CONTOUR MATCHING USING ANT COLONY OPTIMIZATION AND CURVE EVOLUTION

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ABSTRACT

Shape retrieval is a very important topic in computer vision. Image retrieval consists of selecting images that fulfil specific criteria from a collection of images. This thesis concentrates on contour-based image retrieval, in which we only explore the information located on the shape contour. There are many different kinds of shape retrieval methods. Most of the research in this field has till now concentrated on matching methods and how to achieve a meaningful correspondence. The matching process consist of finding correspondence between the points located on the designed contours. However, the huge number of incorporated points in the correspondence makes the matching process more complex. Furthermore, this scheme does not support computation of the correspondence intuitively without considering noise effect and distortions. Hence, heuristics methods are convoked to find acceptable solution. Moreover, some researches focus on improving polygonal modelling methods of a contour in such a way that the resulted contour is a good approximation of the original contour, which can be used to reduce the number of incorporated points in the matching. In this thesis, a novel approach for Ant Colony Optimization (ACO) contour matching that can be used to find an acceptable matching between contour shapes is developed. A polygonal evolution method proposed previously is selected to simplify the extracted contour. The main reason behind selecting this method is due to the use of a stopping criterion which must be predetermined. The match process is formulated as a Quadratic Assignment Problem (QAP) and resolved by using ACO. An approximated similarity is computed using original shape context descriptor and the Euclidean metric. The experimental results justify that the proposed approach is invariant to noise and distortions, and it is more robust to noise and distortion compared to the previously introduced Dominant Point (DP) Approach.

ABSTRAK

Dapatan semula bentuk merupakan topik yang amat penting dalam penglihatan komputer. Dapatan semula imej melibatkan pemilihan dari koleksi-koleksi imej yang memenuhi kriteria tertentu. Tesis ini menjurus kepada dapatan semula imej berasaskan kontur, di mana kami meneroka hanya maklumat yang terletak pada kontur bentuk. Terdapat pelbagai jenis kaedah dalam dapatan semula bentuk. Sehingga kini, kebanyakan penyelidikan di dalam bidang ini tertumpu kepada kaedah-kaedah pemadanan dan cara untuk mencapai kesepadanan hubungan yang lebih baik. Proses pemadanan ini terdiri daripada mendapatkan kesepadanan hubungan antara titik-titik yang terletak pada kontur-kontur yang direka. Walau bagaimanapun, kewujudan jumlah yang besar bagi titik-titik pemadanan di dalam kesepadanan hubungan tersebut sehingga pemadanan menjadi lebih kompleks. Selain itu, kaedah ini tidak mengambilkira kesan hingar dan gangguan-gangguan lain. Oleh itu, kaedah heuristik digunakan untuk mencari penyelesaian yang lebih bermakna. Tambahan pula beberapa penyelidikan memfokuskan kepada penambahbaikan kaedah-kaedah pemodelan poligon kontur supaya kontur yang dihasilkan mempunyai penghampiran yang baik berbanding kontur asal, di mana ia boleh digunakan untuk mengurangkan bilangan titik-titik pemadanan. Dalam tesis ini, satu pendekatan baru untuk pemadanan kontur bagi Pengoptimuman Koloni Semut (PKS), boleh digunakan dalam mencari pemadanan antara bentuk-bentuk kontur. Satu kaedah evolusi poligon yang dicadangkan sebelum ini telah dipilih bagi memudahkan kontur yang telah diekstrak. Tujuan utama di sebalik pemilihan kaedah ini ialah disebabkan penggunaan kriteria pemberhentian yang perlu ditentukan pada peringkat awal. Proses pemadanan yang diformulasikan sebagai Masalah Tugasan Kuadratik (MTK) telah diselesaikan dengan menggunakan PKS. Satu penghampiran keserupaan dikira menggunakan bentuk penghurai konteks asal dan metrik Euclidean. Sebagai kesimpulan dari keputusan eksperimen, pendekatan ini tidak varian kepada bunyi bising dan gangguan-gangguan, dan lebih teguh kepada bunyi bising dan gangguan-gangguan berbanding dengan Pendekatan Titik Dominan (TD) sebelumnya.

PUBLICATIONS

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4. **Younes Saadi**, Rathiah Binti Hashim and Rosmila Abdul-Kahar, Shape Recognition Using Heuristic Algorithm, International Journal of Signal & Image Processing IJSIP. (Accepted, indexed by **DBLP**).
5. **Younes Saadi**, Rathiah Binti Hashim, Rosmila Abdul-Kahar. Using Video Games to Detect Dyslexia Symptoms Malaysian Technical Universities International Conference on Engineering & Technology (MUiCET 2011).

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ABBREVIATIONS

ACO	Ant Colony optimization
DCE	Discrete Curve Evolution
COPAP	Contour matching via the Order preserving assignment Problem
QA	Quadratic Assignment
DP	Dominant Points
US	Ultrasound
CT	Computed Tomography
QAP	Quadratic Assignment Problem
LAP	Linear Assignment Problem
ICP	Iteration Closest Point
MDS	Multi Dimension Scaling
PA	Polygonal Approximation

CHAPTER 1

INTRODUCTION

1.1 Background study

Shape retrieval is a very important topic in computer vision. It is an act of selecting images that fulfil a specific criterion from a collection of images (Nacereddine *et al.*, 2007). It is based on shape matching, which is one of the fundamental techniques in verifying those criteria. Shape matching is classified into two main categories, region based matching and contour based matching. This research focused only on contour based matching.

In contour based matching, only the information located on the shape contour is explored. The contour consists of a collection of points located along the edges, formulating a bipartite graph that can be structured as a Quadratic Assignment (QA) (Kaick *et al.*, 2007). Most of the research in this field has till now concentrated on how to achieve a meaningful correspondence between contour points (Kaick *et al.*, 2007). Based on the distribution of the contour points, a bipartite graph can be formulated, and the problem is structured as Quadratic Assignment Problem (QAP). This is considered as one of the NP-Hard problems. Finding an optimal solution for such problems is so difficult and in some cases, impossible (Ruberto and Morgera, 2011). For this complexity reason, heuristics techniques are convoked to find acceptable solutions instead of using exact methods.

On the other hand, some researches focus on improving polygonal approximation methods of a contour in such a way that the resulted contour contains only the most dominant points that can be used to extract meaningful information (Parvez and Sabri, 2010) which can reduce the correspondence complexity.

1.2 Motivation

Shape matching has been deeply studied by many researchers (Veltkamp *et al.*, 2001). However, challenges still remain. The first challenge is the invariance. Since shape in many applications is often discussed based on the property of invariance, shape matching is expected to be invariant to transformations such as example translation, scale and orientation. The second challenge is tolerance as noise, blur, crack and deformation are usually introduced when the shape of an object is extracted from an image. In this case, a shape matching is required to be robust to these imperfections.

Moreover, in contour based matching, the points along the contours as a bipartite graph can be figured out and formulated as a QA. This is considered as an NP-hard problem (Ruberto and Morgera, 2011). In order to find an acceptable solution, heuristics techniques are often used. The main idea is to compute the mapping between two contours by minimizing the global dissimilarity. Many research studies have been proposed. For example, Hungarian method uses the simple greedy matching (Papadimitriou and Stieglitz, 1982), and COPAP (Scot and Nowak, 2006) takes into account the order preserving. However, the main drawback of these techniques is the omission of proximity information measurement between feature points on the same shape contour. For this reason, Kaick *et al.* (2007) proposed an Ant Colony Optimization (ACO) approach based on incorporating proximity into an optimization framework. However, the huge number of points incorporated in the correspondence makes the matching more complex and less accurate (Ruberto and Morgera, 2011). A modified ACO matching approach has been proposed by Ruberto and Morgera (2011) based on genetic algorithm; only dominant points are used instead of the sampling distribution of contour points, which improves the correspondence accuracy and reduces the complexity.

In this research, a new ACO matching approach based on Discrete Curve Evolution (DCE) was proposed. In order to reduce the number of contour points incorporated in the correspondence, a polygonal approximation proposed previously (Latecki and Lakamper, 2000) was selected to simplify the extracted contour. Mainly, it simplifies the contour by neglecting distortions while at the same time preserving the perceptual appearance at a level sufficient for object recognition. To test the effectiveness of our approach, an MPEG-7 subset described by Ruberto and Morgera (2011) was used to test shape retrieval considering noise and distortions effect. The results were also compared with the previous work of ACO matching by Ruberto and Morgera (2011).

1.3 Objectives

The objectives of this research are:

(i) To design a shape contour matching algorithm based on Ant Colony Optimization and DCE techniques.
(ii) To develop a prototype for shape matching which will be known as `myMatch`
(iii) To test the prototype and evaluate its efficiency by comparing it with an existing approach, namely Dominant Point (DP).

1.4 Scope of study

This research focused only on Ant Colony Optimization approach as a model of 2D contour matching. We used the original shape context which is not invariant to rotation. `Not invariant to rotation` means that the accuracy of the matching process is affected by rotation. Only the invariance to distortion and noise was investigated in this approach. This approach was tested using MPEG-7 dataset following the work of Ruberto and Morgera (2011) which was the only research on matching algorithm based on ACO and approximation method.

1.5 Significance of study

Shape matching has numerous applications in both computer vision and graphics. The significance of this study is to provide a robust matching invariant to noise and distortion, which can be used in many different potential applications:

(i) This research can be used in registration as a matching model. The registration can be used for various applications such as assistance in surgery and also for reconstruction of data from different images.

(ii) It could be used in the field of medical image analysis such as matching two images to identify diseased tissue or to detect tumours.

(iii) It could be used in similarity measure for 2D shape based object retrieval and recognition.

(iv) This technique can be a basis for modelling new computer aided detection software for automated detection of abnormalities.

1.6 Thesis outline

This thesis is organized as a progressive study of our approach using ACO matching and DCE. Each chapter shows a stage in the research and is based on the foundation provided by previous chapters.

In Chapter 2, the basic concepts used in contour matching are initially followed by a review on Ant Colony Optimization (ACO). A review has been done on previous work, focusing on contour correspondence based heuristics techniques and polygonal approximation techniques. Some concepts on polygonal approximation are also presented. This survey is not exhaustive but serves to set the context for the contribution of this thesis and also summarizes the problems and issues of the topic.

The focus in Chapter 3 is on establishing the direction of research that takes into account the issues decorticated in Chapter 2. In this chapter, the proposed approach consists of ACO matching based on DCE is introduced briefly by using original shape context as a descriptor. The main idea is to reduce the number of the contour points along the contour in such a way that the simplified contour preserves the original frame of the shape. This simplification provides a solution for the problem related to the complexity of correspondence and also improves the accuracy of correspondence by reducing noise and distortion effect. Finally, the flowchart of the proposed approach is presented.

Chapter 4 describes the implementation part of this research, which is part of the research contribution. It consists of a detailed explanation of the DCE algorithm used with the ACO matching. The mathematical model of DCE is described and the different functions used in the algorithm are detailed. Furthermore, the original ACO algorithm is explained with its mathematical model. Finally, some problems encountered during the implementation of the approach are discussed.

In Chapter 5, the obtained results are discussed. The experimental results are justified for each class which provide a good analysis for the efficiency of the proposed approach. Moreover, a comparison between the proposed approach and Dominant Point approach (Ruberto and Morgera, 2011) is carried out in order to prove the robustness of the proposed approach.

Finally, Chapter 6 concludes the thesis with a summary of the contributions. Some limitations of the proposed approach are highlighted, and possible directions and future research are presented.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Nowadays, shape analysis has become a hot topic in computer vision. It has been widely adopted in many applications of computer graphics. In particular, shape matching—one of the fundamental techniques of shape analysis—plays a primordial role in shape retrieval, recognition and classification, and medical registration (Tsapanos *et al.*, 2011). Shape matching has reached a state of maturity in which many real products based on shape matching are commercialized in different areas (Aaron *et al.*, 2011). In the commercial domain, shape matching methods are being used to retrieve and classify images, for personal and institutional needs like security and military. In the medical domain, shape matching is used in radiology to diagnose and to assess medical images to determine the progress and the suitable treatment options.

Shape matching is application dependent. Different applications may have different requirements on invariance and tolerance to noise, distortion, blur, transformation, scale and orientation. Thus, it is difficult to design a universal method which is suitable for all applications. Nowadays, many techniques have been proposed but most of them only focus on the applications where shape is invariant to transformation.

Based on the representation techniques, shape matching techniques can be classified into two categories: contour based matching and region based matching (Luciano and Roberto, 2009). This research is focusing on contour based matching techniques, in which only the information located on the shape contour is explored. Most of the past research concentrated on how to achieve a meaningful correspondence (Kaick *et al.*, 2011). On the other hand, some research focused on improving representation methods of a contour in such a way that the resulted contour could be used to extract meaningful information (Luciano and Roberto, 2009).

Over the last years, several correspondence based matching methods have been introduced; some of them are heuristic-based. They represent an acceptable solution for the cases where the matching is formulated as an NP-hard problem (Kaick *et al.*, 2007). Furthermore, many methods for representing shape contours by approximation have been proposed. The rest of this chapter will discuss on correspondence based contour matching, polygonal approximation, Ant Colony Optimization (ACO) and some issues related to the topic.

2.2 Fundamentals of shape representations

Shape plays a crucial role in human object classification and identification (Beeck *et al.*, 2001). It is also regarded as being one of the predominant features determining the perceived similarity of images (Lu *et al.*, 2007). The term shape, however, is used in a variety of meanings. The task of defining shape is complex (Luciano and Roberto, 2009). The point is that if shapes are to be successfully analysed, we must precisely know what they are and what properties they exhibit, and often take into account the way they are perceived by humans. Therefore, this research adopts a definition of shape which tries to take into account what the object is and to decide on shape equivalence under a generic class of transformations were adopted. Regarding this aspect, a shape can be considered as any connected set of points. Many fundamental concepts are described in order to put the reader in the context of this research.

As shown in Figure 2.1, shape representation can be classified into three categories: representation based contour, representation based region and representation based transformations. According to the type application, a representation model is selected. The most useful model is region based representation because using the whole surface gives more efficiency than using contour. However in some particular application, using the whole surface is not significant. Contour based representation is useful in many types of applications especially in medical image processing and image detection.

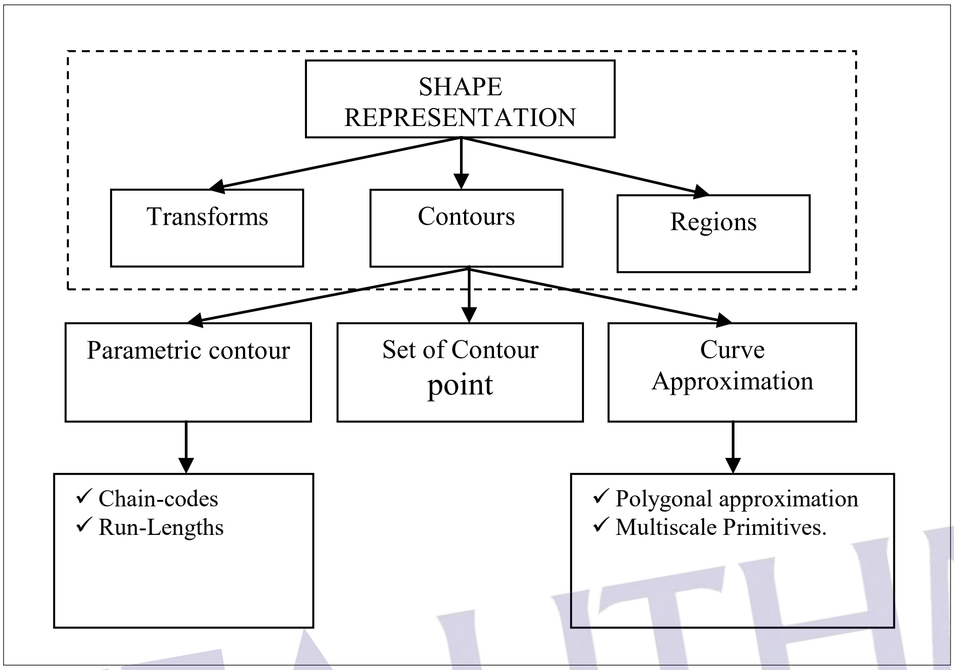

Figure 2.1: A taxonomy of contour based shape representation techniques (Luciano and Roberto, 2009)

2.2.1 Shape representation by contour based method

Shape representation generally consists of finding effective and perceptually important shape features based on either shape boundary information or boundary plus interior content (Zheng and Doerman, 2006). Shape representation and description techniques can be generally classified into three classes of methods (Luciano and Roberto, 2009). The focus in this research is on contour based representation. Figure 2.2 shows fundamental contour points; it comprises set of consecutive points forming the general frame of the shape. It is very important to mention that all the contour tracing techniques used to obtain contour points are for the same results.

This discussion focuses only on contour based methods, which is the focus of this research. According to Luciano and Roberto (2009), contour shape methods only exploit shape boundary information.

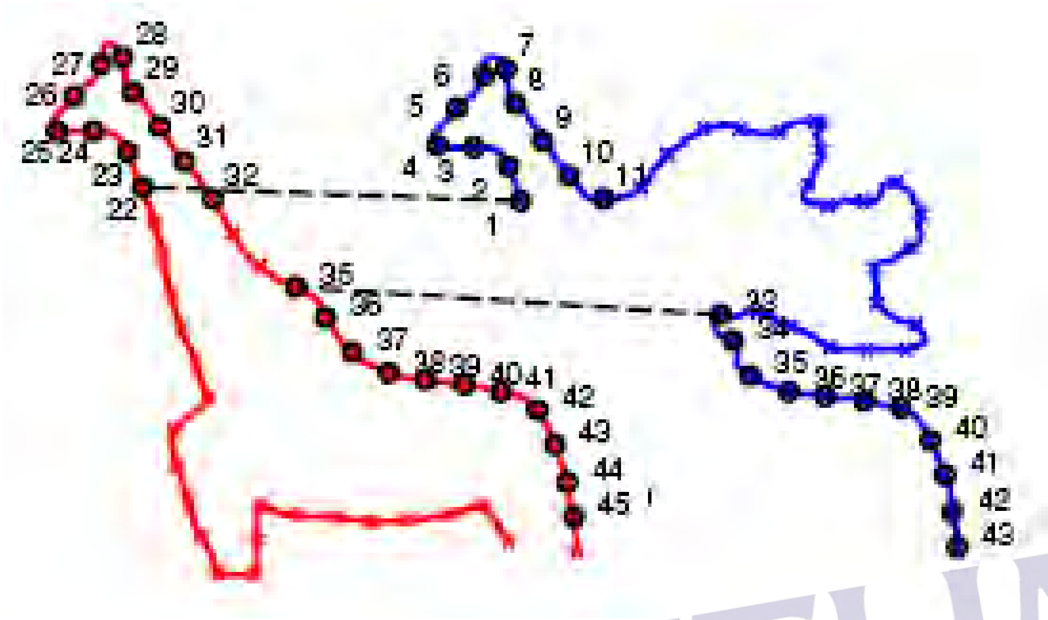

Figure 2.2: Contour based representation (Kaick *et al.*, 2007)

(1) Parametric Contour

Representing shapes directly by binary images implies some drawbacks. For example, it demands large storage space and does not explicitly identify the shape elements. Hence, an alternative approach has been introduced based on parametric representation. This alternative approach involves representing the contour as a parametric curve thus implying a sequential order along it as shown in Figure 2.3.

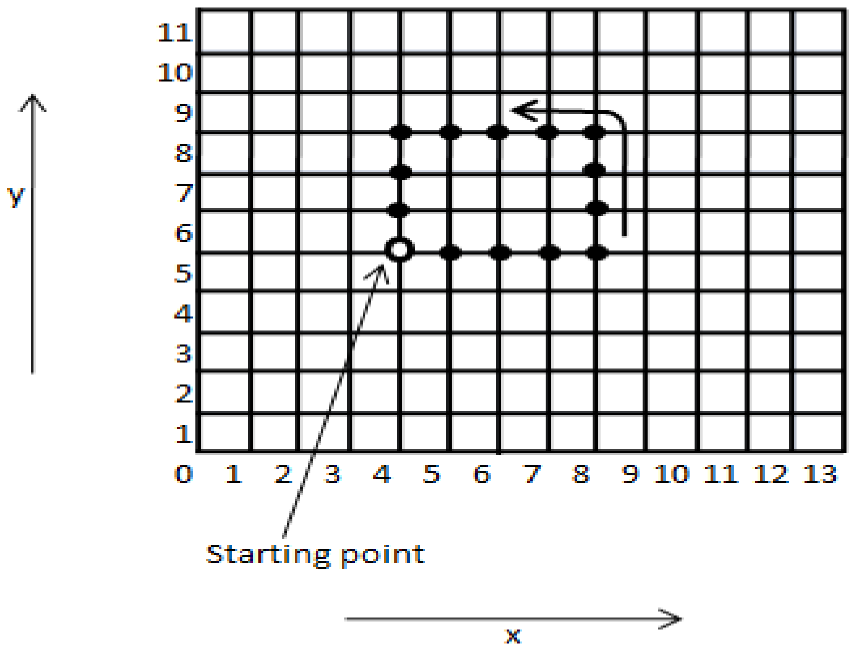

Figure 2.3: Parametric contour representation on a discrete grid.

(2) Set of Contour Points

Several approaches in representing the shape as a parametric contour are based on tracking the shape boundary in a given order. Assuming that $g(p,q)$ is a binary image where $g(p,q)=1$ for shape pixels and $g(p,q)=0$ for background pixels creates a set of contour points stored in the variable cs, which is of the above defined contour set type.

Square tracing algorithm (see Figure 2.4) is adopted to trace the contour. Given a digital pattern and a group of black pixels on a background of white pixels (a grid), a black pixel is located and declared as a "start" pixel. (Locating a "start" pixel can be done in a number of ways. For instance, at the bottom left corner of the grid, each column of pixels is scanned from the bottom going upwards starting from the leftmost column and proceeded to the right until a black pixel encountered. Then, that pixel is declared as a "start" pixel).

In order to extract the contour of the pattern, every time a black pixel is found, the direction is turned to the left, and every time a white pixel is found, the direction is turned to the right, until the start pixel is encountered again. The black pixels found during the loop constitute the contour of the pattern.

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