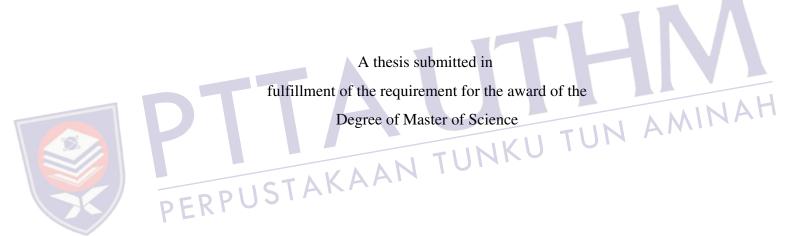
# UNSTEADY BLOOD FLOW IN THE STENOSED ARTERY SUBJECTED TO MAGNETIC FIELD AND INJECTED NANOPARTICLES

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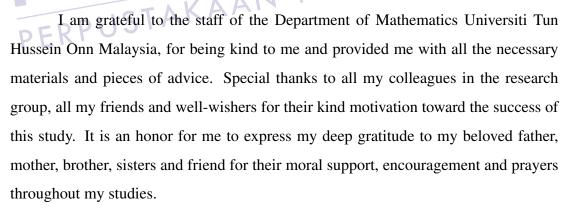
This thesis is consecrated to my beloved parents; Haji Jamil bin Ahiya' and Hajah Gayah binti Panot.



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#### ABSTRACT

The blood flow in the stenosed artery was investigated in the current work. The blood, which was modelled as Newtonian or non-Newtonian fluid, was subjected to an oscillating pressure gradient and a periodic body acceleration. In the first problem, the magnetic field and the porosity were considered. In the second problem, the augmentation of heat transfer due to drug carriers such as nanoparticles ( $Fe_3O_4$ ,  $TiO_2$ , Cu) was modelled. In the first problem, the non-dimensional equation was solved by combining both perturbation and power series methods. However, for the second problem, only the perturbation method was used. The MATHCAD software was adopted to find the numerical figures of the analytical solutions. The presence of magnetic field tended to decelerate the blood flow in the stenosed artery due to strong resistance. However, the blood velocity increased with respect to the body acceleration, the pressure gradient and the permeability parameter. For the second problem, it was observed that the velocity increased with respect to the slip velocity and the body acceleration but decreased as the yield stress and the pressure gradient increased. The temperature of blood mixed with Fe<sub>3</sub>O<sub>4</sub> was higher as compared to those with  $TiO_2$  and Cu nanoparticles, thus implying that  $Fe_3O_4$  could be used as a diagnosis tool for drug delivery in stenosis treatment. The blood temperature increased slightly along the stream wise direction before reaching the constricted region. The temperature distributions were significantly dependent on periodic body acceleration, pressure gradient and Prandtl number.



#### ABSTRAK

Aliran darah di dalam arteri yang tersumbat telah dikaji di dalam thesis ini. Bendalir darah yang telah dimodelkan sebagai cecair Newtonian atau bukan Newtonian adalah tertakluk kepada kecerunan tekanan yang berayun dan pecutan jasad berkala. Dalam masalah pertama, medan magnet dan kesan keliangan telah diambil kira. Masalah kedua pula membincangkan kesan terhadap kehadiran nanopartikel (Fe<sub>3</sub>O<sub>4</sub>, TiO<sub>2</sub>, Cu) di dalam aliran darah. Di dalam kajian ini, persamaan yang diperoleh akan dibentuk ke dalam bentuk yang tak berdimensi dengan menggunakan pembolehubah tak berdimensi yang sesuai. Penyelesaian masalah pertama akan diselesaikan dengan menggunakan kaedah gabungan usikan dan siri kuasa. Permasalahan kedua pula hanya menggunakan kaedah usikan dalam proses penyelesaian. Perisian MATHCAD telah diguna kepada penyelesaian analitik yang diperoleh untuk mendapat data berangka. Kehadiran medan magnet cenderung melemahkan aliran darah dalam arteri disebabkan oleh rintangan yang kuat. Walau bagaimanapun, kelajuan aliran darah meningkat apabila kadar pecutan badan, kecerunan tekanan dan parameter keliangan meningkat. Dalam masalah kedua pula, didapati bahawa kelajuan aliran meningkat dengan kelajuan slip dan pecutan jasad tetapi menurun apabila hasil tekanan dan kecerunan tekanan meningkat. Suhu bendalir darah yang bercampur dengan  $Fe_3O_4$  adalah lebih tinggi berbanding dengan campuran darah dengan nanopartikel TiO<sub>2</sub> dan Cu. Oleh yang demikian Fe<sub>3</sub>O<sub>4</sub> boleh digunakan sebagai alat diagnosis untuk penghantaran dadah dalam rawatan stenosis. Suhu darah meningkat sedikit di sepanjang aliran darah sebelum sampai ke kawasan sempit. Taburan suhu adalah ketara bergantung kepada pecutan badan berkala, kecerunan tekanan dan nombor Prandtl.



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### LIST OF ABBREVIATIONS

- BFD Biomagnetic fluid dynamics
- MDT Magnetic drug targeting
- FHD Ferrohydrodynamics flow
- MHD Magnetohydrodynamic flow
- ADI Alternating direction implicit
- HPM Homotopy perturbation method



## NOMENCLATURE

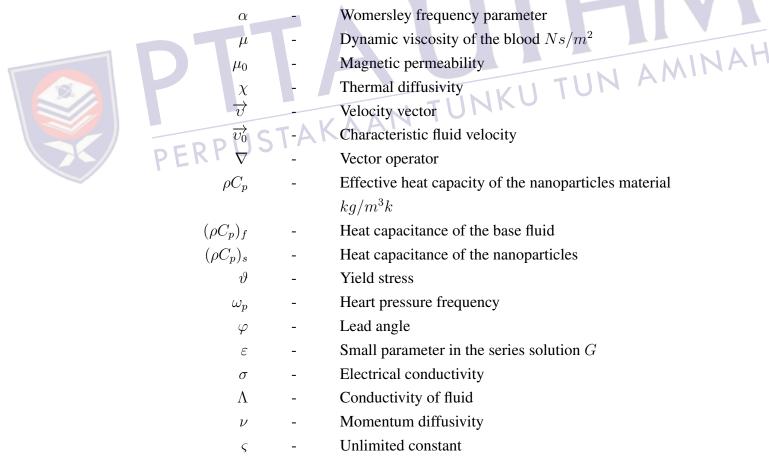
## **Roman Letters**

	$a_n$	-	n th term coefficient
	$A_0$	-	Steady-state part of the pressure gradient
	$A_1$	-	Amplitude of the pressure fluctuation
	$A_g$	-	Amplitude of the acceleration
	$B_0$	-	Externally applied constant magnetic
	$B_0 \over \overrightarrow{B}$	-	Magnetic flux intensity
	С	-	Constant
	$C_p$	-	Heat capacity of the particle
	d	-	Region of the normal artery
	Ec	-	Eckert number
	$\overrightarrow{E}$	-	Electric field intensity
	$\overrightarrow{f}$	-	Body force
	$f_p$	-	Pulse rate frequency
8	f(t)	-	Function due to body acceleration and pressure gradient
			as mentioned in Equation (4.19)
	$F(t) \subseteq T$	AK	Body acceleration
	$F_0, F_1, F_2, \dots$	-	Coefficients in series solution
	g	-	Gravity acceleration
	$G_0$	-	Exact solution of the initial problem
	$G_1, G_2, \dots$	-	Higher order terms
	Ha	-	Hartmann number as mentioned in Equation (4.19)
	$I_0$	-	Modified Bessel function of order zero
	$\overrightarrow{J}$	-	Current density
	k	-	Thermal conductivity of blood
	K	-	Permeability parameter
	$\overrightarrow{K}$	-	Dissipation function representing work done against
			viscous forces
	$K_0$	-	Magnetic permeability
	$K_1$	-	Porosity parameter
	L	-	Length scale characteristics
	$L_0$	-	length of the stenosis
	m	-	Parameter in determining stenosis shape
	Nt	-	Thermophoresis parameter

### **Greek Letters**

p	-	Pressure
Pr	-	Prandtl number
Q	-	Function of Hartmann number and porosity parameter
r	-	Radial coordinate
$R_0$	-	Radius of the normal artery
$R\left(z ight)$	-	Radius of the artery in the stenosed region
t	-	Time
T	-	Temperature
$T_w$	-	Wall temperature
$T_0$	-	Bulk fluid temperature
u	-	Non-dimensional velocity
$u_s$	-	Slip velocity at the stenotic walls
z	-	Axial direction

## **Greek Letters**



 $ho_{nf}$  - Effective density of nanofluid,  $kg/m^3$ 

## **Greek Letters**

$\rho$	-	Density, $kg/m^3$
$ ho_f$	-	Density of blood
$\rho_s$	-	Density of dispersed copper nanoparticles
$\xi_s$	-	Maximum height of the stenosis
au	-	Shear stress
$ au_y$	-	Yield stress
$\mu_{nf}$	-	Effective dynamic viscosity of the nanofluid
$\phi$	-	Solid volume fraction
$\varphi$	-	Lead angle of the body acceleration with respect to
		pressure gradient

## Superscripts

-	* -	Non-dimensional sign while transforming the dimen- sional variables
	· _	First order differentiation with respect to $t$
Subscript	f -	Blood Nanofluid Surface



### **CHAPTER 1**

#### **INTRODUCTION**

#### 1.1 Research background

The vascular system (or cardiovascular framework) circulates blood in order to supply oxygen and nutrients to all organs. The study of hemodynamics in human is gaining popularity nowadays since it is important for clinical analyses of infection and more intricate diseases (e.g. (Buchanan, Kleinstreuer, & Comer, 2000), (Mandal, 2005), (Hernan & Gonzalez, 2007), (Ismail *et al.*, 2008)).

Large number of studies have been available to understand disorders in a blood flow that is induced due to stenosis by treating blood as a Newtonian or non-Newtonian fluid. Blood was normally treated as Newtonian fluid in large artery. Numerical solutions of the blood flow in the stenosed artery have been described by several investigators. Prakash *et al.* (2004) discussed the velocity profile, resistive impedance and the variation of the wall shear stress through an arterial stenosis by assuming blood as a Newtonian fluid. Tzirtzilakis (2005) designed the mathematical model of biomagnetic fluid dynamics which suitable for the description of Newtonian blood flow with the presence of magnetic field. This model was consistent with the principles of ferrohydrodynamics and magnetohydrodynamics and takes into account both magnetization and electrical conductivity of blood.

Nevertheless, the non-Newtonian effect becomes more apparent in medium and small sized arteries. Sankar & Hemalatha (2007) studied the effects of non-Newtonian nature of blood in small arteries by assigning the blood as a Herschel-Bulkley fluid



subjected to various physiological conditions. The influences of velocity distributions, flow rate, wall shear stress and frictional resistance were investigated by assuming the flow to be steady. Rabby *et al.* (2014) conducted a numerical investigation of non-Newtonian modelling effects on the unsteady periodic flows in a two-dimensional pipe with stenosis. The investigation has been carried out to characterize four different non-Newtonian constitutive equations of blood, namely, the Carreau, Cross, Modified Casson, and Quemada models.

So far, many mathematical models on blood flow have been developed ((Maikap, Mahapatra, & Niyogi, 2005), (Layek and Midya, 2007) and (Chakravarty, Mandal, & Layek, 2007)). Bio-magnetic fluid dynamics (BFD) is a branch of fluid mechanics that deals with the application of magnetic field in biological fluids. The bio-magnetic fluid model could be used to study blood rheology as blood can be treated as an electrically non-conducting magnetic fluid (Haik, Pai, & Chen, 1999). Also, blood carries erythrocytes that contain haemoglobin molecules (rich in iron). Its magnetic property is affected by the elements of oxygenated (Sharma *et al.*, 2015).



Some applications of BFD in bioengineering and medical sciences are cell separation magnetic devices, magnetic particles (or drug carriers), wound and cancer tumour treatment, reduction of bleeding during surgeries, provocation of occlusion of the feeding vessels of cancer tumors and development of magnetic tracers ((Ruuge & Rusetski, 1993), (Plavins & Lauva, 1993), (Haik, Pai, & Chen, 1999)). The computational models would give an insight of the feasibility of a medical technique before the real clinical trial is attempted ((Haverkort & Kenjeres, 2008) and (Haverkort & Kleijn, 2009)). The effects of magnetism and Lorentz force should be included in the governing equation of blood flow for accuracy purpose (Kenjeres & Opdam, 2009).

### **1.2 Problem statement**

Nowadays, death due to coronary artery disease or stenosis is common. The blood circulation is disturbed by the formation of plaque inside the wall artery. This study was motivated by a previous study that reported on blood flow in the presence of

stenosed artery. This previous study, however, did not address the effects of external forces (e.g. magnetic field). Stemming from the fact that magnetic field could regulate the blood flow rate (blood flow control), the effects of external magnetic forces and porosity parameter were studied in the current work. Previous researchers have analyzed blood flow in normal artery subjected to body acceleration and oscillating pressure gradient. In the current work, the presence of nanoparticles in the stenosed artery was analyzed instead. The blood was driven by periodic body acceleration and pressure gradient (mimicking the body movement and the heart pumping action).

### **1.3** Objectives of the study

The main objective is to analyze the unsteady blood flow in the stenosed artery subjected to both body acceleration and pressure gradient. The blood flow models are :



i Incompressible Newtonian fluid subjected to the magnetic field;
ii Similar to (i). However, the artery is considered as a porous medium;
iii Incompressible non-Newtonian fluid with injected nanoparticles as drug carriers.

### **1.4** Scope of the study

The flow problem was treated as 2D in the current study. The blood was treated as incompressible as well. For the first problem, the external magnetic field was applied in the stenosed porous artery without considering the effect of slip velocity. However, the effect of slip velocity was considered in the second problem. The magnetic field was assumed to be dependent on the electrical conductivity. Heat transfer due to collisions of drug particles would be significant.

### **1.5** Significance of study

The outcomes of this research are:

- i To characterize the stenosed blood flow via mathematical modelling.
- ii The use of magnetic field in blood flow control is important for the advancement in medical science.
- iii The application of magnetic field could reduce the blood loss during surgery and targeted drug delivery.

### 1.6 Research methodology

The non-dimensional governing equations were obtained by substituting the nondimensional quantities into the dimensional blood flow equations. The analytical approaches of perturbation method and/or power series method were used to solve the non-dimensional governing equations. The computations were executed using the MATHCAD software. The numerical results were then verified.



## 1.7 Thesis organization

This thesis consists of six chapters. The first chapter provides an overview of the current research including the background of research, the objectives, the scope and the significance of the study. The second chapter reviews some important previous works that are related to the current work. The analytical techniques and the non-dimensional numbers are presented in chapter 3. The numerical results are discussed in chapters 4 and 5. Chapter 4 discusses the analytical solutions of the first problem. The governing equation of blood flow has been transformed into the non-dimensional one by considering the similarity transformation quantities. Chapter 5 presents the non-isothermal effect due to the employment of nanoparticles. The same transformation

process as chapter 4 was used in this chapter. Chapter 6 concludes the current work and offers some suggestions for future work.



### **CHAPTER 2**

#### LITERATURE REVIEW

#### 2.1 Introduction

This chapter reviews the previous works that are related to the current research, such as those highlighting on the effects of magnetic field and permeability on blood flow. The effects of nanoparticles on blood flow are highlighted as well.

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## 2.2 Stenosed artery

Artery stenosis (or atherosclerosis) is an arterial disease caused by accumulation of fats and fibrous tissue in the lumen of arterial wall. The presence of stenosis is detrimental to a blood circulatory system. The study of blood flow in stenosed arteries has been reported by many researchers. Chakravarty & Mandal (2000) studied the two-dimensional blood flow through tapered arteries with stenosis. They treated the blood as Newtonian fluid and solved the governing Navier-Stokes equations by using an analytical approach. From the numerical results, the blood flow characteristics were obtained. Liu *et al.* (2004) reported that stenosis restricted the flow field at the end of stenotic and tapered arteries. Sankar & Lee (2008) investigated analytically the pulsatile Herschel Bulkely fluid flow in stenosed arteries with the help of regular perturbation method. Based on their investigations, they found that the velocity and flow rate profiles increased as the yield stress ratio increased while fixing the other governing parameters. The regular perturbation method was used by Akbar (2016a) in order to study the characteristics of Walters B fluid model in tapered arteries with

stenosis. Later, Akbar (2016b) analyzed the tangent hyperbolic blood flow through tapered artery with mild stenosis. The nonlinear governing equations of blood flow were solved analytically by using the perturbation method. The solutions of shear stress, velocity, flow rate and longitudinal impedance were obtained and discussed. Recently, the slip effects on the tapered artery wall with mild stenosis have been studied by Ijaz & Nadeem (2016b) by treating the working fluid as Newtonian.

Flow cases involving different stenosed arteries have been studied by several researchers. Mandal et al. (2007) investigated the effects of periodic body acceleration on blood flow. They considered the arterial wall as an elastic cylindrical tube containing a stenosis on the lumen. By using the finite difference scheme, the effects of periodic body acceleration, blood flow velocity, flow rate, shear stress and flow speed were discussed by using the graphical solutions. Mishra et al. (2011) modelled the blood flow through a composite stenosis in an artery with permeable wall. Based on the numerical results, they found that the impedance increased with respect to the Darcy number and the slip parameter, while the flow resistance increased with respect to the stenosis height. Yadav & Kumar (2012) studied the effects of the stenosis length on the blood flow resistance (a Bingham plastic model) in a generalized artery with multiple stenosis. The flow resistance decreased as the shape parameter increased and the size of stenosis decreased. The study of blood flow subjected to magnetic field in arterial stenosis (with slip effect) has been investigated by Hazarika & Sharma (2014). They observed that as the Hartman number increased, the fluid velocity and the wall shear stress decreased. Also, they claimed that increased Reynolds number would lead to the augmentations of fluid velocity and wall shear stress. Siddiqui et al. (2015) have modelled the blood as non-Newtonian Bingham plastic fluid model. The model was used to study the flow through stenosed artery in the presences of slip velocity and body acceleration. They used the perturbation method in solving the problem. Matlab was used to generate the graphical figure. From the analysis, they noticed that the introduction of body acceleration would enhance the velocity and the flow rate. Meanwhile, the wall shear stress would decrease due to slip velocity. Akbar (2016a) modeled the blood using the Walters B fluid model. This study was carried



out by applying the perturbation method on the cylindrical coordinate system. Several types of tapered arteries such converging, diverging and non-tapered arteries have been evaluated. The influence of periodic body acceleration on the pulsatile blood flow in the porous-saturated stenotic artery was studied by Zaman et al. (2017). In their study, the overlapping w-shape stenosis model was used to represent the fatty cholesterol and the blood clots. The non-dimensional governing equations were solved numerically by using the finite difference method. Based on the results, the resistance rate decreased as the permeability parameter increased. Nevertheless, parameters such as velocity, flow rate, and shear stress increased as the resistance rate decreased. Rekha & Nivedita (2018) investigated the problem of non-symmetrical stenosed artery in the blood flow. With the condition of no-slip condition at the arterial wall, the blood is assumed to be non-Newtonian K-L (Kuang and Luo) model. The equations of flow rate, wall shear stress and resistance to blood flow have been solved numerically by applying the Gauss- Quadrature two point approach. The impacts of non-Newtonian behaviour of AMINA the blood flow through the skin friction and resistance were discussed graphically.



## 2.3 Magnetic blood flow effects in the stenosed porous artery

The effectiveness of magnetic field in treating stenosis has been extensively studied. Sharma *et al.* (2004) examined the effect of magnetic field on the rheological properties of blood. They argued that magnetic field might be useful in treating hypertension. Later on, Tashtoush & Magableh (2008) studied the magnetic blood flow in multistenosed arteries. The finite difference method was applied to solve the stated governing equations. Based on the generated results, the collision of particles inside the arteries would increase the blood temperature and changed the standard blood flow pattern. Ikbal *et al.* (2009) studied the transverse magnetic blood flow in the atherosclerosis arteries. The governing equation for the blood motion was discretized by using the finite difference method and the convective term was modeled using the power-law model. From the results, the blood flow rate decreased due to the magnetic field gradient. Varshney *et al.* (2012) studied the magnetic blood flow in the stenosed

artery. The transformed governing equations were solved using the finite difference technique. All flow characteristics were sensitive to the presence of multiple stenoses and the intensity of magnetic field.

The magnetic blood flow through a radially non-symmetric constricted artery was investigated by Singh (2013). The effects of magnetic field, stenosis height, velocity, flow rate and wall shear stress were studied analytically. As reported, the flow rate decelerated in the presence of magnetic field. The non-Newtonian (Herschel-Bulkley) fluid model was used. Xenos (2013) simulated the magnetized stenosed blood flow using the finite volume method. Parameters such as velocity, pressure, and skin friction were affected by the magnetic field. Bhatnagar (2014) modelled the transversely magnetized blood flow in multiple stenosed arteries using the velocity slip condition. They derived the expressions of wall shear stress, volumetric flow rate, axial velocity and core velocity analytically and used MATLAB 7.0 in generating the graphical solutions for analysis purpose. They concluded that the presence of stenosis in blood flow would affect the flow conditions. Also, they observed that the magnetic field would reduce the flow parameters significantly. Bose & Banerjee (2015) applied the magnetic drug targetting (MDT) technique in treating stenosed aortic bifurcation. The principles of FHD and MHD were combined in order to treat blood as biomagnetic fluid. The simulation (fully incompressible) was performed using ANSYS FLUENT. Due to the presence of external magnetic field, the number of targeted drug carrier (magnetic particles) on the expected zone could be maximized. Shit & Majee (2015)studied the influences of magnetic field and whole body vibration on the overlapping stenosed artery. Moreover, the effect of heat transfer on blood flow was considered as well. The finite difference Crank-Nicolson scheme was used in solving the governing equations in transformed coordinate system. An uniform magnetic field was applied on the blood flow stream by Sharma et al. (2015). They solved the governing equations by using the finite difference scheme. The blood was driven by the pressure gradient and the external magnetic field. Again, the presence of magnetic field would decelerate the magnetic particles. Siddiqui et al. (2015), noticed that parameters such as blood velocity, flow rate and flow resistance would increase



#### REFERENCES

- Ahmed, A., & Nadeem, S. (2016). The study of (Cu, TiO<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub>) nanoparticles as antimicrobials of blood flow through diseased arteries. *Journal of Molecular Liquids*, 216, 615–623.
- Akbar, N., Rahman, S., Ellahi, R., & Nadeem, S. (2014). Nano fluid in tapering stenosed arteries with permeable walls. *International Journal of Thermal Sciences*, 85, 54–61.
- Akbar, N. S. (2014). Metallic nanoparticles analysis for the peristaltic flow in an asymmetric channel with MHD. *IEEE Transactions on Nanotechnology*, 13(2), 357–361.



Akbar, N. S. (2016a). Blood flow suspension in tapered stenosed arteries for Walters B fluid model. *Computer Methods and Programs in Biomedicine*, 132, 45–55.
Akbar, N. S. (2016b). Non-Newtonian model study for blood flow through a tapered

artery with a stenosis. Alexandria Engineering Journal, 55(1), 321–329.

- Akbarzadeh, P. (2016). Pulsatile magneto-hydrodynamic blood flows through porous blood vessels using a third grade non-Newtonian fluids model. *Computer Methods and Programs in Biomedicine*, 126, 3–19.
- Bansi, C., Tabi, C., Motsumi, T., & Mohamadou, A. (2018). Fractional blood flow in oscillatory arteries with thermal radiation and magnetic field effects. *Journal* of Magnetism and Magnetic Materials, 456, 38 – 45.
- Bhargava, R., Rawat, S., Takhar, H., & Anwar, B. (2007). Pulsatile magneto-biofluid flow and mass transfer in a non-Darcian porous medium channel. *Meccanica*, 42, 247–262.
- Bhatnagar, A. (2014). Analysis of MHD flow of blood through a multiple stenosed artery in the presence of slip velocity. *International Journal of Innovative*

Research in Advanced Engineering, 10, 250–257.

- Bhatti, M. M., & Ali Abbas, M. (2016). Simultaneous effects of slip and MHD on peristaltic blood flow of Jeffrey fluid model through a porous medium. *Alexandria Engineering Journal*, 55(2), 1017–1023.
- Bose, S., & Banerjee, M. (2015). Magnetic particle capture for biomagnetic fluid flow in stenosed aortic bifurcation considering particle-fluid coupling. *Journal of Magnetism and Magnetic Materials*, 385, 32–46.
- Buchanan, J., Kleinstreuer, C., & Comer, J. (2000). Rheological effects on pulsatile hemodynamics in a stenosed tube. *Computers Fluids*, 29(6), 695–724.
- Chakravarty, S., Mandal, P., & Layek, G. (2007). Numerical simulation if unsteady generalized newtonian blood flow through differently shaped distensible arterial stenoses. *Journal of Medical Engineering Technology*, 40, 1268– 1281.
- Chakravarty, S., & Mandal, P. K. (2000). Two-dimensional blood flow through tapered arteries under stenotic conditions. *International Journal of Non-Linear Mechanics*, *35*(5), 779–793.
- Choi, S., & Eastman, J. (1995). Enhancing thermal conductivity of fluids with nanoparticles. ASME International Mechanical Engineering Congress and Exposition, 66, 357–361.
- El-Shahed, & Moustafa (2003). Pulsatile flow of blood through a stenosed porous medium under periodic body acceleration. *Applied Mathematics and Computation*, 138(2-3), 479–488.
- Ellahi, R., Rahman, S. U., Nadeem, S., & Akbar, N. S. (2013). Blood flow of nanofluid through an artery with composite stenosis and permeable walls. *Applied Nanoscience*, 4(8), 919–926.
- Haik, Y., Pai, V., & Chen, C. (1999). Biomagnetic fluid dynamics. Fluid Dynamics at Interfaces, edited by W. Shyy and R. Narayanan (Cambridge University Press, Cambridge), pp. 439–452.
- Hatami, M., Hatami, J., & Ganji, D. D. (2014). Computer simulation of MHD blood conveying gold nanoparticles as a third grade non-Newtonian nanofluid in



a hollow porous vessel. *Computer Methods and Programs in Biomedicine*, *113*(2), 632–641.

- Haverkort, J., & Kenjeres, S. (2008). Optimizing drug delivery using nonuniform magnetic fields: A numerical study. *IFMBE Proceedings*, 22, 2623–2627.
- Haverkort, K. S., J.W., & Kleijn, C. (2009). Computational simulations of magnetic particle capture in arterial flows. *Annals of Biomedical Engineering*, 37(12), 2436–2448.
- Hayat, T., Rafiq, M., Ahmad, B., & Asghar, S. (2017). Entropy generation analysis for peristaltic flow of nanoparticles in a rotating frame. *International Journal of Heat and Mass Transfer*, 108, 1775–1786.
- Hazarika, G., & Sharma, B. (2014). Blood flow through a composite stenosis in an artery with permeable wall. *International Journal of Modern Sciences and Engineering Technology*, 1(4), 45–54.

 Hernan, A., & Gonzalez, R. (2007). Numerical implementation of viscoelastic blood flow in a simplified arterial geometry. *Medical Engineering Physics*, 29(4), 491–496.

Ijaz, S., Iqbal, Z., Maraj, E., & Nadeem, S. (2018). Investigation of cu-cuo/blood
 mediated transportation in stenosed artery with unique features for theoretical outcomes of hemodynamics. *Journal of Molecular Liquids*, 254, 421 – 432.

- Ijaz, S., & Nadeem, S. (2016a). Examination of nanoparticles as a drug carrier on blood flow through chaterized composite stenosed artery with permeable walls. *Computer Methods and Programs in Medicine*, 133, 83–94.
- Ijaz, S., & Nadeem, S. (2016b). Slip examination on the wall of stenosed artery with emerging application of nanoparticles. *International Journal of Thermal Science*, 109, 401–412.
- Ijaz, S., & Nadeem, S. (2016c). Slip examination on the wall of tapered stenosed artery with emerging application of nanoparticles. *International Journal of Thermal Sciences*, 109, 401–412.
- Ikbal, M., Chakravarty, S., Wong, K. K., Mazumdar, J., & Mandal, P. (2009). Unsteady response of non-Newtonian blood flow through a stenosed artery in magnetic



field. Journal of Computational and Applied Mathematics, 230(1), 243–259.

- Ismail, Z., Abdullah, I., Mustapha, N., & Amin, N. (2008). A power-law model of blood flow through a tapered overlapping stenosed artery. *Journal of Applied Mathematics and Computation*, 195, 669–680.
- Kenjeres, S., & Opdam, R. (2009). Computer simulations of a blood flow behaviour in simplified stenotic artery subjected to strong non-uniform magnetic fields, 4th european conference of the international federation for medical and biological engineering. *IFMBE Proceedings*, 22(22), 2604–2608.
- Liu, G. T., Wang, X. J., Liu, L. G., & Ai, B. (2004). Numerical study of pulsatile flow through a tapered artery with stenosis. *Chinese Journal of Physics*, 42(4), 401–409.
- Maikap, T., Mahapatra, T., & Niyogi, a. G. A., P. (2005). Numerical investigation of laminar separated flow through a channel with symmetric double expansion. *Acta Mechanica*, 179, 197–210.
- Majee, S., & Shit, G. C. (2017). Numerical investigation of MHD flow of blood and heat transfer in a stenosed arterial segment. *Journal of Magnetism and Magnetic Materials*, 424, 137–147.
- Mandal, P. K. (2005). An unsteady analysis of non-Newtonian blood flow through tapered arteries with a stenosis. *International Journal of Non-Linear Mechanics*, 40(1), 151–164.
- Mandal, P. K., Chakravarty, S., Mandal, A., & Amin, N. (2007). Effect of body acceleration on unsteady pulsatile flow of non-Newtonian fluid through a stenosed artery. *Applied Mathematics and Computation*, 189(1), 766–779.
- Mishra, S., Siddiqui, S. U., & Medhavi, A. (2011). Blood flow through a composite stenosis in an artery with permeable wall. An International Journal of Applications and Applied Mathematics, 6(11), 1798–1813.
- Nadeem, S., & Ijaz, S. (2016a). Theoretical examination of nanoparticles as a drug carrier with slip effects on the wall of stenosed arteries. *Journal of Magnetism* and Magnetic Materials, 93, 1137–1149.

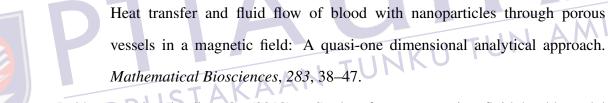
Nadeem, S., & Ijaz, S. (2016b). Impulsion of nanoparticles as a drug carrier for the



theoretical investigation of stenosed arteries with induced magnetic effects. Journal of Magnetism and Magnetic Materials, 410, 230–241.

- Plavins, J., & Lauva, M. (1993). Study of colloidal magnetite binding erythrocytes: Prospects for cell separation. *Journal of Magnetism and Magnetic Materials*, 122, 349–535.
- Ponalagusamy, R., & Priyadharshini, S. (2017). Nonlinear model on pulsatile flow of blood through a porous bifurcated arterial stenosis in the presence of magnetic field and periodic body acceleration. *Computer Methods Programs Biomedicine*, 142, 31–41.
- Prakash, J., Makinde, O. D., & Ogulu, A. (2004). Magnetic effect on oscillary blood flow in a constricted tube. *Chinese Journal of Physics*, 13(1), 45–50.
- Rabby, M. G., Shupti, S. P., & Molla, M. (2014). Pulsatile non-newtonian laminar blood flows through arterial double stenoses. *Journal of Fluids*, 2014, 01–13.

Rahbari, A., Fakour, M., Hamzehnezhad, A., Vakilabadi, M. A., & Ganji, D. D. (2017).



- Rekha, B., & Nivedita, G. (2018). Study of non-newtonian fluid by kl model through a non-symmetrical stenosed narrow artery. *Applied Mathematics and Computation*, 320, 358 – 370.
- Ruuge, E., & Rusetski, A. (1993). Magnetic fluid as drug carriers: Targeted transport of drugs by a magnetic field. *Journal of Magnetism and Magnetic Materials*, 122, 335–339.
- Sankar, D., & Hemalatha, K. (2007). A non-newtonian fluid flow model for blood flow through a catheterized arterysteady flow. *Journal of Applied Mathematical Modelling*, 31, 1847–1864.
- Sankar, D. S., & Lee, U. (2008). Two-fluid Herschel-Bulkley model for blood flow in catheterized arteries. *Journal of Mechanical Science and Technology*, 22(5), 1008–1018.

- Schlichting, H., & Gersten, K. (2017). *Boundary-Layer Theory*. Springer-Verlag Berlin Heidelberg, 9 ed.
- Shah, N., Vieru, D., & Fetecau, C. (2016). Effects of the fractional order and magnetic field on the blood flow in cylindrical domains. J. Magnetism and Magnetic Materials, 409, 10–19.
- Shahzadi, I., & Nadeem, S. (2017). Inclined magnetic field analysis for metallic nanoparticles submerged in blood with convective boundary condition. *Journal of Molecular Liquids*, 230, 61–73.
- Sharma, G., Jain, M., & Kumar, A. (2004). Performance modeling and analysis of blood flow in elastic arteries. *Mathematical and Computer Modelling*, 26(3), 345–354.
- Sharma, S., Singh, U., & Katiyar, V. (2015). Magnetic field effect on flow parameters of blood along with magnetic particles in a cylindrical tube. *Journal of Magnetism and Magnetic Materials*, 377, 395–401.
- Shit, G. C., & Majee, S. (2015). Pulsatile flow of blood and heat transfer with variable viscosity under magnetic and vibration environment. *Journal of Magnetism* and Magnetic Materials, 388, 106–115.

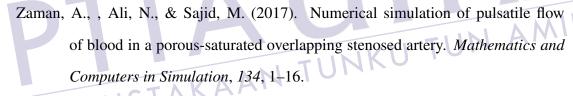
Siddiqui, S. U., Shah, S. R., & Geeta (2015). A biomechanical approach to study the effect of body acceleration and slip velocity through stenotic artery. *Applied Mathematics and Computation*, 261, 148–155.

- Singh, A. K. (2013). MHD Flow of Blood through Radially Non-symmetric Stenosed Artery: a Hershcel-Bulkley Model. *International Journal of Engineering*, 26(8 (B)), 859–864.
- Sud, V., & Sekhon, G. (1985). Arterial flow under periodic body acceleration. Bulletin of Mathematical Biology, 47(1), 35–52.
- Tanwar, V. K., Varshney, N. K., & Agarwal, R. (2014). Effect of porous medium on blood flow through an artery with mild stenosis under the influence. *International Journal of Mathematical Archive*, 5, 181–188.
- Tashtoush, B., & Magableh, A. (2008). Magnetic field effect on heat transfer and fluid flow charecteristics of blood flow in multi-stenotic arteries. *Heat and Mass*



Transfer, 44(3), 297–304.

- Tzirtzilakis, E. E. (2005). A mathematical model for blood flow in magnetic field. *Physics of Fluids*, *17*, 1–15.
- Ullah, I., Rahim, M. T., Khan, H., & Qayyum, M. (2016). Analytical analysis of squeezing flow in porous medium with mhd effect. UPB Scientific Bulletin, Series A: Applied Mathematics and Physics, 78(2), 281–292.
- Varshney, G., Katiyar, V., & Kumar, S. (2012). Effect of magnetic field on the blood flow in artery having multiple stenosis: a numerical study. *International Journal of Mathematics and Mathematical Sciences*, 6(2), 71–77.
- Xenos, M. A. (2013). MHD Effects on Blood Flow in a Stenosis. Advances in Dynamical Systems and Applications, 8(2), 427–437.
- Yadav, S. S., & Kumar, K. (2012). Bingham plastic characteristic of blood flow through a generalized atherosclerotic artery with multiple stenoses. Advances in Applied Science Research, 3(6), 3551–3557.



Zaman, A., Ali, N., & Sajid, M. (1999). Pulsatile flow of blood with periodic body acceleration. *International Journal of Engineering Sciences*, 29(1), 113–121.

Zaman, A., Ali, N., & Sajid, M. (2015). Numerical simulation of pulsatile flow of blood in a porous-saturated overlapping stenosed artery. *Mathematics and Computers in Simulation*, 134, 1–16.

