Abstract—This paper describes the $H_\infty$ control design for robust speed control for the Permanent Magnet Synchronous Motor. The speed feedback control is used to maintain the speed during load changing. Due to the load changing it will create the disturbance to the motor. To solve this problem the $H_\infty$ control theory has been used to determine the robustness of the controller and to have good tracking performance for the speed. The model and the controller value have been developed in MATLAB/Simulink where the results show the controller able to maintain the speed despite the load is changed.

Index Terms—Permanent Magnet Synchronous Motor, robust control, MATLAB, $H_\infty$.

I. INTRODUCTION

Permanent Magnet Synchronous Motors (PMSMs) have been used widely because of the high torque applications, accurate positioning, lower inertia, large power rate, no spark and lower noise [1], [2], [3]. These advantages can be extracted if the PMSMs have minimum effect to the disturbances due to the changed of the mechanical torque at the input of the PMSM. To meet this condition, uncertainties such as perturbations, disturbances and load changing [2], [4], [5] must be considered when designing the controller. It also must have high sensitivity while at the same time maintain the stability of the system.

As known, speed is a state variable that can be controlled in the PMSM. Due to this variable this paper is focused in speed control design. The control theory which able to response or manipulate this situation is the robust control where the aims are to achieve the robustness performance and to have a close boundary for the differences between the true model and the nominal model [6]. One of the techniques is using the $H_\infty$ or non-linear $H_\infty$ control theory. The standard block diagram for the $H_\infty$ control is shown in Fig. 1. It shows that the controller is able responded to the disturbance input in generating the required output.

This paper explains the $H_\infty$ control design for speed control to generate suitable signal for the motor current output. The signal that will be generated is based on the how the load changed will affected the speed. The load is looked as a the disturbance variable ($T_m$) when designing the controller. A typical PMSM control drive consists of a speed control and the current generator with a closed loop feedback is shown in Fig.2.

II. MODELING OF THE PERMANENT MAGNET SYNCHRONOUS MOTOR

In this section the model of PMSM is explained and referred to [1], [2], [4], [7]. Papers [1], [2], [4], [7] give a clear view to determine the outputs and how the torque is developed inside the PMSM. The model is in $dq$ transformation where only the $q$ component is appeared while the $d$ and zero components will not given any significant to the model. The mathematical model of $q$ component is expressed in the equation below.

$$\frac{di_q}{dt} = \frac{v_q}{L_q} - \frac{R i_q}{L_q} - \frac{\omega M_f}{L_q}$$

$$\frac{d\omega}{dt} = \frac{T_e}{J} - \frac{T_m}{J} - \frac{B \omega}{J}$$

$$T_e = k_n i_q$$

where

$v_q$: q axis stator voltage
$i_q$: q axis stator current
$L_q$: q axis stator impedance
$R$: stator resistance
$M_f$: flux linkage
$\omega$: inverter frequency
$\omega$: motor speed
$T_e$: electrical torque
$T_m$: mechanical torque
$k_n$: $\frac{2}{\pi} \times \text{Pole} \times M_f$
Eq.1 is used to model the current control for the PMSM and will not be discussed here while Eq.2 is used to model the speed controller with the disturbance affect. Eq.3 shows the generated electrical torque in the PMSM. Fig.3 shows the speed control block diagram that has been modeled using Eq.2 where it consists of two inputs and one output where the inputs are the \( T_m \) and \( i_{qref} \) while the output is \( \omega \). In this model \( T_m \) is known as one of the disturbances or the mechanical torque input or load effect. The speed control that will be designed will generate the current reference \( i_{qref} \) to the input for ramp comparator for the Pulse Width Modulation (PWM) generation for the motor current.

### III. Design of \( H_\infty \) Robust Controller

A robust control design with respect to the plant parameters variations and \( T_m \) variations are proposed using \( H_\infty \) control theory. The \( H_\infty \) control matrix consists of generalized plant \( P(s) \) and the new design controller \( K_\infty \) where it can be determined refer to Fig.4. The new plant \( P(s) \) consists of the weighting functions \( W_1, W_2, W_3 \) and the nominal plant \( P \). \( W_1 \) is known as error performance, \( W_2 \) is the robust performance and \( W_3 \) is the input performance constant function to the plant. The \( \Delta \) is known as the relative plant uncertainty that determined the boundary of \( W_2 \) should have. The output of the \( P \) is the \( \omega \) where it should be controlled.

#### A. Definition of the Generalized Plant

The generalized plant \( P(s) \) is shown in Fig.4, where the outputs are \( z_1, z_2, z_3 \). These outputs are the new output where have included the weighting functions. The disturbance \( T_m \) and the reference signal \( \omega_{qref} \) are also included in the \( P(s) \). The input of \( P \) is the output of the \( K_\infty \) control transfer function. The state-space equation for the model is given by,

\[
\frac{d\omega}{dt} = \begin{bmatrix} \frac{-B}{J} & [\frac{1}{J} \frac{L_2}{J}] \end{bmatrix} \begin{bmatrix} \omega \end{bmatrix} + \begin{bmatrix} \omega_{qref} \end{bmatrix} \]

\[
T_m \]

(4)

The aim of the feedback control is to make sure the speed \( \omega \), as the same as the \( \omega_{qref} \) value when the disturbance is been applied in the nominal plant. The generalized plant \( P(s) \) can be written as,

\[
\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ e \end{bmatrix} = \begin{bmatrix} W_1 & -W_1P_{11} & -W_1P_{12} \\ 0 & W_2P_{11} & W_2P_{12} \\ 0 & W_3 & 0 \\ 1 & -P_{11} & -P_{12} \end{bmatrix} \begin{bmatrix} \omega_{qref} \\ \omega \\ T_m \end{bmatrix} \]

(5)

\[
P(s) = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \end{bmatrix} \]

(6)

The weighting functions \( W_1, W_2, W_3 \) are the designed parameters to force the closed loop response to meet certain specifications that will be discussed later in this paper. \( W_1 \) is chosen to be low pass filter in order to reject the output disturbance \( W_2 \) is to determine the plant relative uncertainty \( \Delta \) boundary condition \( \Delta \). The value of \( W_1 \) must be chosen as small as possible, to make sure the \( D_{12} \) in generalized plant is full rank, required by the \( H_\infty \) control.

#### B. Weighting functions selection

As known \( W_1 \) is used for error tracking performance where it indicates the error from the reference value to the actual value. \( W_1 \) is selected base on the Proportional Integrator (PI) which has high gain. This filter will reject the disturbance at the low frequency range [1]. It can be written as

\[
W_1 = \frac{0.5s + 35}{s + 0.0001} \]

(7)

\( W_2 \) is the robust performance for the speed output which includes the disturbance. In dealing with this the original plant \( P \) has been tested under the plant uncertainty \( \Delta \) condition. This is to make sure the new \( W_2 \) responded to the rapid torque changing. The \( \Delta \) has been chosen to be in range of 80% to 120% from the nominal model \( \Delta \) and with the time delay response of \( 0 < \theta_{sec} < 1 \). The general transfer function for \( W_2 \) can be written as \( [9] \). In this application the \( \Delta \) has been selected to the value of 100% \( \approx 1 \). The value for \( W_2 \) is selected based on the boundary condition \( [2], [4], [5] \) where shows in Eq 8. \( W_3 = 0.001 \) value has been selected to make sure the generalized plant is in full rank.

\[
W_2 = 0.000068s + 0.7 \]

(8)
IV. SIMULATION RESULTS

The simulations were done in MATLAB and the results are the motor current ($i_{abc}$), speed ($\omega$) and the speed error ($e$) in which to determine and observe the performance of the controller to the PMSM. The values that were used to model the PMSM is given in Table I.

**TABLE I**

<table>
<thead>
<tr>
<th>PMSM PARAMETERS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Speed</td>
<td>1000 (rpm)</td>
</tr>
<tr>
<td>Torque</td>
<td>5 (Nm)</td>
</tr>
<tr>
<td>Pole Pairs</td>
<td>4</td>
</tr>
<tr>
<td>J (moment of inertia)</td>
<td>$1.05 \times 10^{-4} (kg.m^2)$</td>
</tr>
<tr>
<td>$k_{in}$</td>
<td>$5.536 \times 10^{-1} (N.m/s)$</td>
</tr>
<tr>
<td>$B$ (viscous damping coefficient)</td>
<td>$4.5498 \times 10^{-4} (N.m.s/rad)$</td>
</tr>
</tbody>
</table>

The $K_{in}$ controller value has been generated using hinfsys function in MATLAB. The generated $H_{in}$ controller transfer function is given by

$$K_{in} = \frac{2.609(s + 345.6)}{s + 0.05048} \quad (9)$$

Eq.9 shows the controller is in stable region because all the poles and zeros are at the left hand side of the stability region. The bode diagram of this controller is shown in Fig.5. From the bode diagram the controller responses to the lower input frequency and reject the high frequency.

The complete simulation block diagram is shown in Fig.6. The feedbacks are the $i_{abc}$ and the $\omega$ where both of them are used to generate the desire input to the motor. From the block diagram, only $q$ component is considered while the other components are selected to 0 . The results from this simulation are shown in Fig.7.

Fig.7 shows the outputs of the motor which are the motor current and the speed of the motor. As can be seen, at the time of $1sec$ the load is increased and the motor current is dropping. At this stage if there is no speed control the speed will drop. Due to the robustness speed control that has been modeled, the controller is able to maintain the speed before and after the load changing. This is shown in motor speed result in Fig.7. For the $\omega$, the speed is maintained at the target value during the simulation time. It shows that the controller is suitable for the speed control.

Fig.8 shows the speed error where it graph has been limited from $-3$ to $2$. It indicates that when the load changed at $1sec$ point, the controller needs $0.2sec$ to restore back to the normal value. It shows that the controller is robust enough to response in quick time when the disturbance affected the PMSM.
As a conclusion, this paper shows that the speed controller for PMSM can be designed and modeled with the help of $H_{\infty}$ control theory where it is included the disturbance effect or load changing to generate the desired signals to maintain the speed of the motor. As a result, the robustness speed controller that has been proposed can be implemented in the motor control application without any additional controller to the PMSM.

**REFERENCES**


