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ABSTRACT

Disc brakes have been used for many years in automobiles and are still undergoing further development in terms of the temperatures that they can reach and operate safely at. Many methods have been introduced in the past to simulate and predict the temperature history of the disc brake, such as lumped analysis, one-dimensional analytical method and numerical method for analysis in 2-D or 3-D. These numerical simulations range from finite differences to finite elements, each with a variety of assumptions.

In this paper, we show that the method of order-of-magnitude analysis, originally proposed by Ludwig Prandtl in his analysis of fluid flow and boundary layers, could be advantageously applied to the study of temperature distribution in disc brakes. The governing equations are formulated in their entirety sans simplifying assumptions, as a 3-D transient problem, in the axial, radial and peripheral directions, also accounting for enthalpy flow and fin effect. The equations are then normalized with respect to variables of physical significance in the problem. The normalized equations are then examined for the order of magnitude of the coefficients of the terms in order to determine their relative importance. This approach has been applied with success in fluid flow, convective heat transfer and transient conduction.

The results of the analysis of a disc brake based on the order-of-magnitude approach are compared with previously obtained solutions in the literature and found to have very good agreement. The value of the method is that, even without a physical understanding of the underlying phenomena, it is possible to make appropriate problem simplifications and get efficient solutions. Further, the analysis has the potential to be extended to include thermal cracking, warping and thermal stress in the disc brake.

Keywords: Disc Brake, Thermal Analysis, Order-of-Magnitude Approach.

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INTRODUCTION

A safe motor vehicle should have continuous adjusting of its speed to changing traffic conditions. The brake along with the tires and steering system is the most critical safety measure, in avoiding accident in motor vehicles. Its must perform safely under a variety of operating conditions including slippery, wet, and dry roads; whether a vehicle is lightly or fully loaded; when braking on a straight or in a curve; with new or worn brake lining; with wet or dry brakes; when applied by the novice or experienced driver; when braking on smooth or rough roads; or when pulling a trailer.

These general uses of the brakes can be formulated in terms of three basic functions that a braking system must provide:

- Decelerate a vehicle including stopping.
- Maintain vehicle speed during downhill operation
- Hold a vehicle stationary on a grade.

This study is about obtaining the temperature variation in brake discs where a mathematical equation is formulated and solved in order to predict the thermal behavior of the disc brake. The result is then obtained with an analysis of the temperature distribution in disc brake using the developed code. The formulation can be extended to other analyses such as thermal cracking, warping and thermal stress in disc brakes.

The objective of this project is to develop the equation of the temperature variations of the brake disc axially, radially and circumferentially with respect to time in 3 dimensions. By using the Order of Magnitude Analysis, this equation will be analyzed to determine the relative importance of three directions in predicting the temperature variations of the disc with respect to time. Finally, the equation is solved using appropriate software to get the temperature variations of the disc in graphical form.

This problems is an unsteady thermal distribution where all three directions (r, θ, z) are involved in heat flow. From the derived equations it is then required to find which independent direction or axis of heat flow is most important determinant of the temperature of the brake disc. To find this direction, the order of magnitude analysis plays its part to analyzing all the three directions of heat flow. This study is complicated by the conditions at the boundary where forced convection, free convection due to disc rotation and also radiation occur. The forced convection is due to the movement of the car. Free convection is due to the centrifugal forces and temperature difference between the disc and the surrounding air. Due to the above complexity, numerical methods of solution must be resorted to.

LITERATURE REVIEW

Analysis of the temperature distribution of a disc brake has been studied via experimental, analytical and numerical methods by previous researches. One of the earliest analytical was a 1-D approach by Newcomb (1960). Continuing from Newcomb, R Limpert (1975) introduced analytical work of new thermal performance measurement in term of weight and area effectiveness of the disc. He also found the equations for determining the cooling capacity of solid and ventilated discs.

Numerical methods based on finite difference and finite element became popular among researchers. Noyes and Vickers (1969) use 3-D finite difference methods in determining the heat distribution and temperature reach of the ventilated disc. Finite difference were also used for 1, 2 and 3-D analysis (Sheridan, 1988) for determining which dimension is important for the situation at hand. Gao and Lin (2002) used 3-D finite element method to determine the temperature reach in disc brake, which differed from others authors.

Heat distribution was also investigated experimentally by Yano and Murata (1993) and the temperature reach of the disc compared with the analytical 1-D result.

ORDER OF MAGNITUDE ANALYSIS

As the Order of Magnitude Approach is applied in our work, we will first introduce its role in the simplification of the Navier-Stokes equations. Prandtl is credited with the introduction of Order of Magnitude in deriving the Boundary layer equations which were, till then, not solvable for a majority of real-life situations. Notably, the characteristic of the partial differential equations (PDE) becomes parabolic, rather than the elliptical form of the full Navier-Stokes equations. This greatly simplifies the solution of the equations. By making the boundary layer approximation, the flow is divided into an inviscid part and the viscous region at the boundary, which is governed by an easier-to-solve PDE. The continuity and Navier-Stokes equations for a 2-D steady incompressible flow in Cartesian coordinates are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2})$$

where u and v are the velocity components, ρ is the density, p is the pressure, and ν is the kinematic viscosity of the fluid at that point.

The approximation states that, for a sufficiently high Reynolds number the flow over a surface can be divided into an outer region of inviscid flow unaffected by viscosity (the majority of the flow), and a region close to the surface where viscosity is important (the boundary layer). The streamwise and transverse (wall normal) velocities inside the boundary layer are, respectively, u and v. Using asymptotic analysis, it can be shown that the above equations of motion reduce within the boundary layer to become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}$$

and the remarkable result that

$$\frac{1}{\rho}\frac{\partial p}{\partial y}=0$$

The asymptotic analysis also shows that v, the wall normal velocity, is small compared with u the streamwise velocity, and that variations in properties in the streamwise direction are generally much lower than those in the wall normal direction.

Since the static pressure p is independent of y, then pressure at the edge of the boundary layer is the pressure throughout the boundary layer at a given streamwise position. The external pressure may be obtained through an application of Bernoulli's Equation. If u_0 be the fluid velocity outside the boundary layer, where u and u_0 are both parallel, upon substituting for p, the result is

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_0\frac{\partial u_0}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}$$

with the boundary condition

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

For a flow in which the static pressure p also does not change in the direction of the flow then

$$\frac{\partial p}{\partial x} = 0$$

Therefore u_0 remains constant.

Then the equation of motion is simplified to become

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

These approximations are used in a variety of practical flow problems of scientific and engineering interest. The Order of Magnitude Analysis is applicable for any instantaneous laminar or turbulent boundary layer. It is also applicable for a variety of other analogous problems, for example in convective heat transfer, conductive heat transfer, mass transfer etc.

METHODOLOGY

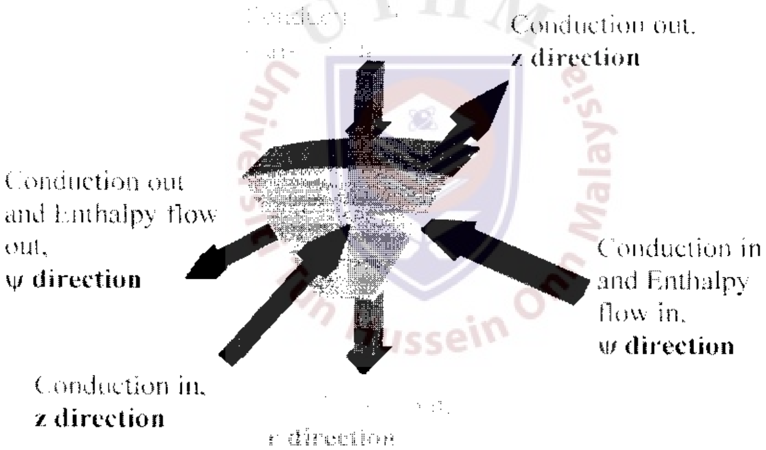

FIGURE 1. Energy Balance In Control Volume Selected From The Disc.

Derivation of energy balance in the disc:

$$q_z + q_r + q_\psi + enthalpy_{in} =$$

$$q_{z+dz} + q_{r+dr} + q_{\psi+d\psi} + enthalpy_{out} + \frac{\partial E}{\partial t}$$

Hence the net energy flow in the disc:

$$krd\,\psi dr\,\frac{\partial^2\theta}{\partial z^2}dz + kd\,\psi dzdr\,\frac{\partial\theta}{\partial r} + krd\,\psi dzdr\,\frac{\partial^2\theta}{\partial r^2} + kdrdz\,\frac{\partial^2\theta}{r\partial\psi^2}d\psi$$

$$+\,\rho r\omega C_p drdz\,\frac{\partial\theta}{\partial\psi}d\psi = \rho rd\,\psi drdzC_p\,\frac{\partial\theta}{\partial t}$$

After simplification:

$$\frac{\partial^2\theta}{\partial z^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2\theta}{\partial\psi^2} + \frac{\rho\omega C_p}{k}\frac{\partial\theta}{\partial\psi} = \frac{\rho C_p}{k}\frac{\partial\theta}{\partial t} \tag{1}$$

The above equation is normalized by substituting:

$$Z = \frac{z}{L},\ \beta = \frac{r}{R}\ \text{and}\ \frac{k}{\rho C_p} = \alpha$$

Equation (1) becomes

$$\frac{\partial^2\theta}{\partial Z^2} + \frac{L^2}{R^2\beta}\frac{\partial\theta}{\partial\beta} + \frac{L^2}{R^2}\frac{\partial^2\theta}{\partial\beta^2} + \frac{L^2}{R^2\beta^2}\frac{\partial^2\theta}{\partial\psi^2} + \frac{\omega L^2}{\alpha}\frac{\partial\theta}{\partial\psi} = \frac{L^2}{\alpha}\frac{\partial\theta}{\partial t} \tag{2}$$

Substituting the value of the physical quantities of relevance:

$$L = 6.35mm, R = 158.75mm, \beta_{min} = 0.25, \rho = 7228\frac{kg}{m^3}, C_p = 419\frac{J}{kg^oC}, k = 48.46\frac{W}{m^oC}$$ T

$$\alpha = 1.6\times10^{-5}\frac{m^2}{s}, \omega(t=0) = 25.06\frac{rev}{s}$$

he dimensionless energy equation becomes:

$$\frac{\partial^2\theta}{\partial Z^2} + (0.01)\frac{\partial\theta}{\partial\beta} + (0.0025)\frac{\partial^2\theta}{\partial\beta^2} + (0.01)\frac{\partial^2\theta}{\partial\psi^2} + (63.16)\frac{\partial\theta}{\partial\psi} = \frac{\partial\theta}{\partial t^*} \tag{3}$$

By using Order of Magnitude Analysis (OOM) on the result above, enthalpy flow direction gives large value which is 63.16 followed by z-direction, r-direction and angular direction of heat flows. This means that the enthalpy flows and z-direction (thickness of the disc) are the more important terms of the transient heat transfer in the disc.

To validate the Order of Magnitude Analysis above, 2-D finite difference simulations were done, subject to the following boundary conditions;

Initial Condition:

$$T(0) = T_\infty$$

In z-direction:

At the angle of $0 < \psi < \frac{\pi}{3}$; $\quad \frac{\partial T}{\partial z} = -\frac{mV^2}{16kt}$

At the angle of $\frac{\pi}{3} < \psi < 2\pi$; $\frac{\partial T}{\partial z} = -\frac{h}{k}(T(z) - T_{\infty})$

In r-direction:

At the $r = 0$; $\frac{\partial T}{\partial r} = 0$ (symmetry)

At the $r = R$; $\frac{\partial T}{\partial r} = -\frac{h}{k}(T(r) - T_{\infty})$

The results are shown in the next section on Result and Discussion.

RESULT AND DISCUSSION

For the validation of the simulation, results from R. Limpert is compared which is shown below;

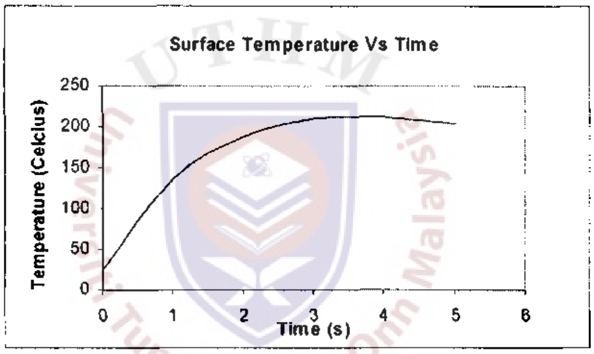

FIGURE 2. Theoretical surface temperature computed from R. Limpert[3].

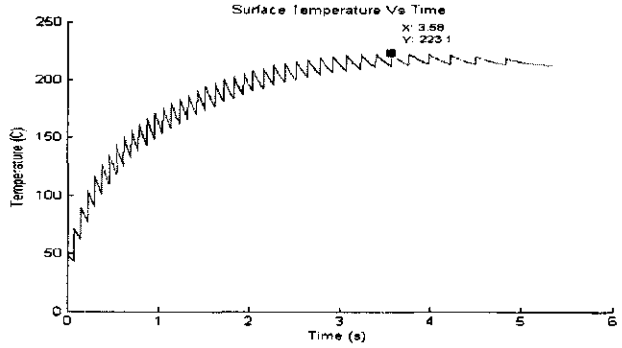

FIGURE 3. One-Dimensional simulation of temperature distribution in the disc brake computed by finite difference calculation

For 1-D finite difference simulation with linear velocity deceleration, the difference in maximum temperature reached is about 7 percent between our work and Limpert's results, which is acceptable. The maximum temperature reached is at about half of the total braking process. From this result it can be concluded that the simulation by finite differences can be used to validate the Order Of Magnitude Analysis presented in this paper.

In order prove that the order of magnitude is correct, 1-D and 2-D simulations are done in r (radial direction of the disc) and z (thickness of the disc) for the heat flow and temperature reach. Figure 2 shows the temperature reach during 2-D simulations in r and z directions. It shows that the maximum temperature reach is about 266.9 °C. This results is compared with simulation of 1-D finite difference simulations shown in figure 3. the maximum temperature reached for 1-D simulation differs only by about 0.6°C which shows that most of the heat is flowing into z (along thickness of the disc) direction. The temperature increase indicates that most of the heat flow is in that direction.

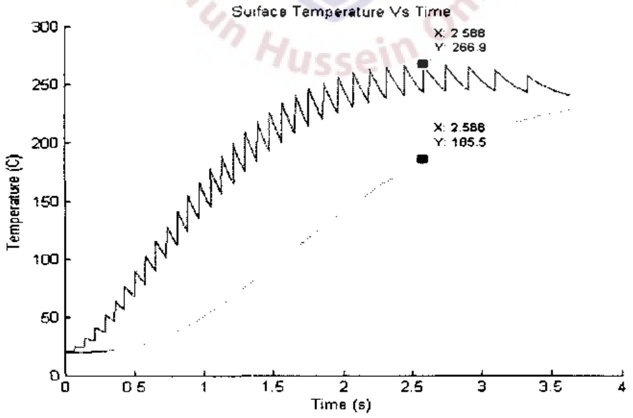

FIGURE 2 Two dimensional simulation of disc using finite difference method

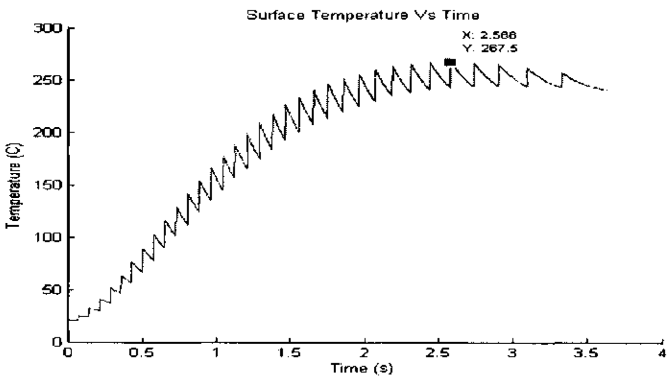

FIGURE 3 One dimensional simulation of disc using finite difference method

From the two results above it can be seen that the order of magnitude analysis proves that the z (axial) direction of the disc is the important direction of heat flow in the disc. This also indicates that inclusion of the r direction only gives a small percentage of solution accuracy of the temperature reach of the disc and heat flows.

It has been proved that 1-D analysis can be used to predict the worst case increase in temperature of disc and it is inherently safe. For manufacturing purposes, 2-D analysis gives a more accurate temperature profile. 3-D analysis, that is mathematically involved, is not necessary.

CONCLUSION

An appraisal of all the analysis and results of simulation shows that enthalpy flow and conduction in the z-direction play an important role in calculation of the heat flow in disc brakes. It indicates that these terms in the governing equation can provide the worst case scenario for designing the brake disc. One dimensional analysis proves to be convenient to determine the surface temperature increase in disc brake and this analysis should be in the z-direction, i.e. the thickness direction of the disc brake. With the results shown in the results and discussion, it is established that the Order Magnitude of Analysis can be used to predict the important directions of heat flow in a brake disc. This has been proved by the small difference between the 2-D (r,z) simulation and 1-D (z direction) simulation during single stop braking.

NOTATIONS

z	axial direction	R	radius of the disc
r	radial direction	Θ	temperature difference
ψ	angular direction	P	density of the material of the disc
k	thermal conductivity	C_p	Specific heat capacity
h	heat transfer coefficient	ω	angular velocity
L	thickness of the disc brake	α	thermal diffusivity

REFERENCES

Newcomb, T. P. (1960). "Temperatures Reach In Disc Brake." Journal of Mechanical Engineering Science.

Prakash Agrawal, O. M. (2004). "Application of Fractional Derivatives in Thermal Analysis of Disc Brakes." Nonlinear Dynamic Kluwer Academic Publisher.

Limpert, R. (1975). "The Thermal Performance of Automotive Disc Brakes." SAE Technical Paper.

Sheridan, D. C. et. al. (1988). "Approaches to the Thermal Modeling of Disc Brakes." SAE Technical Paper.

Noyes, R. N. and Vickers, P. T. (1969). "Prediction of Surface Temperature in Passenger Car Disc Brakes." General Motor Corp.

Masataka Yano and Masahiko Murata. (1993). "Heat Flows on Disc Brakes." SAE Technical Paper

Gao, C. H. and Lin, X. Z. (2002). "Transient Temperature Field Analysis Of A Brake In A Non-Axisymmetric Three-Dimensional Model." Journal of Materials Processing Technology Elsevier.

Limpert, R. (1975). "Cooling Analysis of Disc Brake Rotors." SAE Technical Paper.

Jancirani, J. et al. (2003). "Virtual Heat transfer Analysis of the Automotive." SAE India Technical Paper.