

**PATH PLANNING ALGORITHM FOR A CAR LIKE ROBOT  
BASED ON VORONOI DIAGRAM METHOD**

HADIE INUN

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PERPUSTAKAAN TUNKU TUN AMINAH

Faculty of Electrical & Electronic Engineering  
Universiti Tun Hussien Onn Malaysia

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## ABSTRACT

The purpose of this study is to develop an efficient offline path planning algorithm that is capable of finding optimal collision-free paths from a starting point to a goal point. The algorithm is based on Voronoi diagram method for the environment representation combined with Dijkstra's algorithm to find the shortest path. Since Voronoi diagram path exhibits sharp corners and redundant turns, path tracking was applied considering the robot's kinematic constraints. The results has shown that the Voronoi diagram path planning method recorded fast computational time as it provides simpler, faster and efficient path finding. The final path, after considering robot's kinematic constraints, provides shorter path length and smoother compared to the original one. The final path can be tuned to the desired path by tuning the parameter setting; velocity,  $v$  and minimum turning radius,  $R_{min}$ . In comparison with the Cell Decomposition method, it shows that Voronoi diagram has a faster computation time. This leads to the reduced cost in terms of time. The findings of this research have shown that Voronoi Diagram and Dijkstra's Algorithm are a good combination in the path planning problem in terms of finding a safe and shortest path.



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## ABSTRAK

Kajian ini bertujuan untuk membangunkan satu algoritma perancang laluan secara luar talian yang cekap serta berupaya mencari laluan selamat yang optimum dari titik permulaan ke titik sasaran. Algoritma tersebut adalah berdasarkan kepada kaedah gambar rajah Voronoi untuk mewakili persekitaran dan digabungkan dengan algoritma Dijkstra untuk mendapatkan laluan terdekat. Oleh kerana laluan yang diperoleh menggunakan gambar rajah Voronoi mempunyai sudut-sudut tajam dan pusingan yang berlebihan, penjejakan laluan digunakan dengan mengambil kira kekangan kinematik robot. Hasil kajian menunjukkan bahawaperancang laluan menggunakan kaedah gambar rajah Voronoi merekodkan masa pengiraan terpanjang dan merupakan kaedah lebih mudah, lebih cepat dan cekap. Selepas mengambil kira kekangan kinematik robot, laluan yang diperoleh adalah lebih pendek dan licin berbanding dengan laluan asal. Laluan tersebut boleh ditala dengan menala parameter halaju,  $v$  dan jejari pusingan minimum,  $R_{\min}$ . Perbandingan dengan kaedah *Cell Decomposition*, menunjukkan bahawa kaedah gambar rajah Voronoi mempunyai masa pengiraan yang lebih pantas. Ini membawa kepada pengurangan kos dari segi masa. Penemuan kajian ini menunjukkan bahawa gambar rajah Voronoi dan Algoritma Dijkstra adalah satu gabungan yang baik dalam menyelesaikan masalah perancangan laluan iaitu untuk mencari satu laluan terdekat yang selamat.



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**LIST OF ABBREVIATIONS**

CD	Cell Decomposition
$C_{\text{free}}$	Free Space
$C_f$	Free Space
RM	Roadmap
VD	Voronoi Diagram
Ed	Voronoi Edge
PRM	Probabilistic roadmap
$Q_{\text{free}}$	Configuration Space Area
RRT	Rapidly-exploring Randomise Tree
PF	Potential field
BFS	Breadth-First Search
DFS	Depth-First Search
GUI	Graphical User Interface
2D	Two Dimensional Space
$C_{\text{min}}$	Minimum Free Space



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## CHAPTER 1

### INTRODUCTION

#### 1.1 Background

The path planning is the basic element of the mobile robot. The so-called mobile robot path planning is searching for an optimal or sub-optimal collision-free path connecting the initial position and target position. According to the environmental information that collected by the robot, the path planning can be divided into two categories: path planning with fully known global environmental information and completely unknown or partially unknown environmental information [1]. The common methods that have been used, such as grid method and artificial potential field method and so on, but all these algorithms have limitations. The shortcomings of the grid method are when the space increases, it will lead to dramatic increase in storage space that required, and decision is made slowly. The structure of artificial potential field is simple, convenient for real-time control, but may generate minimal path and oscillation in front of obstacles.

Based on Voronoi diagram method for path planning the obstacles nodes and edges will be used to construct the Voronoi diagram. Then the shortest path is calculated using a search algorithm. The advantages of the Voronoi diagram are that it can generate the origin robot path points, and reduce the computational time.

## 1.2 Problem Statement

The problem of path planning occurs in many areas, such as computational biology, computer animations and computer-aided design. It is of particular importance in the field of robotics. The basic path planning problem is concerned with finding a good-quality path from a source point to a destination point that does not result in collision with any obstacles. Depending on the amount of the information available about the environment, which can be completely or partially known or unknown, the approaches to path planning vary considerably. Also, the definition of a good-quality path usually depends on the type of a mobile device (a robot) and the environment (space), which has fostered the development of a rich variety of path-planning algorithms, each catering to a particular set of needs.

The path planning problem criteria may include distance, time and energy. The distance is the most common criterion. However common path planning approaches do not take into consideration path safety or smoothness. Safety constraints are important to both the robot and its surrounding objects. Smoothness is also another important constraint. Most mobile robots need to consider this constraint because of the bounded turning radius. For example car-like robots have this constraint due to mechanical limitations and its steering angle.

Path planning algorithms are classified according to completeness as exact and heuristic [4]. Exact algorithms aim to find an optimal solution if one exist, or prove that there is no feasible solution. On the other hand, heuristic algorithms aim to search for a good quality solution in a short time. Exact algorithms are usually computationally expensive; however heuristic algorithm may fail to find a good solution for difficult problems.

There are three basic types of robot path planning algorithms [14] based on potential field, cell decompositions and roadmaps. The potential field methods are used rarely because they often converge to a local minimum. Cell decompositions avoid this drawback and, in a special case with the scene divided into squares, then they can be simply used for 8-directional (horizontal, vertical and diagonal) robot motion. Their main drawbacks are combinatorial explosion, limited granularity and generation of

infeasible solutions. The combinatorial explosion and time of computation may be partially reduced using a case-based reasoning procedure [2].

There are several different methods for developing the roadmap such as visibility graphs and Voronoi diagrams [3]. These methods do not have the drawbacks of the previously-mentioned ones and thus are more promising for the task under investigation. However, all these methods generate non-smooth trajectories and not be suitable for real robots with kinematic and dynamic restrictions. Therefore, this research presents a path planning algorithm based on Voronoi Diagram, considering those constraints for car-like robot.

The goal of this project is to design a path planning algorithm using Voronoi Diagram for a car-like robot, considering the car kinematic constraint. This induces two main problems: the paths generated by this algorithm have to avoid obstacles of the environment, but the vehicle must also be able to follow them precisely. The following are the assumptions made in order to address the stated problem:

- i. The start and goal configurations  $(x_s, y_s, \theta_s)$  and  $(x_g, y_g, \theta_g)$  of a car-like robot in a plane are given;
- ii. Dimensions of the robot-length  $L$ , width  $W$ , minimum turning radius  $R$  are known;
- iii. A collection  $O = \{o_1, o_2, \dots, o_n\}$  of non-overlapping obstacles described by simple polygons, each having a set of vertices  $V_i = \{v_{ij}, j = 1, \dots, m_i\}$ ,  $i = 1, \dots, n$ , and such that the length of each edge is at least  $l_0$  and the angle subtended at each concave vertex is in the range  $[\pi/2, \pi]$ . (Note that  $O$  can be the empty set.)
- iv. A set of vertices  $B = \{b_1, b_2, \dots, b_N\}$  describing a simple polygon that defines a boundary such that the length of each edge is at least  $l_0$  and the angle subtended at each convex vertex is in the range  $[\pi/2, \pi]$ . (Note that  $B$  can also be the empty set.)

The car-like robot need to find a set of way points  $P = \{P_1, \dots, P_k\}$  starting with the start position and ending with the goal position, that is an image of an admissible trajectory of minimal distance for a robot moving only forward.

## 1.4 Objectives

The main objective of this project is to provide an algorithm of path planning for a car-like robot moving through an environment containing obstacles bounded by simple polygons. It will involve the following:

- i. To develop an efficient offline path planning algorithm based on Voronoi Diagram method that is capable of finding optimal collision free paths from a starting point to a goal.
- ii. To extend the path planning algorithm in order to find the shortest path from a start point to a goal, considering the robot's kinematic constraint.

## 1.5 Scope of the Project

The project will be implemented based on the following consideration:

- i. The robot considered is a car-like robot represented by a single point or a circle. Algorithms may identify a robot with a mere point enlarging the obstacles in the workspace accordingly to avoid having to consider the robot's size.
- ii. The car-like robot will only move in forward direction. There will be no capability for the car-like robot to move backward.
- iii. The proposed algorithm is based on Voronoi diagram method for the path planning and using the Dijkstra Algorithm to find the shortest path.
- iv. The path planning algorithm will only effective in a static obstacle environment.
- v. The location of the start point, destination point and the coordinate of the obstacles have been identified.
- vi. The shape of the obstacles are considered to be rectangular
- vii. The area of the environment will be given.
- viii. No path smoothing will be implemented.



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## 1.6 Organization of Report

As a whole, this report aims to documents all the concepts, activities and works related to the implementation of this project. This report stressed on the aspects of designing and developing an efficient offline path planning algorithm for a car-like robot.

This report consists of three chapters. In chapter 1, it describes the introduction of the project, problem background, problem statement and project scope. Chapter 2 describes about the literature review that includes an introduction to some basic concepts and a survey of existing work in the areas of path planning or car-like robot. Meanwhile in chapter 3 describes the methodology of the project which is provides a detailed description of the algorithms for path planning based on Voronoi diagrams.



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## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 Introduction

Path planning is the basic element of an autonomous mobile vehicle for both static and dynamic environments and many researchers have worked on it since 80's. It deals with the search and execution of collision free paths by vehicles performing specific tasks. Path planning is often broken down into two stages which are path planning and path tracking. The path planning stage involves the search for a collision free path, taking into consideration the geometry of the vehicle and its surroundings, the vehicle's kinematic constraints and any other external constraints that may affect the planning of a path. The path tracking stage involves the actual navigation of a planned path, taking into consideration the kinematic and dynamic constraints of the vehicle.

In robotics, one can envision a robot moving autonomously in a maze of obstacles from a start to a goal configuration without collisions. An *obstacle* can be defined as a region inaccessible to the robot while *free space* is the area external to the obstacles. The problem becomes even more challenging when the nature of the terrain is allowed to vary and the robot has to pick a path that is most suitable for it. Many techniques have been researched to utilize multiple path schemes for different applications as outlined in [21]. These applications have been dealt within two strategies. One strategy is to use pre-known global environment information and robot characteristics, while another builds up local environment with sensor information and using robot characteristics.



## 2.2 Classical Path Planning Methods

The successful traditional computational geometry-based approaches to robot path planning can be classified into three basic categories [14]:

- i. Cell Decomposition
- ii. Roadmap
- iii. Potential Field

This chapter will present the basic ideas and a few different realizations for each approach. For these methods, an explicit representation of  $C$  is assumed to be known.

## 2.3 Representation Techniques in Path Planning

There are a number of environment representations based on  $C$ -space. The most common representations are explained in the following sections, based on the scenario represented by the  $C$ -space as depicted in Figure 2.1 (b).

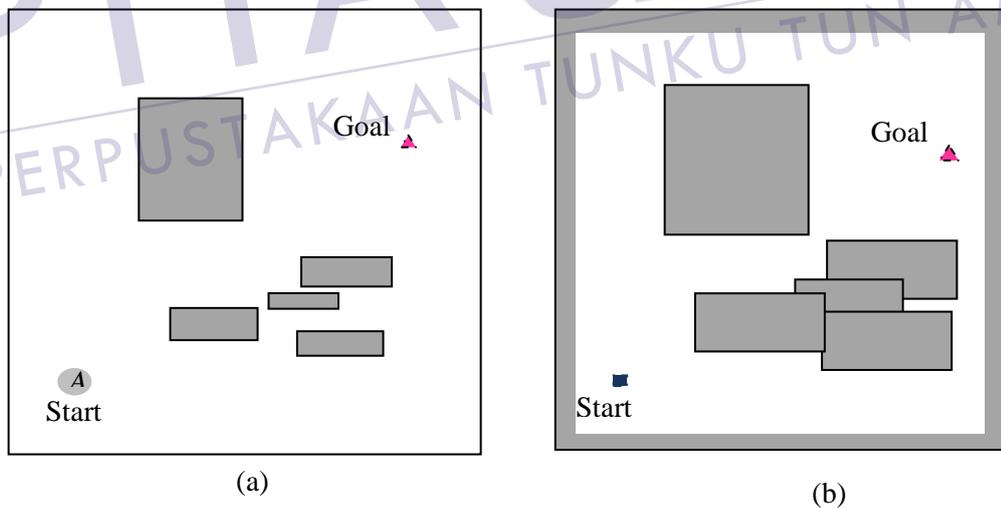
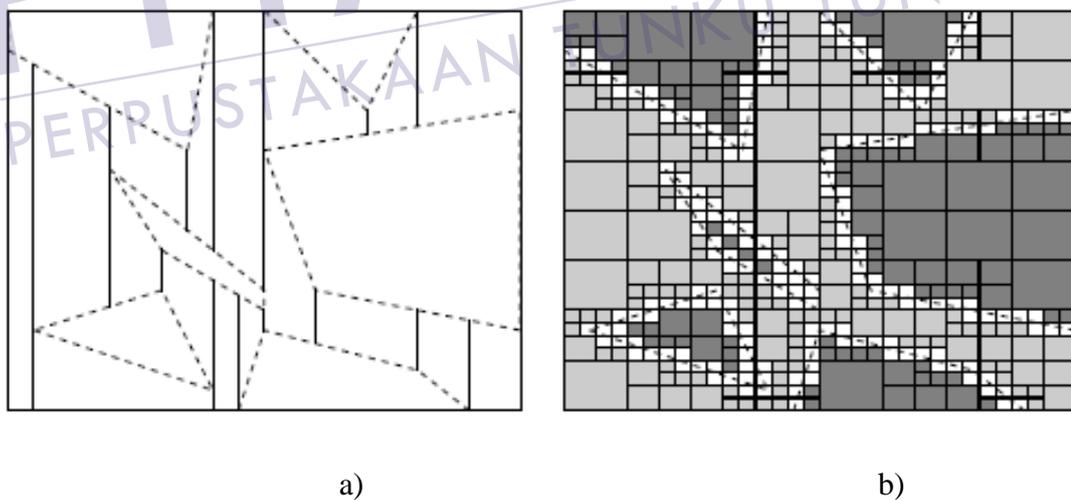


Figure 2.1: A scenario represented in (a) original form (b) configuration space. The darker rectangles in (a) are those with actual dimensions while in (b) are those enlarged according to the size of vehicle A. The white areas denote the free space.

## 2.4 Cell Decomposition (CD)

The cell decomposition method uses non overlapping cells to represent the free-space ( $C_f$ ) connectivity. The decomposition can be exact or approximate. An exact decomposition divides  $C_f$  exactly [8]. An approximation scheme Kambhampati discretizes  $C_f$  with cells. It decomposes the free space recursively, stopping when a cell is entirely in  $C_f$  or entirely inside an obstacle. Otherwise, the cell is further divided. Because of memory and time constraints, the recursive process stops when a certain degree of accuracy has been reached. The cell decomposition method, although simple to implement, seldom yields high-quality paths. The exact cell decomposition technique is faster than the approximate one, but the path obtained is not optimal. The approximate cell decomposition can yield near-optimal paths by increasing the grid resolution, but the computation time will increase drastically. There is also the known problem of digitization bias associated with using a grid. This stems from the fact that while searching for the shortest path in a grid, the grid distance is measured and not the Euclidean distance. A visualization of this approach is shown in Figure 2.2.



**Figure 2.2:** a) Exact cell decomposition,  $C_{free}$  exactly decomposed into trapezoids;  
 b) Approximate cell decomposition, mixed cells are divided until a series of free cells connects the start with the goal cell; free cells: light gray, obstacle cells: dark gray, mixed cells: white, obstacles (- -)

## 2.5 Roadmap (RM)

The roadmap method is a popular approach to motion planning. Roadmap (RM) represents the C-space of obstacles and vehicle with edges and nodes by constructing graphs or maps. A graph  $G$  is made of a set of vertices or nodes  $V(G)$  as well as a set of edges/lines  $E(G)$ .  $E(G)$  is an unordered pair of distinct vertices of  $G$  [40]. RM typically takes several steps to build a graph or map for path planning purpose, starting with establishment of nodes' connections with edges within the free C-space area. Next the starting point  $p_{start}$  and target point  $p_{target}$  of the vehicle are combined to the network to complete the graph or map. A collision-free path through a series of line segments is then searched from the  $p_{start}$  to  $p_{target}$  [41] using graph search algorithm. Roadmaps overweigh the cell decompositions method in the number of nodes as path planner needs to search through (in cell decomposition method) in order to find a path.

### 2.5.1 Voronoi Diagram

Voronoi diagram (VD) is a popular roadmap method in path planning. The idea behind VD is to generate a line segment called Voronoi Edge ( $Ed$ ) which is equidistant from all the points of the obstacle area in C-space. The point where  $Ed$  joins is called Voronoi Vertex ( $Vd$ ). An example of VD representation, used for path planning is shown in Figure 2.3. The resultant path is shown in solid black line. As illustrated in the figure, VD has edges that give a maximum clearance path among set of obstacles in the C-space. If a vehicle traverses the planned path, it is guaranteed that the vehicle will not intersect any obstacle. One major disadvantage of VD is the generated paths are not optimal in terms of path length as it produces path which is undesirably long at areas where obstacles are far apart. In addition, the path are also has many unnecessary turns and the length of the path may be undesirably long at regions where obstacles are far apart. In fact, it is worth noting that minimizing path length and maximizing clearance seemingly contradict each other as increasing the clearance results in a longer path while reducing the path length necessarily reduces the clearance from obstacles. It would be highly beneficial for many applications if an algorithm could be developed that would accept the minimum clearance required

as an input parameter and produce a path that would be shortest while satisfying the minimum clearance requirement. The shortest path problem on its own can be viewed as only a special case when we set the clearance required to zero.

The Voronoi diagram of the configuration space is the set of collision-free configurations, whose minimal distance to  $C_{\text{obst}}$  is achieved with atleast two points on the boundary of  $C_{\text{obst}}$  [14].

$$v = \{q \in C_{\text{free}} \mid d = \min_{q' \in C_{\text{obst}}} \text{dist}(q, q'), \exists q', q'' \in C_{\text{obst}} q' \neq q'', d = \text{dist}(q, q') = \text{dist}(q, q'')\}$$
(2.1)

As can be seen from the example shown in Figure 2.3, if the agent moves along the Voronoi diagram, it keeps a maximum distance to all  $C$ -obstacles.

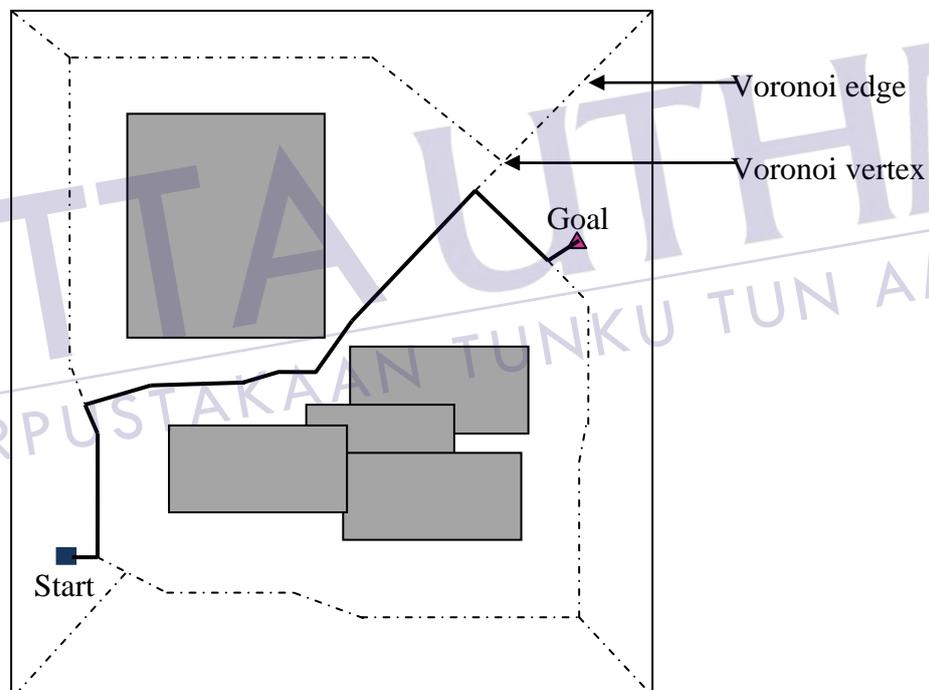


Figure 2.3: A Voronoi diagram. The dashed lines are the set of points equidistant to obstacles. The path is shown in solid darker lines.

### 2.5.2 Probabilistic Roadmap

Probabilistic Roadmap (PRM) is a popular method for path planning as it is easy to apply. It makes path planning in large or high-dimensional spaces tractable and provides a good approximation of the connectivity of the configuration space area  $Q_{\text{free}}$ . This method consists of two phases i.e. learning phase and query phase. Learning phase constructs a graph  $G$  whose nodes are on the free  $Q_{\text{free}}$  and the edges connect the nodes without intersecting any obstacle.

On the other hand, query phase connects the starting point  $p_{\text{start}}$  and target point  $p_{\text{target}}$  to  $G$ . A search algorithm is then used to find a path from  $p_{\text{start}}$  to  $p_{\text{target}}$ . Figure 2.4 shows an example of PRM used in path planning. A path connecting the starting point and target point is illustrated in solid black line. However, the construction of roadmap is computationally expensive as it might sample thousands of nodes to ensure a path exists. Furthermore the generated path often has poor quality as a result of the randomness inherent in the graph that represents the  $Q_{\text{free}}$  connectivity. This method may also be incomplete i.e. do not find a path between two locations although there exist a path connecting them, in the present of narrow passage. In addition, there is no way to know whether a path exists as long as no path has been found [42].

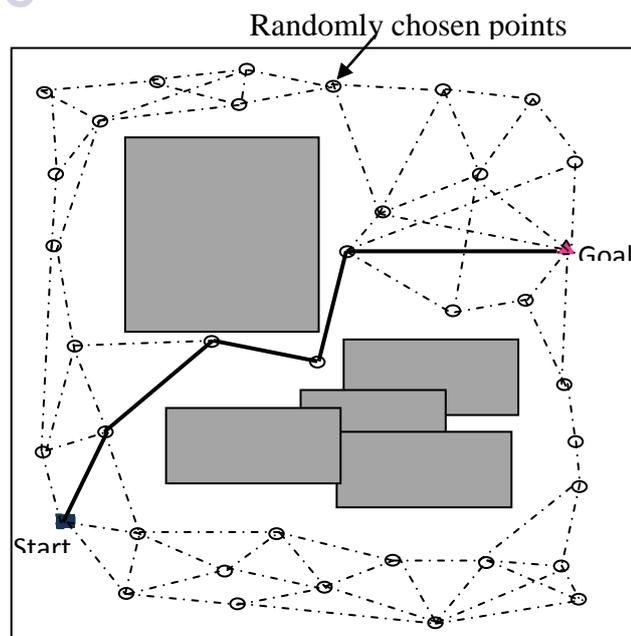


Figure 2.4: A PRM which nodes are chosen randomly

### 2.5.3 Rapidly-exploring Randomise Tree

Rapidly-exploring Randomise Tree (RRT) which is introduced by [43] is a variation of PRM. RRT begins at both starting point  $p_{start}$  and target point  $p_{target}$ , and randomly expands tree for the whole configuration space. The idea is to incrementally construct a roadmap which expands the connected paths toward the areas which have not been covered yet. In order to construct a map using RRT, consider a tree,  $T$  which is one of  $T_{ps}$  and  $T_{pt}$  originated at  $P_{start}$  and  $P_{target}$ , respectively.  $T$  is then extended incrementally by adding a random node,  $q_r$  in  $Q_{free}$  in uniform manner at each iteration by sampling a nearest node  $q_n$  to  $q_r$ . The  $T$  tries to connect  $q_r$  to  $q_n$  subject to kinematic constraints which result in a new node  $q_w$ . The new edge connecting  $q_n$  and  $q_r$  is included in the set of edges of  $T$  [44]. Readers are referred to [43] for a further explanation of RRT. One example of multiple RRTs which  $T$  started from both  $p_{start}$  and  $p_{target}$  is shown in Figure 2.5 where the planned path is shown in solid dark lines. There are several advantages of RRT including relatively simple, suitable for finding a path for vehicle with dynamic and physical constraints, the expansion of RRT is heavily biased toward unexplored areas of C-space and it is minimal in terms of the number of edges, to name just a few. However, the resulted path from RRT is always not optimal.

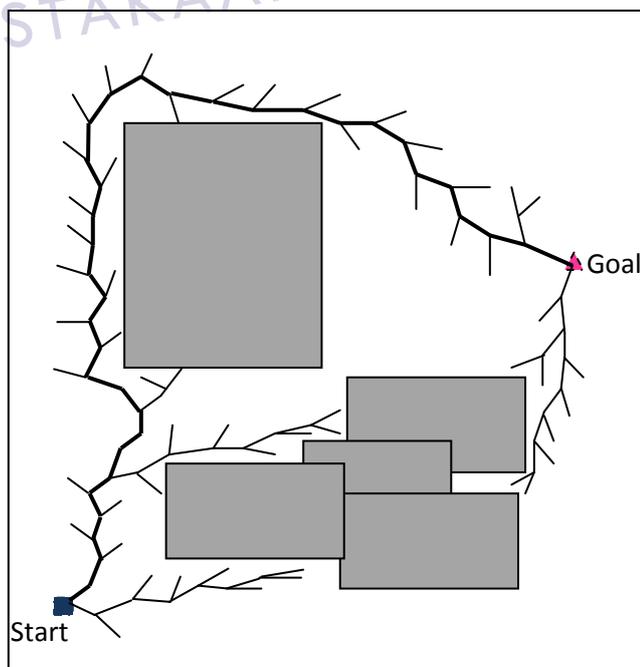
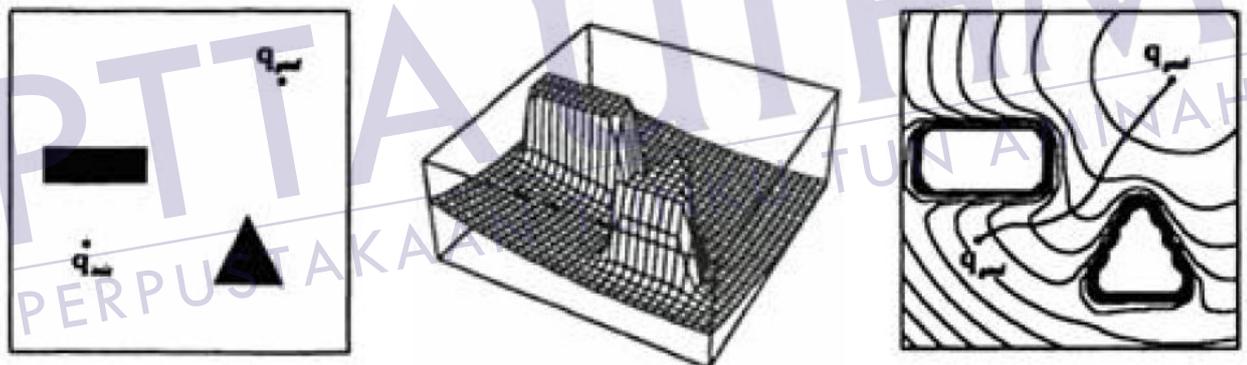


Figure 2.5: Path Planning using multiple RRTs

## 2.6 Potential Field

The idea behind the potential field (PF) method is to assign a function similar to the electrostatic potential to each obstacle and then derive the topological structure of the free space in the form of minimum potential valleys. The robot is pulled toward the goal configuration as it generates a strong attractive force. In contrast, the obstacles generate a repulsive force to keep the robot from colliding with them. The path from the start to the goal can be found by following the direction of the steepest descent of the potential toward the goal [11]. The strength of this approach is that path planning can be done in real time by considering only the obstacles close to the robot. Information on the locations of all obstacles is not required beforehand. However, as only local properties are used in planning, the robot may get stuck at local minima and never reach the goal. The potential field approach is illustrated in Figure 2.6.



**Figure 2.6** The potential field approach

## 2.7 Bezier Curve

Bezier curves were introduced by Paul de Casteljau in 1959. The curves were then patented and popularised by Pierre Bezier, an engineer at Renault in 1960s [32]. Bezier used the curve to design vehicles body. The curves are then widely used to model smooth curves. Bezier curve results from the weighted sum of a number of control points,  $P$ . Unlike the curves that are generated by splines, Bezier curve does not go through the control points. An  $n$  degree of Bezier curve is specified by  $n+1$

control points,  $P_0, P_1, \dots, P_n$ . The generalised Bezier curve,  $C_B(l)$  is defined as follows [109]:

$$C_B(l) = \sum_{i=0}^n B_{n,i}(l)P_i, \quad B_{n,i}(l) = \frac{n!}{i!(n-i)!} (1-l)^{n-i}l^i \quad (2.2)$$

where  $l \in [0,1]$  and  $B_{n,i}(s)$  are Bernstein basis polynomials.

In order to illustrate the curves generated by Bezier curve technique, consider two piece-wise paths which are shown in blue in Figures 2.7 and 2.8 with one and two waypoints, respectively. The Bezier curves (dotted magenta lines) are then generated by second and third orders as shown in Figure 2.7 and Figure 2.8, respectively.

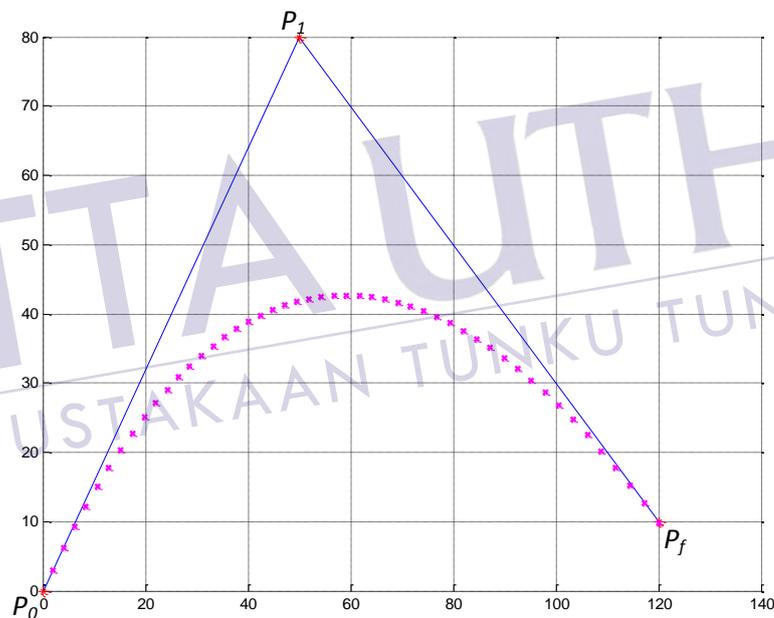


Figure 2.7: A 2<sup>nd</sup> order (quadratic) Bezier curve with three control points

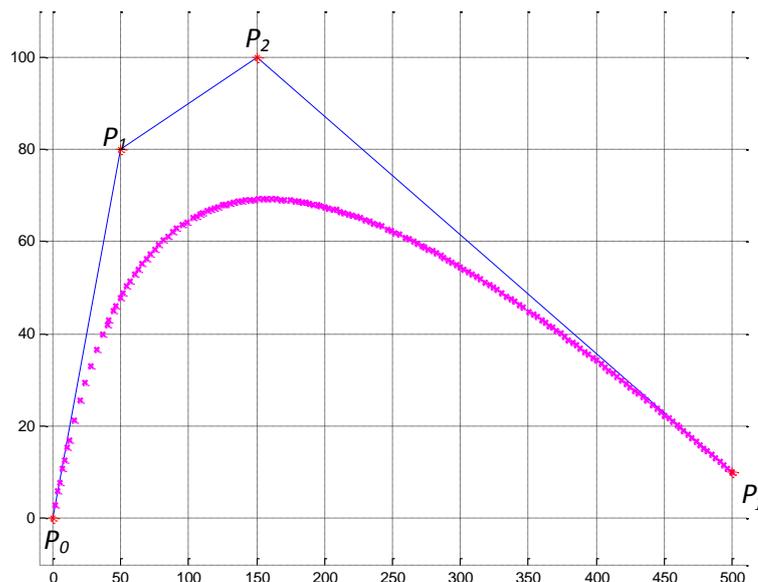


Figure 2.8: A 3<sup>rd</sup> order (cubic) Bezier curve with four control points

Both Figures 2.7 and 2.8 clearly show that the curves pass through the first,  $P_0$  and last control points,  $P_f$  and pulled in the direction of the middle points. The curve is also tangent to the  $P_0P_1$  line segment at the start and the last line segment,  $P_{f-1}P_f$  at the end. These characteristics present for any order of Bezier curve and are advantageous for path smoothing as these will make the robot to gradually leave the first line segment and go to the last line segment smoothly.

## 2.8 Comparison of the Method

Based on Voronoi diagram method for path planning, firstly, there needs known static obstacles which construct the Voronoi diagram, then obtained the shortest path by optimal search algorithm. The Voronoi diagram can generate the origin robot path points, and reduce the path search time.

The advantage of using a Voronoi diagram as a roadmap over a visibility graph is that the Voronoi diagram can be constructed in just  $O(n \log n)$  time whereas even the fastest known algorithm for constructing visibility graph [9] can take  $O(n^2)$  time in worst case when the visibility graph has  $O(n^2)$  edges. Since a Voronoi

diagram has  $O(n)$  edges, querying for a path in a Voronoi diagram based roadmap is also much faster than querying in a visibility graph.

However, as mentioned before, the quality of path obtained directly from the Voronoi diagram may not be good and is usually far from optimal. So in recent years, improving the quality of the path has been an active area of research. In [15], the authors combine the Voronoi diagram, visibility graph and potential field approaches to path planning into a single algorithm to obtain a tradeoff between safest and shortest paths. The algorithm is fairly complicated and although the path length is shorter than those obtained from potential field method or Voronoi diagram, it is still not optimal. The path exhibits bumps and rudimentary turns and is not smooth. Another recent work on reducing the length of the path obtained from a Voronoi diagram can be observed in [7]. The method involves constructing polygons at the vertices in the roadmap where more than two Voronoi edges meet. The path is smoother and shorter than that obtained directly from the Voronoi diagram but there has been no attempt to reach optimality. In [20], the authors create a new diagram called the  $VV(c)$  diagram (the Visibility-Voronoi diagram for clearance  $c$ ). It is similar to this project which is to obtain an optimal path for a specified clearance value. The diagram evolves from the visibility graph to the Voronoi diagram as the value of  $c$  increases.

Unfortunately, as the method is visibility based, the processing time is  $O(n^2 \log n)$  which renders it impractical for large spatial datasets. Apart from roadmap based techniques, the potential field approach [10],[19] and cell decomposition method [12], [5] are two popular path planning approaches. The potential field approach is simple but has several potential problems, such as being trapped in local minima or failure to pass through small openings [13]. In particular, the robot may get stuck at a local minimum. The paths obtained by cell decomposition are not optimal because of the connectivity limitations in a grid. The method is also not suitable for spatial datasets as it becomes particularly difficult to correctly estimate the grid resolution.



## 2.9 Search Algorithm

At its core, the search algorithm or a path finding method searches a graph by starting at one vertex and exploring adjacent nodes until the destination node is reached, generally with the intent of finding the shortest route. To find a path between two nodes in a graph and to find the shortest path problem this algorithm will be used. During the search, each configuration is allowed to have only one parent, otherwise, the robot keeps moving in a cycle [33].

This section presents a brief review of basic graph searching principles for edge detection applications. A graph consists of a set of points called nodes and a set of links that determine how the nodes can be connected [34]. The connections between start node and end node are defined as a set of links. There are several search methods developed here are several search methods developed in artificial intelligence such as Breadth First, Depth First, A\* and Dijkstra's Algorithm [33] as shown in Figure 2.9.

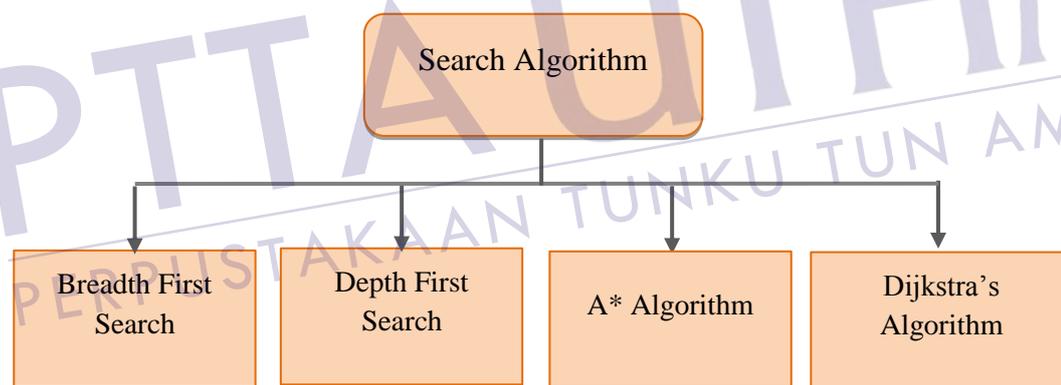
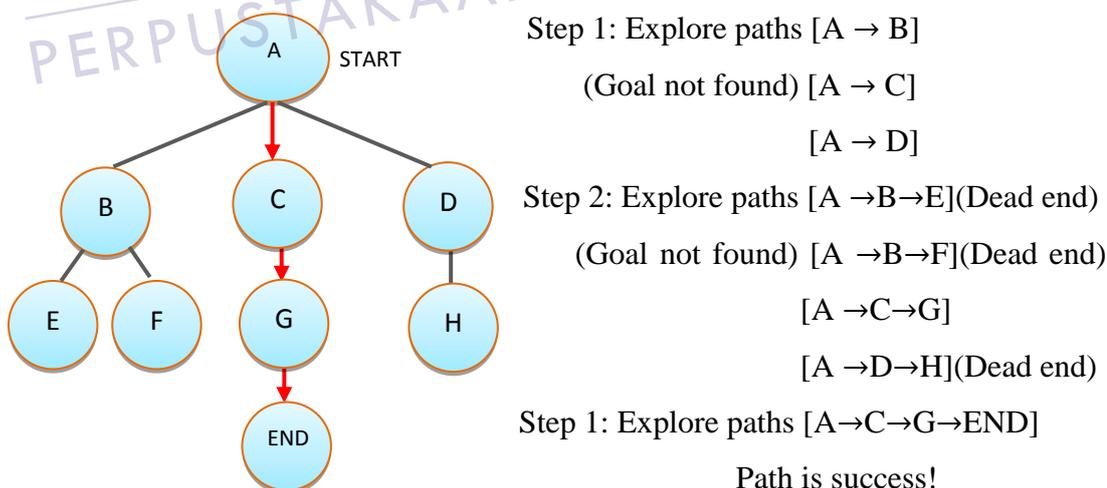


Figure 2.9: Classifications of the Search Algorithms

### 2.9.1 Breadth-first Search

In graph theory, breadth-first search (BFS) is a graph search algorithm that begins at the root node and explores all the neighboring nodes. Then for each of those nearest nodes, it explores their unexplored neighbor nodes, and so on, until it finds the goal [36]. The theory support in [37] that states the search starts at the root and then visits all of the children of the root first. Next, the search then visits all of the grandchildren, and so forth. The belief here is that the target node is near the root, so this search would require less time. BFS considers each of the nodes that are the same link length from the start node before going onto second stages [37]. This method seem like Dijkstra's algorithm, it is to find the shortest paths, but with every edge having the same length. This algorithm more simple and does no need any data structures.

Breadth-First searches keep all the possible paths to be keeping in memory at once, evaluating them simultaneously. These algorithms will always find the shortest path on its first run and are more appropriate when there are a small number of solutions, which take a relatively short number of steps [36]. However, it will waste time considering irrelevant cycles. These operations will typically require more time and must be count as part of the algorithm's complexity in practice [39].



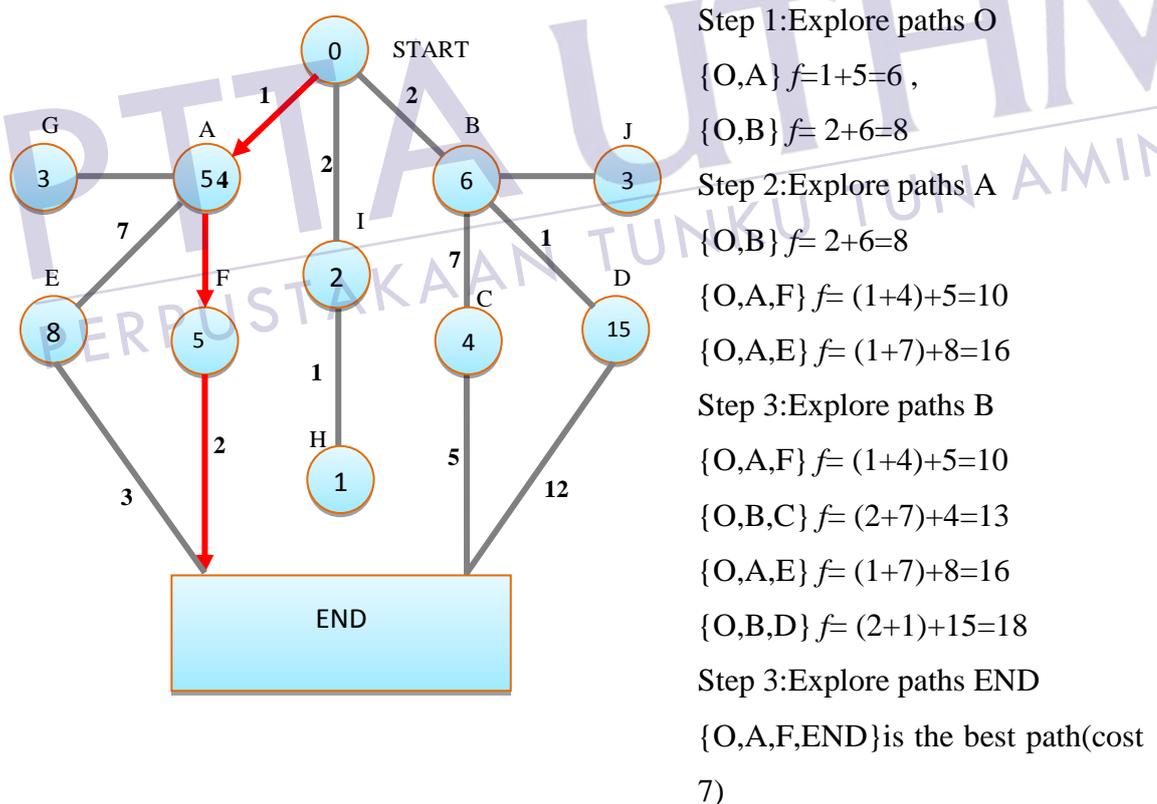
**Figure 2.10:** Breadth First Search. (Adapted from [36]). The gray line is visited line while the red line is the path.



### 2.9.3 A\* Algorithm

The A\* algorithm is one of the search algorithms which is quite popular among programmer. This algorithm provides a solution that efficiently enough for optimal path finding process [35]. This algorithm used when a solution with a minimum cost desired and there must underestimate of the cost from the current configuration to the goal. The better the cost-to-go approximates the actual cost, the more A\* can prune out partially generated paths, making it more efficiently [33, 37].

The advantages of this algorithm are it is complete provided that the node does not underestimate how close the node is to goal and it is optimal in that it will provide the fastest search of any other shortest path algorithm which uses the same heuristic [36]. Although this algorithm can find a path in a relatively short time but in fact the path are not optimal and smooth. It is also required a lot of memory.



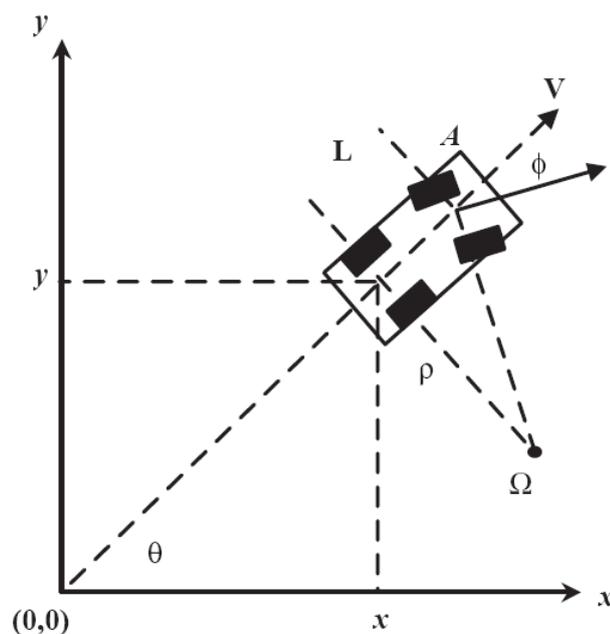
**Figure 2.12:** A\* Algorithm. (Adapted from [38]). The gray line is visited line while the red line is the path.



## 2.10 Car-Like Robot Model

Firstly, a ‘standard model’ for the vehicle and its workspace is introduced. This is the representation of the car-like robot used throughout the development of the path planning, obstacle avoidance and tracking systems. The model given below is based upon those described in [14] and [31].

Let  $A$  be a car-like robot, capable of only forward motion, modeled as a rigid rectangular body moving on a planar (two-dimensional) workspace,  $W \equiv \mathbb{R}^2$ , that is free of obstacles.  $A$  is supported by four wheels making point contact with the ground, while it has two fixed rear wheels and two directional (steerable) front wheels. The wheelbase (distance between front and rear wheels) is denoted by  $L$ .  $\Lambda$  is the configuration of  $A$  within  $W$  defined by the following quartet,  $(x, y, \theta, \kappa)$  where  $(x, y)$  are the coordinates of the rear axle midpoint.  $\theta$  (the vehicle’s orientation) is the angle between the  $x$ -axis and the main axis of the vehicle ( $-\pi < \theta < \pi$ ). Steering angle is denoted as  $\phi$  and determines the instantaneous centre of rotation  $\Omega$  of the robot. The position of  $\Omega$  determines the instantaneous radius of curvature  $\rho$  of the vehicle’s path, given by  $L / \tan \phi$  [14]. The instantaneous curvature  $\kappa$  of the robot is the inverse of the instantaneous radius of curvature ( $\kappa = \rho^{-1}$ ). Finally, the vehicle’s velocity at the centre of its rear axle is defined as  $V$ . This vehicle model is shown in Figure 2.14.



**Figure 2.14:** Car-Like Robot Model

## 2.11 Previous Study on Voronoi Diagrams

This section will review several recent papers that are using Voronoi diagram method for path planning. [17] addresses the problem of on-line path following for a car working in unstructured outdoor environments. The partially known map of the environment is updated and expanded in real time by a Simultaneous Localization and Mapping (SLAM) algorithm. This information is used to implement global path planning. A cost graph is initially constructed followed by a search to find the near-optimal path considering uncertainty in both vehicle location and map. Selected points in the global path are connected by continuous-curvature paths. An improved feedback linearization technique is presented to guide the car along the defined path.

[24] gives an algorithm for generating the Voronoi diagram from the points set on a cylinder by modification process to improve the efficiency of the weld testing for acylindrical oilcan performed by the wall-climbing robot. Based on this algorithm the paper also provides a method about the design of cylindrical tank wallboards and the weld testing path planning from Delaunay Triangulation. A software simulation platform is also developed. The simulation results show that the method is effective to the stand cylindrical tank design and the wall-climbing robot weld testing path planning.

A new motion planning technique, which is built on the generalized Voronoi diagram, for a robot with kinematic or dynamic constraints is proposed in [18]. The generalized Voronoi diagram serves this task effectively as it maintains the largest (the safest) possible distance from surrounding obstacles. Moreover, a novel approximation geometric algorithm, which embodies a trade-off between the efficiency of computation, implementation difficulty, and robustness, for computing this diagram is presented.

[21] present a simulated annealing (SA) based algorithm for robot path planning. The kernel of the SA engine is based on Voronoi diagram and composite Bezier curve to obtain the shortest smooth path under given kinematic constraints. In the algorithm, a Voronoi diagram is constructed according to the global environment. The piecewise linear path in the Voronoi diagram which keeps away from the obstacles is obtained by performing Dijkstra's shortest path algorithm. The control points on the reference path are used to create the control variables for the SA engine. The SA engine then updates the control variables to obtain the shortest



composite Bezier curve path while satisfying given kinematic constraints. Experimental results on two maps containing sharp turns demonstrate the effectiveness of the proposed SA-based smooth path planning algorithm.

Meanwhile [16] presents a new sensor based global Path Planner which operates in two steps. In the first step the safest areas in the environment are extracted by means of a Voronoi diagram. In the second step Fast Marching Method is applied to the Voronoi extracted areas in order to obtain the shortest path. In this way the trajectory obtained is the shortest between the safe possible ones. This two-step method combines an extremely fast global planner operating on a simple sensor based environment modeling, while it operates at the sensor frequency. The main characteristics are speed and reliability, because the map dimensions are reduced to a unidimensional map and this map represents the safest areas in the environment for moving the robot.



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