1 and thus $K = \sigma\sqrt{\pi a}$. In this case, referring to Equation (2.12). Now, combining both formulas for K and G, which leads to the relationship between both of them as follows [46]:

$$G = \frac{K^2}{E} \tag{2.17}$$

Since $K = \sigma\sqrt{\pi a}$, for a central crack in an infinite plate, it follows from the result of Griffith's energy balance approach, Equation (2.8), that crack will propagates when K, reaches a certain critical value. This value is K_c, equal to $\sqrt{2E\gamma_s}$, or by considering Irwin's modification, $\sqrt{2E(\gamma_s + \gamma_p)}$. Therefore, in terms of K, the criterion for a crack extension is:

$$K = \sigma\sqrt{\pi a} > K_c \tag{2.18}$$

As a result, the parameter regulating fracture could be stated as either a critical energy release rate, G_c, or a critical stress intensity, K_c. For tensile loading the relationships between G_c and K_c are:

$$G_c = \frac{K_c^2}{E} \tag{2.19}$$

In conclusion, it could be said that the major parameters of the LEFM were introduced, as well as the most important research performed in this concept. Where, G measures the net alteration in potential energy that escorts an increment of crack extension, while K quantity depicts the stresses, strains, and displacements close to the crack tip. It should be noted, in the current study, the stress intensity factor parameter chosen to be the driving force for the multiple crack interaction problem.

It should be remarked that all the stresses in the regions nearby the crack tip could be divided into three basic types; each is associated with a local mode of crack surface displacements. These types are called modes of failure, mode I representing the opening mode, mode II denoting to sliding mode, and mode III describing the tearing mode.

Since this study is concerned with an elastic loading type, to keep as much as possible, the plastic deformation is confined to a small region. However, the next subsection presents a brief summary of the “Elastic-Plastic Fracture Mechanics”, (EPFM) and the most important parameters used in this field.

2.4.4 Elastic-plastic fracture mechanics, EPFM

As it is known, Linear Elastic Fracture Mechanics, LEFM considered valid until the nonlinear deformations of the material precede the failure. Generally, it is almost not possible to describe the fracture behavior with LEFM, and an alternate fracture mechanics model is necessary.

Elastic-Plastic Fracture Mechanics, EPFM applicable to the materials show time-independent, nonlinear behavior (plastic deformations). Two parameters of EPFM are introduced, Crack Tip Opening Displacement (CTOD), and the J-integral. Both of these parameters define the crack tip condition in elastic-plastic materials; therefore, each of them can be used as a fracture criterion. Similar to the other criterion of LEFM, CTOD and J could be treated as a measure of the material fracture toughness depending on their critical values even for relatively large amounts of crack tip plasticity. The following subsections introduce the basics of each criterion and the way the linked to each other. In addition, the relationships between the criterion of LEFM and EPFM are introduced as well.

2.4.4.1 Crack Tip Opening Displacement

It is considered a significant fracture mechanics criterion, it is first proposed in [12] by Wells at the British Welding Institute. Firstly, Wells referred to this parameter as the crack opening displacement (COD), but more recently, the name has been changed to CTOD to differentiate the quantity from the crack mouth opening displacement (CMOD) [48], a physical crack opening displacement measured crosswise the crack mouth at the sample surface. Wells established the CTOD approach in order to expand

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