ROBOTIC PATH PLANNING USING RAPIDLY-EXPLORING RANDOM TREES

FAHAD SHERWANI

A dissertation submitted in fulfillment of the requirement for the award of the

Degree of Master of Electrical Engineering.

Faculty of Electrical and Electronic Engineering,

Universiti Tun Hussein Onn Malaysia.

January 2013

ABSTRACT

This study concerns the implementation of Rapidly-Exploring Random Trees (RRTs) algorithm for an autonomous robot path planning. RRTs possesses a number of advantages such as relatively simple, suitable for finding a path for a robot with dynamic and physical constraints, the expansion of RRT is heavily biased toward unexplored areas of search space and the number of edges is minimal. However, the planned path by using basic RRT structure might not always be optimal in terms of path length. Therefore, a path pruning method has been proposed to address this issue and improve the overall performance of the RRTs. Through simulations, the path pruning method has been proven to reduce paths lengths while preserving the aforementioned advantages of RRTs. A Graphical User Interface (GUI) has also been developed to demonstrate the RRTs technique in planning a path for an autonomous robot. The GUI package is designed to be interactive and user-friendly even for the users with minimal or no guidance and practice.

ABSTRAK

Kajian ini adalah berkaitan dengan pelaksaan algoritma *Rapidly-Exploring Random Trees* (RRTs) untuk perancangan laluan bagi sebuah robot automatic. RRTs mempunyai beberapa kebaikan seperti ringkas secara relatifnya, sesuai untuk robot yang mempunyai kekangan dinamik dan fizikal, pengembangan RRT sangat berpandukan kawasan yang belum diterokai dan bilangan sisi adalah minimum. Walaubagaimanapun, laluan yang dirancang oleh struktur RRT yang asas tidak optimum dari aspek kepanjangan laluan. Oleh itu, satu kaedah pemendekan jarak diutarakan untuk mengatasi masalah tersebut dan memperbaiki prestasi RRT secara keseluruhannya. Melalui simulasi, kaedah pemendekan jarak telah terbukti mengurangkan panjang laluan yang dirancang di samping mengekalkan kebaikan-kebaikan RRT seperti yang tersebut di atas. Satu pengantaramuka penggguna bergrafik (GUI) juga telah dibangunkan untuk mendemonstrasi teknik RRT di dalam perancangan laluan bagi sebuah robot automatik. Pakej GUI direkabentuk supaya ia interaktif dan mesra pengguna hatta kepada mereka yang belum pernah menggunakannya sebelum ini.

Table Of Contents

List of Tables

List of Figures

LIST OF SYMBOLS AND ABBREVIATIONS

$\mathcal{A}$	-	Autonomous Vehicle
$\mathcal{W}$	-	Workspace
O	-	Obstacles
C-SPACE	-	Configuration Space
$\mathcal{Q}_{free}$	-	Free Space
$\mathcal{Q}O_i$	-	Space Containing Obstacles
q	-	Random Point
$V(q)$	-	Set of Points
RRT	-	Rapidly-Exploring Random Trees
GUIDE	-	Graphical User Interface Development Environment
p_{start}	-	Start Point
p_{target}	-	End Point

CHAPTER 1

INTRODUCTION

1.1 Motivation

Robotic path planning is a key concern in robotics dynamics and kinematics as it allows a robot to autonomously acquire the shortest path from point A to point B. There have been many ways to address this issue and implement efficient path planning techniques. Commonly, path planning algorithms are judged by their computational complexity. The implication of real-time motion planning depends upon the accuracy of the map, on robot localization and on the number of obstacles. Topologically, the problem of path planning is related to the shortest path problem of finding a route between two nodes in a graph.

An important part of any robot application is the determination of a collision-free path through the environment for the manipulator. Most simply, robotic path planning is to predict this collision-free motion from initial through final points. To solve this issue we use different efficient algorithms. These algorithms have many significant applications which include: industrial robotics, service robots, computer animation, drug design and automated surveillance. This is why that research in this area had been very active and gradually increasing within previous years.

One such efficient algorithm to find a collision-free path for the movement of autonomous bodies is Rapidly exploring random trees (RRT). RRTs may be defined as the randomized data structures which can be used to address a wide range of path planning problems. RRTs are being designed using the inspiration of previous algorithms but it is specified to tackle non-holonomic constraints and is implied to higher degrees of freedom.

1.2Problem Statement

Path planning algorithms have many significant applications within robotics which include: industrial robotics, service robots, computer animation, drug design and automated surveillance. This is why that research in this area had been very active and gradually increasing within previous years. Computational time, path length and completeness are considered as three basic and most important factors for path planning. These key factors need to be calculated and defined before any path planning technique/algorithm design process takes place. Most commonly, algorithms lack at efficiently searching high-dimensional search spaces. The response time from the algorithms is high due to their computational complexity, making them not suitable for applications where a quick response is required. The cost of the planned path is high in terms of computational efficiency. While, some path planning algorithms are not complete in terms of path prediction, and optimal in terms of path length. RRT is a data structure and algorithm designed for efficiently searching high-dimensional search spaces. The RRT algorithm is extremely simple and computationally efficient. The response time from this algorithm is competitively low, even in large environments, making it suitable for fast moving bodies. The path planned using RRTs is usually complete. A path pruning technique is proposed in order to provide optimality in terms of path length.

1.3Project Objectives

The aim of this project is to present and implement an efficient robotic path planning algorithm using Rapidly exploring random trees (RRT). In order to achieve that, the following objectives are outlined:

a) To implement an algorithm for robotic path planning applications using RRTs.
b) To simulate the algorithm using MATLAB.
c) To test the functionality and reliability of proposed algorithm under different conditions.
d) To observe the overall performance of the algorithm.
e) To propose a path pruning method for robotic path planning using RRTs.

1.4 Project Scopes

The scope of this project is primarily concerned to focus on generating an efficient algorithm to address the path planning issues in robotic path prediction using RRTs.

The scope of the proposed project includes:

a) Using MATLAB, generate a path planning algorithm based on RRTs.
b) Simulate the algorithm in order to investigate its practical feasibility and performance.
c) To test and evaluate the presented algorithm using different scenarios.

1.5 Path Planning Overview and Issues

Technically, path planning is about the determination of a safe path for an automated vehicle which can guide the vehicle to traverse collision-free from a starting point to an end point. It must be ensured that the vehicle steer free from the obstacles present in its vicinity, and the planned path must comply with the vehicle's physical/kinematic constraints [25]. In a report by [12], some of the basic terminologies that relate to path planning are as follows:

- ***Motion planning*** The term "Motion planning" is generally associated to manipulator robotics. It relates to the different ways of high level and low level programming of a robotic manipulator.

- ***Trajectory planning***
  It can also be referred to as pre-motion planning. It is concerned with the prediction of the robots' next movement. Trajectory planning can also be considered as motion planning.

- ***Navigation***
  Navigation means " to move from one place to another". Navigation is very broad in its meanings. It also includes path planning, motion planning, obstacle avoidance and localization.

- ***Global path planning***
  In this, a path is planned before the actual movement of the vehicle. This is done by gathering the information from the surrounding world, from a starting point to a set target point. It can be computationally extensive and may take a longer time to process but the resulting path is usually optimal.

- ***Local navigation***
  In this process, the data regarding surrounding environment of the vehicle is constantly updated to avoid any upcoming or moving obstacles. This

approach is used to handle the vehicles' movements in real time, assuring the stability and safety of the vehicle all along its course of movement.

1.6 Criteria of Path Planning

Issues related to Path Planning have been widely studied and discussed by many researchers [7-10]. The research area of keen interest, for path planning in robotics, has been robotic manipulators and autonomous ground robots. Computational time, path length and completeness are considered as three basic and most important factors for path planning. All these criterions vary according to application specifications, such as, real time applications require prompt response and continuous update of the upcoming situation, hence a path planning algorithm with less computational time is preferred. While, the factor of completeness justified, if it is able to find a path using the path planning approach.

Therefore, tradeoffs regarding these path planning criterion have to be considered. For instance, a computationally efficient path planning algorithm lacks to predict the shortest available path i.e. optimal path. It may be a priority in a scenario, where finding a slightly longer path with less computational time would have been desirable. In contrast, the computational complexity of path planning is higher if it is preferable to construct an optimal path. These key factors need to be calculated and defined before any path planning technique/algorithm design process takes place.

1.7 Path Planning Steps

Most commonly, all path planning algorithms for any vehicle $\mathcal{A}$ comprises of two phases. In the first phase, which is known as pre-processing phase, a workspace environment $\mathcal{W}$for vehicle $\mathcal{A}$containing obstacles $\mathcal{O}$is created and defined. The concept of configuration space ($\mathcal{C}$-Space) is implied in this phase to represent $\mathcal{A}$ and $\mathcal{O}$ in $\mathcal{W}$[9, 12]. In $\mathcal{C}$-space, the vehicle is transformed into a point , and similarly the obstacles' sizes are enlarged with respect to the size of $\mathcal{A}$. After that, representations techniques are used to generate maps of graphs. The way of defining the nodes and edges depends on different representation techniques.

Secondly, in the query phase, a search for a collision-free path is performed from a starting point to an end point, using (graph) search algorithms. Consequently, a path may be planned for the vehicle to traverse through its initial position to final position while steering free of obstacles in vicinity.

1.8 $\mathcal{C}$-Space Representation

$\mathcal{C}$-Space environment in path planning for an object can be represented by various methods; some of them which are widely used are potential field (PF) [21-24], cell decomposition (CD) [13-16] and roadmap (RM) [17-20]. Potential field (PF), as the name suggests, covers the environment with potential fields and simulates the object as a particle manipulating under these potential fields throughout the $\mathcal{C}$-space. The magnitude of these fields can be calculated by the generation of fields at starting p_{start}, target point p_{target} and the obstacles O in the $\mathcal{C}$-space. These points produce repulsive and attractive forces in the $\mathcal{C}$-space, where p_{start} and $\mathcal{O}$ are repulsive surfaces (which generate repulsive forces), while the p_{target} is the attractive pole which generates attractive forces [21]. The calculation of the path is based on the resulting potential fields from a point with the highest magnitude of the resultant potential field, i.e. p_{start}, to a point with the minimum potential, i.e. p_{target}. Among various advantages of PF, is that it is able to plan the path in real time when the vehicle is under motion. It is also capable of generating a smooth path. One of the drawbacks of the typical PF methods is that sometimes the vehicle got stuck in the way before reaching its target point due to local minima, hence it might not satisfy the completeness criterion.

One of the widely used path planning methods includes CD-based techniques. In this technique, the environment of free space and obstacles are represented in the form of cells. This makes it a comparatively straightforward technique [29] and very useful in outdoor environments as well [12]. Firstly, the C-space is divided into simple, connected regions called cells [35]. These cells may possess square, rectangular or polygonal geometry. They are discrete, non-overlapping but adjacent to each other. A cell is marked as occupied, if it contains an obstacle or part of obstacle, otherwise it is marked as obstacle free. In the second step, A path through this CD environment is then found using connectivity graph and a graph search algorithm from the starting point to the target point. The quality of CD can be increased by dividing the environment into smaller cells, the smaller the cells, the better the quality, but consequently, the grid's resolution is also increased, which in turn increases the computational time. In order to increase the quality of the path, the size of the cells has to be made smaller, which in turn increases the grid's resolution, and hence computational time. Many variants of CD method have been proposed in literature, these include Approximate Cell Decomposition, Adaptive Cell Decomposition and Exact Cell Decomposition.

In contrast, RM-based path planning technique characterize the C-space in terms of graphs or maps by using sets of nodes and edges. Commonly used RM based methods are Voronoi diagrams (VD) and Visibility Graphs (VG). These methods differ from each other by the way of defining the nodes and edges to build roadmaps. In VD, all the nodes are defined that are at the equal distance from all the surrounding obstacles of the point. The edges of the paths are positioned as far as possible from the obstacles, hence the planned paths produced using VD are comparatively highly safe. Though, because of being very far from obstacles, the paths are not optimal in terms of path length and are inefficient at the same time [12]. Conversely, the vertices of the obstacles including the starting and target points in the C-space are used as nodes in VG method. Pairs of mutually-visible nodes are then connected by a set of lines $\mathcal{E}$, forming the VG network. A pair of mutually-visible nodes means that those nodes can be linked by a line/edge that does not intersect with any edge of obstacles in the C-space. Additionally, there is a cost associated with each $\mathcal{E}$, possibly in terms of Euclidean distance. VG network or VL network is capable of producing the shortest length path.

1.9 Graph Search Algorithms

It has been discussed before that path planning using graph search algorithm is the second step of any path planning method. Breadth-First Search (BFS) and Depth-First (DFS) are two of the widely used essential search algorithms. BFS focuses to search paths in a systematic manner while ensuring that the solution must be found in the least number of iterations [34]. While, DFS is also systematic but it concentrates to explore only one direction and completely misses large portions of the C-space so that the number of iterations become very high.

Both BFS and DFS take a relatively long time for a large environment with a large number of nodes because they need to consider every node in the graph in calculating the best possible path [12]. In order to address this issue, there are a variety of search algorithms such as Dijkstra's and A* (pronounced A-star) algorithms [12] which consider only a subset of the nodes. Dijkstra's algorithm generates the shortest path by considering the costs from the current node to the starting point. A* on the other hand calculates a path based on the costs from current node to both starting and target points.

1.10 Real time and Off-Line Path Planning

A real time path planner is the one that can find and modify the path of the robots while it is in motion. This is done by detecting the presence of oncoming obstacles in the course of robots' path by using an appropriate sensor. The information from sensor is fed to the path planner and is constantly updated, which process this information to generate a collision-free path. Alternatively, an offline path planner produces the planned path before the actual movement of the autonomous vehicle. This path is normally planned using the previous data extracted from the environment of the robots' movement, acquired either by satellite, surveillance or other means.

CHAPTER 2

PATH PLANNING

2.1 Introduction

Path planning is one of the main concerns present in the development of autonomous vehicles. In its most general form, the path planning problem for an autonomous vehicle $\mathcal{A}$ in an Euclidean space $\mathcal{W}$ can be stated in the following way [45]: For a vehicle of geometry $\mathcal{A}$, determine if there exists a continuous collision-free motion from p_{start} to p_{target} through a set of obstacles O. If there exists a continuous obstacle-avoiding motion, construct the path for such a motion. Note that $\mathcal{W}$ is called the workspace, represented as R^N , with N=2 or 3 for 2D and 3D, respectively.

Path planning is used to guide an autonomous vehicle to steer clear through obstacles in order to adopt a safe path from p_{start} to p_{target}. Since the evolution of autonomous robots, Various research has been done on path planning issues to generate efficient algorithms, which are being able to provide an obstacle-free route to the robots. Therefore, many path planning techniques, which are categorized under geometric-based, grid-based or potential field, to name just a few, have been documented in ground robotics and manipulators systems [23, 38, 51-57].

The process of path planning and prediction of collision-free paths usually can be divided into two steps. The first phase of path planning is pre-processing phase. In this phase, workspace $\mathcal{W}$is defined ad 2-D or 3D using appropriate graph or map. Then the p_{start} and p_{target} are incorporated into the graph or map and a path is calculated through the represented environment using a (graph) search algorithm. This step is called the query phase. However there are several path planning methods that don't require the graph search algorithms to find paths such as Mixed Integer Linear Programming (MILP) [4, 65,79 -90] and Evolutionary Algorithm (EA) [87-90].

In this chapter, path planning aspects in general are discussed starting with the introduction of the configuration space (C-space) followed by a discussion on the path planning technique using Rapidly Exploring Random Trees(RRTs) in $\mathcal{W}$. Then several existing graph search algorithms are discussed.

2.2 Workspace Representation

The primary step of the path planning process is to identify the obstacles or objects in the surroundings of the robots' path and to discover free space for its movement. A map or graph is being constructed in this phase with respect to the configuration of vehicle and obstacles. Note that a configuration of an object is defined as a position specification of all points of this object relative to a fixed reference frame [14]. The concept of configuration space (C-space) has been developed with the help of path planning through polygonal obstacles, which allows the specification of the obstacles and the vehicle positions.

2.2.1 Configuration Space ($\mathcal{C}$-Space)

Configuration space (C-Space) can be defined as the common idea to represent the workspace $\mathcal{W}$ within most path planning methods. All possible specifications of vehicle $\mathcal{A}$ and obstacles region Oin $\mathcal{W}$ are defined in C-Space ($\mathcal{Q}$). The concept of C-space is used to ensure the safe traversing of vehicle $\mathcal{A}$, through obstacles region Oin workspace $\mathcal{W}$. C-space is an essential structural and pre-defined idea for path planning, and is broadly used in path planning problems as it also provides a consistent reference structure, allowing evaluation and assessment of different algorithms [12, 29]. Configuration of $\mathcal{A}$can be representedby defining its centre point at$q$ = (x, y) relative to some fixed coordinate frame [41, 49]. The set of points occupied by $\mathcal{A}$ can be determined from the configuration q, if the radius r (or a distance from the furthest point to the centre) of $\mathcal{A}$ is known. If the notation $V(q)$ represent the set of points, then

$$V(x,y) = \{(x',y')|(x-x')^2 + (y-y')^2 \leq r^2\}$$

The above notation shows that, it is sufficient for x and y to completely specify the configuration of $\mathcal{A}$. In C-space with an obstacle region $O = \{o_1, o_2,\ldots,o_p\}$, the set of configuration of obstacle region at which A will intersect $O_i$is defined as,

$$QOi = \{q \in Q|V(q) \cap Oi \neq 0\}$$

Conversely, the free configuration space in which the vehicle will traverse is,

$$Qfree = Q/(\cup\, iQOi)$$

2.2.2 C-Space Representation Techniques

Once, the idea of C-space is implied to the environment, the next step is to represent the C-space. Representation techniques for C-space can be fundamentally categorized in to three categories namely, Cell decomposition (CD), Roadmap (RM) and Potential fields (PF). Mainly, path planning methods fall under one of these categories. Fig. 2.1 shows the categorized C-space representation techniques.

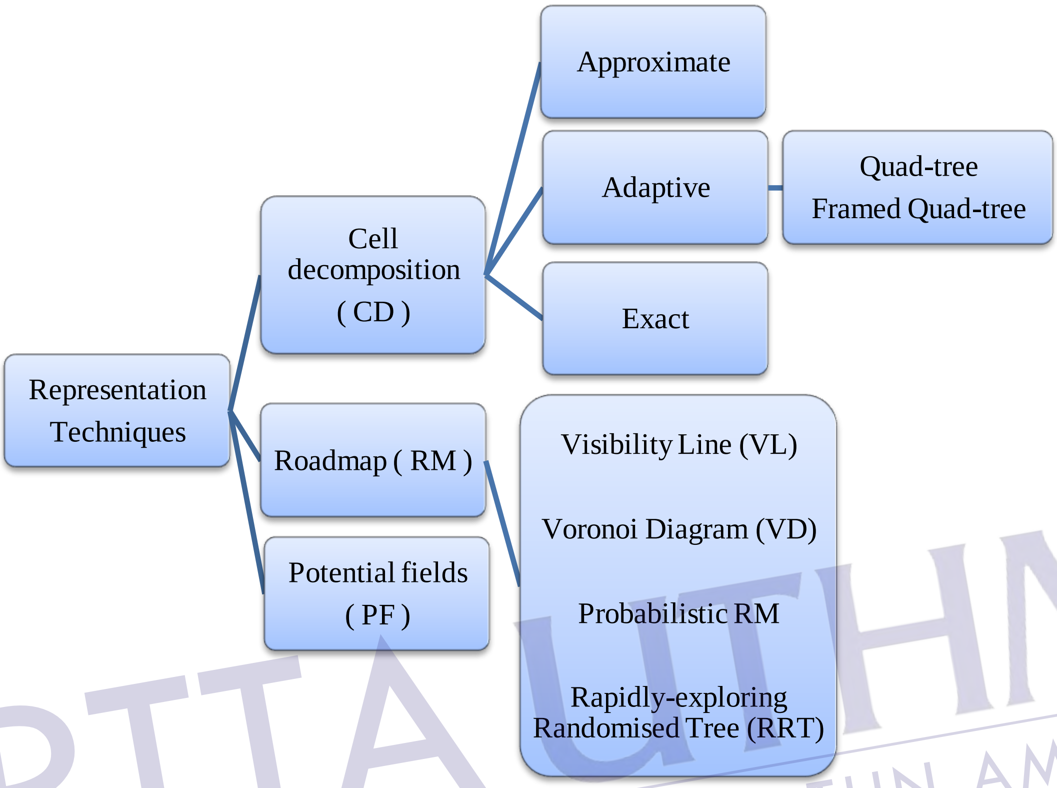

Figure 2.1Path planning representation techniques categories

2.3 Rapidly-Exploring Random Trees (RRTs)

A Rapidly-exploring Random Tree (RRT) is a data structure and algorithm that is designed for efficiently searching non-convex high-dimensional spaces. RRTs are constructed incrementally in a way that quickly reduces the expected distance of a randomly-chosen point to the tree. RRTs are particularly suited for path planning problems that involve obstacles and differential constraints (non-holonomic or Kino-dynamic). RRTs can be considered as a technique for generating open-loop trajectories for nonlinear systems with state constraints. An RRT can be intuitively

REFERENCES

[1] Tian, Yu, et al. "Application of RRT-based local path planning algorithm in unknown environment." *Computational Intelligence in Robotics and Automation, 2007.CIRA 2007.International Symposium on*.IEEE, 2007.

[2] Kothari, Mangal, Ian Postlethwaite, and Da-Wei Gu. "A Suboptimal Path Planning Algorithm Using Rapidly-exploring Random Trees." *International Journal of Aerospace Innovations* 2.1 (2010): 93-104.

[3] Kothari, Mangal, Ian Postlethwaite, and Da-Wei Gu. "Multi-UAV path planning in obstacle rich environments using rapidly-exploring random trees."*Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009.Proceedings of the 48th IEEE Conference on*.IEEE, 2009.

[4] Glavina, Bernhard. "Solving findpath by combination of goal-directed and randomized search."*Robotics and Automation, 1990.Proceedings., 1990 IEEE International Conference on*. IEEE, 1990.

[5] Henrich, Dominik, and Xiaoqing Cheng. "Fast distance computation for on-line collision detection with multi-arm robots." *Robotics and Automation, 1992.Proceedings., 1992 IEEE International Conference on*. IEEE, 1992.

[6] Challou, Daniel, et al. "Very fast motion planning for dexterous robots." *Assembly and Task Planning, 1995.Proceedings., IEEE International Symposium on*. IEEE, 1995.

[7] Henrich, Dominik, and Thomas Honiger. "Parallel processing approaches in robotics." *Industrial Electronics, 1997.ISIE'97., Proceedings of the IEEE International Symposium on*. Vol. 2.IEEE, 1997.

[8] Kala, Rahul, and Kevin Warwick. "Planning of multiple autonomous vehicles using RRT."*Cybernetic Intelligent Systems (CIS), 2011 IEEE 10th International Conference on*.IEEE, 2011.

[9] Rodriguez, Samuel, et al. "An obstacle-based rapidly-exploring random tree." *Robotics and Automation, 2006.ICRA 2006.Proceedings 2006 IEEE International Conference on*.IEEE, 2006.

[10] Esposito, J. M. "A simplified model of RRT coverage for kinematic systems." *Intelligent Robots and Systems (IROS), 2011 IEEE/RSJ International Conference on*.IEEE, 2011.

[11] Karaman, Sertac, et al. "Anytime motion planning using the RRT*." *Robotics and Automation (ICRA), 2011 IEEE International Conference on*.IEEE, 2011.

[12] LaValle, Steven M., and James J. Kuffner Jr. "Rapidly-exploring random trees: Progress and prospects." (2000).

[13] Alcázar, Vidal, Manuela Veloso, and Daniel Borrajo. "Adapting a Rapidly-Exploring Random Tree for Automated Planning."*Fourth Annual Symposium on Combinatorial Search*. 2011.

[14] Bry, Adam, and Nicholas Roy. "Rapidly-exploring random belief trees for motion planning under uncertainty." *Robotics and Automation (ICRA), 2011 IEEE International Conference on*.IEEE, 2011.

[15] Van Den Berg, Jur, Pieter Abbeel, and Ken Goldberg. "LQG-MP: Optimized path planning for robots with motion uncertainty and imperfect state information." *The International Journal of Robotics Research* 30.7 (2011): 895-913.

[16] Fulgenzi, Chiara, Anne Spalanzani, and Christian Laugier. "Probabilistic rapidly-exploring random trees for autonomous navigation among moving obstacles."*Workshop on safe navigation, IEEE International Conference on Robotics and Automation (ICRA)*. 2009.

[17] Kim, Jongwoo, and James P. Ostrowski. "Motion planning a aerial robot using rapidly-exploring random trees with dynamic constraints." *Robotics and Automation, 2003.Proceedings.ICRA'03.IEEE International Conference on*.Vol. 2.IEEE, 2003.

[18] Cheng, Peng, and Steven M. LaValle. "Resolution complete rapidly-exploring random trees." *Robotics and Automation, 2002.Proceedings.ICRA'02.IEEE International Conference on*.Vol. 1.IEEE, 2002.

[19] Rodriguez, Samuel, et al. "An obstacle-based rapidly-exploring random tree." *Robotis and Automation, 2006.ICRA 2006.Proceedings 2006 IEEE International Conference on*.IEEE, 2006.

[20] Xu, Jijie, et al. "Motion planning for steerable needles in 3D environments with obstacles using rapidly-exploring random trees and backchaining." *Automation Science and Engineering, 2008.CASE 2008.IEEE International Conference on*.IEEE, 2008.

[21] Fulgenzi, Chiara, et al. "Probabilistic navigation in dynamic environment using Rapidly-exploring Random Trees and Gaussian Processes." *Intelligent Robots and Systems, 2008. IROS 2008.IEEE/RSJ International Conference on*.IEEE, 2008.

[22] KuffnerJr, James J., and Steven M. LaValle. "RRT-connect: An efficient approach to single-query path planning." *Robotics and Automation, 2000.Proceedings.ICRA'00.IEEE International Conference on*.Vol. 2.IEEE, 2000.

[23] Bruce, James, and Manuela Veloso. "Real-time randomized path planning for robot navigation."*Intelligent Robots and Systems, 2002.IEEE/RSJ International Conference on*.Vol. 3.IEEE, 2002.

[24] LaValle, Steven M., and James J. Kuffner Jr. "Randomized kinodynamic planning." *Robotics and Automation, 1999.Proceedings.1999 IEEE International Conference on*.Vol. 1.IEEE, 1999.

[25] Ferguson, Dave, NidhiKalra, and Anthony Stentz. "Replanning with RRTs."*Robotics and Automation, 2006.ICRA 2006.Proceedings 2006 IEEE International Conference on*.IEEE, 2006.

[26] Urmson, Chris, and Reid Simmons. "Approaches for heuristically biasing RRT growth."*Intelligent Robots and Systems, 2003.(IROS 2003). Proceedings.2003 IEEE/RSJ International Conference on*.Vol. 2.IEEE, 2003.

[27] Szádeczky-Kardoss, Emese, and Bálint Kiss. "Extension of the rapidly exploring random tree algorithm with key configurations for non-holonomic motion planning."*Mechatronics, 2006 IEEE International Conference on*. IEEE, 2006.

[28] Garcia, Ian, and Jonathan P. How. "Improving the efficiency of rapidly-exploring random trees using a potential function planner."*Decision and Control, 2005 and 2005 European Control Conference.CDC-ECC'05. 44th IEEE Conference on*.IEEE, 2005.

[29] Fulgenzi, Chiara, Anne Spalanzani, and Christian Laugier. "Probabilistic rapidly-exploring random trees for autonomous navigation among moving obstacles."*Workshop on safe navigation, IEEE International Conference on Robotics and Automation (ICRA)*. 2009.

[30] Atramentov, Anna, and Steven M. LaValle. "Efficient nearest neighbor searching for motion planning."*Robotics and Automation, 2002.Proceedings.ICRA'02.IEEE International Conference on*.Vol. 1.IEEE, 2002.

[31] Ferguson, Dave, and Anthony Stentz. "Anytime rrts."*Intelligent Robots and Systems, 2006 IEEE/RSJ International Conference on*.IEEE, 2006.

[32] Han, Li, and Nancy M. Amato. "A kinematics-based probabilistic roadmap method for closed chain systems." (2000).

[33] Yershova, Anna, et al. "Dynamic-domain RRTs: Efficient exploration by controlling the sampling domain." *Robotics and Automation, 2005.ICRA 2005.Proceedings of the 2005 IEEE International Conference on*.IEEE, 2005.

[34] Bertram, Dominik, et al. "An integrated approach to inverse kinematics and path planning for redundant manipulators." *Robotics and Automation, 2006.ICRA 2006.Proceedings 2006 IEEE International Conference on*.IEEE, 2006.

[35] Haitao, Guo, Zhu Qingbao, and XuShoujiang. "Rapid-Exploring Random Tree Algorithm for Path Planning of Robot Based on Grid Method [J]." *Journal of Nanjing Normal University (Engineering and Technology Edition)* 2 (2007): 014.

[36] Kuffner, James, et al. "Motion planning for humanoid robots under obstacle and dynamic balance constraints." *Robotics and Automation, 2001.Proceedings 2001 ICRA.IEEE International Conference on*.Vol. 1.IEEE, 2001.

[37] Krejsa, Jirı, and StanislavVechet. "Rapidly Exploring Random Trees Used for Mobile Robots Path Planning." *Engineering Mechanics* 12.4 (2005): 231-237.

[38] KANG, Liang, Chun-Xia ZHAO, and Jian-HuiGUO. "Improved Path Planning Based on Rapidly-Exploring Random Tree for Mobile Robot in Unknown Environment [J]".*Pattern Recognition and Artificial Intelligence* 3 (2009).

[39] Lindemann, Stephen R., and Steven M. LaValle. "Incrementally reducing dispersion by increasing Voronoi bias in RRTs."*Robotics and Automation, 2004.Proceedings. ICRA'04.2004 IEEE International Conference on*.Vol. 4.IEEE, 2004.

[40] Bekris, Kostas E., et al. "Multiple query probabilistic roadmap planning using single query planning primitives."*Intelligent Robots and Systems, 2003.(IROS 2003). Proceedings.2003 IEEE/RSJ International Conference on*.Vol. 1.IEEE, 2003.

[41] Esposito, Joel M., Jongwoo Kim, and Vijay Kumar. "Adaptive RRTs for validating hybrid robotic control systems." *Algorithmic Foundations of Robotics VI* (2005): 107- 121.

[42] Li, Tsai-Yen, and Yang-ChuanShie. "An incremental learning approach to motion planning with roadmap management."*Robotics and Automation, 2002.Proceedings. ICRA'02.IEEE International Conference on*.Vol. 4.IEEE, 2002.

[43] VandeWeghe, M., Dave Ferguson, and Siddhartha S. Srinivasa."Randomized path planning for redundant manipulators without inverse kinematics."*Humanoid Robots, 2007 7th IEEE-RAS International Conference on*.IEEE, 2007.

[44] Zucker, Matt, James Kuffner, and Michael Branicky. "Multipartite RRTs for rapid replanning in dynamic environments."*Robotics and Automation, 2007 IEEE International Conference on*.IEEE, 2007.

[45] Melchior, Nik A., and Reid Simmons. "Particle RRT for path planning with uncertainty." *Robotics and Automation, 2007 IEEE International Conference on*.IEEE, 2007.

[46] Kallman, M., and MajaMataric. "Motion planning using dynamic roadmaps."*Robotics and Automation, 2004.Proceedings.ICRA'04.2004 IEEE International Conference on*. Vol. 5.IEEE, 2004.

[47] Akinc, Mert, et al. "Probabilistic roadmaps of trees for parallel computation of multiple query roadmaps." *Robotics Research* (2005): 80-89.

[48] Jaillet, Léonard, et al. "Adaptive tuning of the sampling domain for dynamic-domain RRTs." *Intelligent Robots and Systems, 2005.(IROS 2005). 2005 IEEE/RSJ International Conference on*.IEEE, 2005.

[49] Branicky, Michael S., and Michael M. Curtiss. "Nonlinear and hybrid control via RRTs." *Proc. Intl. Symp. on Mathematical Theory of Networks and Systems*. Vol. 750. 2002.

[50] Spero, Dorian J., and Ray A. Jarvis. "Path planning for a mobile robot in a rough terrain environment."*Robot Motion and Control, 2002.RoMoCo'02.Proceedings of the Third International Workshop on*.IEEE, 2002.

[51] Vonásek, Vojtěch, et al. "RRT-path–A Guided Rapidly Exploring Random Tree." *Robot Motion and Control 2009* (2009): 307-316.

[52] Kuwata, Yoshiaki, et al. "Motion planning for urban driving using RRT." *Intelligent Robots and Systems, 2008.IROS 2008.IEEE/RSJ International Conference on*.IEEE, 2008.

[53] Yoshida, Eiichi. "Humanoid motion planning using multi-level DOF exploitation based on randomized method." *Intelligent Robots and Systems, 2005.(IROS 2005). 2005 IEEE/RSJ International Conference on*.IEEE, 2005.

[54] Pepy, Romain, and Alain Lambert. "Safe path planning in an uncertain-configuration space using RRT."*Intelligent Robots and Systems, 2006 IEEE/RSJ International Conference on*.IEEE, 2006.

[55] Kuffner, James J., et al. "Dynamically-stable motion planning for humanoid robots." *Autonomous Robots* 12.1 (2002): 105-118.

[56] Burns, Brendan, and Oliver Brock. "Single-query motion planning with utility-guided random trees."*Robotics and Automation, 2007 IEEE International Conference on*.IEEE, 2007.

[57] Kagami, Satoshi, et al. "Humanoid arm motion planning using stereo vision and RRT search." *Intelligent Robots and Systems, 2003.(IROS 2003). Proceedings. 2003 IEEE/RSJ International Conference on*. Vol. 3.IEEE, 2003.

[58] Carpin, Stefano, and Enrico Pagello. "On parallel RRTs for multi-robot systems."*Proc. 8th Conf. Italian Association for Artificial Intelligence*. 2002.

[59] Lindemann, Stephen R., and Steven M. LaValle. "Current issues in sampling-based motion planning."*Robotics Research* (2005): 36-54.

[60] Berenson, Dmitry, et al. "Manipulation planning on constraint manifolds." *Robotics and Automation, 2009.ICRA'09.IEEE International Conference on*.IEEE, 2009.

[61] Levine, Daniel S. *Information-rich path planning under general constraints using rapidly-exploring random trees*. Diss. Massachusetts Institute of Technology, 2010.

[62] Wzorek, Mariusz, and Patrick Doherty. "Reconfigurable path planning for an autonomous unmanned aerial vehicle."*Hybrid Information Technology, 2006.ICHIT'06.International Conference on*.Vol. 2.IEEE, 2006.

[63] Rodriguez, Samuel, Jyh-Ming Lien, and Nancy M. Amato. "Planning motion in completely deformable environments."*Robotics and Automation, 2006.ICRA 2006.Proceedings 2006 IEEE International Conference on*.IEEE, 2006.

[63] Branicky, Michael S., et al. "RRTs for nonlinear, discrete, and hybrid planning and control." *Decision and Control, 2003.Proceedings.42nd IEEE Conference on*.Vol. 1.IEEE, 2003.

[64] Lee, Heon-Cheol, et al. "Visual route navigation using an adaptive extension of Rapidly-exploring Random Trees." *Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on*.IEEE, 2010.

[65] Shkolnik, Alexander, Matthew Walter, and Russ Tedrake. "Reachability-guided sampling for planning under differential constraints."*Robotics and Automation, 2009.ICRA'09.IEEE International Conference on*.IEEE, 2009.

[66] Ferguson, Dave, and Anthony Stentz. "Anytime, dynamic planning in high-dimensional search spaces."*Robotics and Automation, 2007 IEEE International Conference on*.IEEE, 2007.

[67] KANG, Liang, Chun-xia ZHAO, and Jian-hui GUO. "Path Planning Based on Fuzzy Rolling Rapidly-exploring Random Tree for Mobile Robot." *Journal of Nanjing University of Science and Technology (Natural Science)* 5 (2010): 012.

[68] Saunders, Jeffery B., et al. "Static and dynamic obstacle avoidance in miniature air vehicles." *Proc. Infotech@ Aerospace Conf*. 2005.

[69] Alterovitz, Ron, SachinPatil, and Anna Derbakova. "Rapidly-exploring roadmaps: Weighing exploration vs. refinement in optimal motion planning." *Robotics and Automation (ICRA), 2011 IEEE International Conference on*.IEEE, 2011.

[70] Carpin, Stefano, and GianluigiPillonetto. "Motion planning using adaptive random walks."*Robotics, IEEE Transactions on* 21.1 (2005): 129-136.

[71] Abbasi-Yadkori, Yasin, Joseph Modayil, and CsabaSzepesvari. "Extending rapidly-exploring random trees for asymptotically optimal anytime motion

planning." *Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on*.IEEE, 2010.

[72] Plaku, Erion, et al. "Sampling-based roadmap of trees for parallel motion planning." *Robotics, IEEE Transactions on* 21.4 (2005): 597-608.

[73] Ettlin, Alan, and HannesBleuler. "Randomised rough-terrain robot motion planning."*Intelligent Robots and Systems, 2006 IEEE/RSJ International Conference on*.IEEE, 2006.

[74] Levine, Daniel, Brandon Luders, and Jonathan P. How. "Information-rich path planning with general constraints using rapidly-exploring random trees." (2010).

[75] Yang, Kwangjin, and Salah Sukkarieh. "Real-time continuous curvature path planning of UAVs in cluttered environments." *Mechatronics and Its Applications, 2008. ISMA 2008.5th International Symposium on*.IEEE, 2008.

[76] Fraichard, Thierry, and Hajime Asama. "Inevitable collision states—a step towards safer robots?." *Advanced Robotics* 18.10 (2004): 1001-1024.

[77] Zhang, Liangjun, and Dinesh Manocha. "An efficient retraction-based RRT planner."*Robotics and Automation, 2008.ICRA 2008.IEEE International Conference on*.IEEE, 2008.

[78] Sugiura, Hisashi, et al. "Real-time collision avoidance with whole body motion control for humanoid robots." *Intelligent Robots and Systems, 2007.IROS 2007.IEEE/RSJ International Conference on*.IEEE, 2007.

[79] Nakhost, Hootan, and Martin Müller. "Monte-Carlo exploration for deterministic planning."*IJCAI*. 2009.

[80] Jaillet, Léonard, and Thierry Siméon. "A PRM-based motion planner for dynamically changing environments."*Intelligent Robots and Systems, 2004.(IROS 2004). Proceedings.2004 IEEE/RSJ International Conference on*.Vol. 2.IEEE, 2004.

[81] Yang, Libo, and Steven M. LaValle. "A framework for planning feedback motion strategies based on a random neighborhood graph." *Robotics and Automation, 2000.Proceedings.ICRA'00.IEEE International Conference on*.Vol. 1.IEEE, 2000.

[82] Rao, Dushyant, and Stefan B. Williams. "Large-scale path planning for Underwater Gliders in ocean currents."*Australasian Conference on Robotics and Automation (ACRA), Sydney*. 2009.

[83] WANG, Hua, et al. "A new path planning method based On RRT." *Journal of Harbin Institute of Technology* 7 (2004): 039.

[84] Neto, Armando Alves, Douglas G. Macharet, and Mario FM Campos. "On the generation of trajectories for multiple UAVs in environments with obstacles."*Selected papers from the 2nd International Symposium on UAVs, Reno, Nevada, USA June 8–10, 2009*.Springer Netherlands, 2010.

[85] Cerven, W. Todd, Francesco Bullo, and Victoria L. Coverstone. "Vehicle motion planning with time-varying constraints."*Journal of guidance, control, and dynamics* 27.3 (2004): 506-509.

[86] Yang, Kwangjin, and Salah Sukkarieh. "3D smooth path planning for a UAV in cluttered natural environments." *Intelligent Robots and Systems, 2008.IROS 2008.IEEE/RSJ International Conference on*.IEEE, 2008.

[87] Dalibard, Sébastien, and Jean-Paul Laumond. "Control of probabilistic diffusion in motion planning."*Algorithmic Foundation of Robotics VIII* (2009): 467-481.

[88] Jarvis, Ray. "Robot path planning: complexity, flexibility and application scope." *ACM International Conference Proceeding Series*.Vol. 213. 2006.

[89] Jouandeau, Nicolas. "Rapidly-exploring sorted random tree: a self adaptive random motion planning algorithm." *Informatics in Control, Automation and Robotics* (2009): 63-73.

[90] Frazzoli, Emilio, Munther A. Dahleh, and Eric Feron. "Real-time motion planning for agile autonomous vehicles."*IEEE CONFERENCE ON DECISION AND CONTROL*.Vol. 1.IEEE; 1998, 2000.

[91] Lee, Jinhan, Charles Pippin, and Tucker Balch. "Cost based planning with RRT in outdoor environments." *Intelligent Robots and Systems, 2008.IROS 2008.IEEE/RSJ International Conference on*.IEEE, 2008.

[92] Melchior, Nik A., Jun-young Kwak, and Reid Simmons. "Particle RRT for Path Planning in very rough terrain." *NASA Science Technology Conference 2007 (NSTC 2007)*. 2007.

[93] Vahrenkamp, Nikolaus, et al. "Adaptive motion planning for humanoid robots." *Intelligent Robots and Systems, 2008.IROS 2008.IEEE/RSJ International Conference on*.IEEE, 2008.