FUZZY CONTROLLER DESIGN FOR POSITION CONTROL

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The position control system one of the interesting term in control system engineering. Nowadays, several control system algorithm have been applied in that application. PID controller is a well known controller and widely used in feedback control in industrial processes. In position control system, PID controller sometimes cannot accurate for this application because of nonlinear properties. Therefore in this research, the fuzzy logic controller is proposed to overcome the problem of PID controller. Fuzzy logic controller has ability to control the nonlinear system because of the algorithm is concentrate by emulating the expert and implemented in language. Based on the experimental result, the fuzzy logic controller designed able to improve the performance of the position control system compared to the PID controller in term of rise time ($T_r$) is 50%, settling time ($T_s$) is 80% and percentage overshoot (%OS) is 98% can be reduced.
Sistem kawalan kedudukan salah satu istilah yang menarik dalam bidang kejuruteraan sistem kawalan. Kini, beberapa sistem algoritma kawalan telah diguna pakai dalam aplikasi itu. Pengawal PID telah digunakan secara meluasnya dalam kawalan maklum balas dalam proses perindustrian. Dalam sistem kawalan kedudukan, pengawal PID kebiasaanya tidak tepat kerana sifat tidak lurus. Oleh itu, dalam kajian ini pengawal logik kabur dicadangkan untuk mengatasi masalah pengawal PID. Pengawal logik kabur mempunyai keupayaan untuk mengawal sistem tidak lurus kerana algoritma menumpukan perhatian dengan mencontohi kepakaran dan dilaksanakan dalam bahasa. Berdasarkan hasil uji kaji, pengawal logik kabur direka mampu untuk memperbaiki prestasi sistem kawalan kedudukan berbanding dengan pengawal PID dalam 50% untuk jangka masa naik (Tr), 80% untuk menyelesaikan masa (Ts) dan 98% untuk terlajak peratusan (% OS) dapat dikurangkan.
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<td>Proportional Integral Derivative Controller</td>
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<tr>
<td>R_a</td>
<td>Armature resistance</td>
<td>Ω</td>
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<tr>
<td>L_a</td>
<td>Armature inductance</td>
<td>mH</td>
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<tr>
<td>E_a(t)</td>
<td>Supply voltage</td>
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<td>$T_r$</td>
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<td>$T_s$</td>
<td>Settling time</td>
<td>s</td>
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<tr>
<td>$O_s%$</td>
<td>Percentage of overshoot</td>
<td>—</td>
</tr>
<tr>
<td>$e_{ss}$</td>
<td>Steady state error</td>
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CHAPTER I

INTRODUCTION

1.1 Introduction

Several control system algorithm have been applied in control system engineering. The one of the interesting term in that application is position control. Position control for servo motor exists in a great variety of automatic processes. However, the performance of servo motor is influenced by uncertainties such as nonlinear properties, mechanical parameter variation, external disturbance, unstructured uncertainty due to nonideal field orientation in the transient state and unmodelled dynamics.

From a practical point of view, complete information about uncertainties is difficult to acquire in advance. To deal with these uncertainties, much research has been carried out in recent years to apply various approaches in the position control [1]. When it occurs, the fuzzy control method begins to be used instead of the PID control method [2].
As a very good method, a fuzzy controller method can adopt expert knowledge into regulations of control system and the regulations can be used to determine the output value by logical inference. So, it is also does not need precision system model and has high robust ability. In recent years, fuzzy controllers have been widely developed and a variety of methods have been proposed to improve the performance of fuzzy controller [3]-[5].

1.2 Problem Statement

There are wide of control techniques that can be applied to meet the control objectives of the system and these depend on the factors of which the proposed design objective might on. There are factors such as behaviors in terms of nonlinearity system, time response and lastly engineering goals such as cost and reliability. The factors that motivate many researchers to use conventional control theory and techniques which are systems are nonlinear and may contain unknown parameters.

That unknown parameters may not be estimated accurately if reliable experimental data is absent and also the delays present in the process of system might complicate achieving high performance control. Fuzzy logic controller has been use in control system when the mathematical model of the interested process is vague of exhibits uncertainties. The advantages of use the fuzzy logic control that the developed controller can deal with increasingly complex system. It also can implement without precise knowledge of the model structure of dynamic system.
1.3 Objectives

The main objectives of the project are:

1) To design the fuzzy logic controller for position control to control the servo motor system.
2) To investigate the application of fuzzy logic controller in servo motor control.
3) To test the performances of fuzzy logic controller by experiment.

1.4 Project Scope

The scope of this project are:

1) Understanding of the servo motor control background, analysis the problem and investigated fuzzy logic controller theory which has been applied to the servo motor system.
2) Design and implementation of the control algorithm is carried by fuzzy logic controller.
3) Implement fuzzy logic controller on the servo motor control by using MATLAB simulation.
CHAPTER II

LITERATURE REVIEW

2.1 Technology Developments

From the previous research work, knowledge of how others people construct their project and how they specified particular application. These researches that are relevant to this project discussed next to demonstrate continuity from previous researches.

2.1.1 Design of Self-Tuning Fuzzy Controllers for Nonlinear Systems

Jain (2011) proposed about to design of self-tuning fuzzy controllers for nonlinear systems [6]. Nonlinearities present in the systems make their controller design a non-trivial task. The difficulty further increases in case of multi-input–multi-output (MIMO) systems with increased number of variables and interactions between them. In this project, fuzzy based intelligent control schemes are designed for control of nonlinear single-input–single-output (SISO) and MIMO systems.
The comparative study of the designed self-tuning fuzzy controller with a standard Takagi Sugeno fuzzy controller is discussed with application to a shell and tube heat exchanger (nonlinear SISO system) and a coupled two tank system (nonlinear MIMO system). Online tuning of the membership functions and control rules of fuzzy controller is carried out using simulated annealing to obtain improved performance by minimizing the error function.

2.1.2 An Improved Fuzzy System for Position Control of Permanent Magnet Linear Motor

Chen (2005) presented a fuzzy system with stable value is proposed to promote the traditional fuzzy controller and to be applied to position control of the permanent magnet linear motor [7]. By a partition of error and change of error, the proposed fuzzy system can be used in a traditional fuzzy controller and can be switched to preceding a stable output control smoothly.

For permanent magnet linear motor (PMLM) control system, the output of position control can be divided into a stable value and a changing one when the controlled system is close to stable state. A stable parameter value is acquired from the output of a traditional fuzzy controller when the controlled system comes into the stable state and the stable parameter value can be tuned by a designed integral part so as to alleviate the static error.

Simulation results show the improved fuzzy system has a higher performance than the traditional PID and the traditional fuzzy controller. The experiment results also verify that it has high precision in position control of permanent magnet linear motor (PMLM).
2.1.3 Adaptive Fuzzy Position Control for Electrical Servo drive Via Total-Sliding-Mode Technique

K.H. Su (2005) designs an adaptive fuzzy total-sliding-mode controller (AFTSMC) with a total switching surface is proposed to control the position of an electrical servo drive [8]. The proposed controller comprises a special fuzzy total-sliding-mode controller (FTSMC) and an adaptive tuner. The FTSMC acts as the main tracking controller, which is designed via a fuzzy system to mimic the merits of a total-sliding-mode controller (TSMC).

In addition, a translation width is embedded into fuzzy rules to reduce the chattering phenomena. The adaptive tuner, which is derived in the sense of Lyapunov stability theorem, is utilized to adjust the translation width on line for further assuring robust and optimal performance. In the AFTSMC, the fuzzy-control-rules base is compact and only one parameter needs to be adjusted.

An electrical servo drive with the proposed position control system possesses the advantages of a simple control framework, reduced chattering, stable performance and robustness to uncertainties. The effectiveness is demonstrated by simulating and experimental results, and its advantages are indicated in comparison with conventional TSMC and adaptive TSMC for a field oriented control induction motor (IM) drive.

2.1.4 Design of an Adaptive Interval Type-2 Fuzzy Logic Controller for the Position Control of a Servo System with an Intelligent Sensor

Type-2 fuzzy logic systems are proposed by Kayacan (1981), as an alternative solution in the literature when a system has a large amount of uncertainties and type-1 fuzzy systems come to the limits of their performances [9]. In this study, an adaptive type-2 fuzzy-neuro system is designed for the position control of a servo system with an intelligent sensor.
The sensor gives different resistance values with respect to the stretch of it, and it is supposed to be used in a robotic arm position measurement system. These kinds of sensors can be used in human assistance robots that have soft surfaces in order not to damage the humans.

However, these sensors have time varying gains and uncertainties that are not very easy to handle. Moreover, they generally have a hysteresis on their input-output relations. The simulation results show that the control algorithm developed gives better performances when compared to conventional type I fuzzy controllers on such a highly nonlinear uncertain system.

2.1.5 An Improved Method for Designing Fuzzy Controllers for Position Control Systems

This paper presented by Hwang (1977), with a simulation method for determining fuzzy rules in fuzzy control of track systems [10]. The method is based upon minimizing the mean squared error for the combination of the rules in the original position control system. The resulting key rules for adjusting the gain yield a fuzzy controller which gives a performance that is superior to traditional controllers.

It is shown that vagueness and uncertainty of this tracking system can be handled adequately by using a fuzzy controller to obtain a mathematical model for the plant and the controller. This model represents the a priori information about the system. But many real world systems do not have a precise or known structure and have highly complex and nonlinear characteristics.
2.2  Servo Motor

A servo motor is an automatic device that uses error sensing feedback to correct the performance of a mechanism. The term correctly applies only to the systems where the feedback or error correction signals help to control mechanical position or other parameters.

A common type of servo provides position control. Servos are commonly electrical or partially electronic in nature, using an electric motor as the primary means of creating mechanical force. Other types of servos use hydraulics, pneumatics, or magnetic principles. Usually, servos operate on the principle of negative feedback, where the control input is compared to the actual position of the mechanical system as measured by some sort of transducer at the output. Any difference between the actual and wanted values (error signal) is amplified and used to drive the system in the direction necessary to reduce or eliminate the error.

Today, servo motor are used in automatic machine tools, satellite tracking antennas, remote control airplanes, automatic navigation systems on boats and planes, and antiaircraft gun control systems.

2.3  Fuzzy Logic Controller

A fuzzy control is a controller that is intended to manage some vaguely known or vaguely described process. The controller can be used with the process in two modes:

i. Feedback mode when the fuzzy controller determination act as a control device.

ii. Feed forward mode where the controller can be used as a prediction device.
Figure 2.1 shows a fuzzy logic control system in a closed loop control. The plant output are denoted by \( c(t) \), the input are denoted by \( u(t) \), and the reference input to the fuzzy controller is denoted by \( r(t) \) [11].

![Fuzzy logic control system](image)

Figure 2.1: Fuzzy logic control system

The fuzzy controller has four main components:

i) **Fuzzification**

The first block of the fuzzy controller is fuzzification, which converts each piece of input data to degrees of membership by a lookup in one or several membership functions. The fuzzification block thus matches the input data with the conditions of the rules to determine how well the condition of each rule matches that particular input instance. There is a degree of membership for each linguistic term that applies to that input variable.
ii) Inference Mechanism

Inference Mechanism or Engine is the processing program in a fuzzy control system. It derives a conclusion from the facts and rules contained in the knowledge base using various human expert techniques.

iii) Rule-Base

A group of rules may use several variables both in the condition and the conclusion of the rules. They are based on a set of rules that a human expert would follow in diagnosing a problem. Rule-base also where the knowledge is stored.

iv) Defuzzification

Defuzzification is a process that maps a fuzzy set to a crisp set and has attracted far less attention than other processes involved in fuzzy systems and technologies. Four most common defuzzification methods:

• Max membership method
• Center of gravity method
• Weight average method
• Mean-max membership method
CHAPTER III

METHODOLOGY

3.1 Project Methodology

This project begins with gathering information regarding the technical issue in order to develop the position control system. The researches have been design and implement after gathering information from internet and journal. The flowchart that used to develop fuzzy controller design for position control describes in Figure 3.1.
Figure 3.1: The fuzzy controller design for position control project methodology
3.2 Servo Motor Modeling

Servo motor is used for position or speed control in closed loop control systems. It has implemented proportional integral, fuzzy logic and adaptive neuro fuzzy inference system respectively at the variable working situations to the simulation model which has prepared at the Matlab programmers for improvement the servo motor performance.

The equivalent circuit diagram of servo motor is presented in Figure 3.2. The armature is modelled as a circuit with resistance, $R_a$ connected in series with an inductance, $L_a$ and a voltage source, $V_b(t)$ representing the back emf in the armature when the rotor rotates.

![Figure 3.2: Servomotor system (a) Schematic diagram (b) Block diagram](image)
Kirchhoff’s Voltage Law is used to map the armature circuitry dynamic of the motor. Thus, assume the inductance $L_a$ can be ignored, which in the case for servo motor.

The supply voltage ($E_a(t)$) is the product of motor input power by the armature current ($I_a(t)$).

$$E_a(t) = I_a(t) R_a + V_b(t) \quad (3.1)$$

Since the current carrying armature is rotating in a magnetic field, its back electromotive force is proportional to speed. $V_b(t)$ is the velocity of the conductor normal to the magnetic field.

$$V_b(t) = K_B s \Theta_m(t) \quad (3.2)$$

The typical equivalent mechanical loading on a motor connected to the motor shaft including total moment of inertia, $J_m$ and total viscous friction, $B$. Assume that $T(t)$ is the torque developed by the motor.

$$T(t) = J_m s^2 \Theta_m(t) + B s \Theta_m(t) \quad (3.3)$$

The developed motor output torque for this servo motor can be given by,

$$T(t) = K_T I_a(t) \quad (3.4)$$

By using Laplace transforms on the equation above, become,

$$E_a(t) = R_a I_a(t) + V_b(t) \quad (3.5)$$
From equation (3.3) and equation (3.4), it clearly shown the relationship of the motor output torque given on equation (3.6).

\[ K_T I_a(t) = J_m s^2 \Theta_m(t) + B s \Theta_m(t) \] (3.6)

Substitute equation (3.5) into equation (3.6) is as equation (3.7).

\[ K_T \left( \frac{E_a(t) - V_b(t)}{R_a} \right) = J_m s^2 \Theta_m(t) + B s \Theta_m(t) \] (3.7)

By simplifying equation (3.7), the final transfer function can be obtained as equation (3.8).

\[ \frac{\Theta_m(t)}{E_a(t)} = \frac{K_T}{J_m R_a s^2 + (B R_a + K_T K_B) s} \] (3.8)

Where the parameters for servo motor are:

- \( K_T \) (N.m/A) = 0.121
- \( K_B \) [V/(rad/s)] = 0.121
- \( R_a \) (Ω) = 2.23
- \( J_m \) (kg.m²) = 0.00006286
- \( B \) [N.m/(rad/s)] = 0.0000708

Thus, when substitute these parameter values into equation (3.8), the transfer function of servo motor as below.

\[ \frac{\Theta_m(t)}{E_a(t)} = \frac{863.19}{s^2 + 105.58 s} \] (3.9)
3.3 PID Controller Design

A PID (proportional-integral-derivative) controller is one of the most commonly used controllers because it is simple and robust. This controller is extremely popular because they can usually provide good closed loop response characteristics, can be tuned using relatively simple rules and easy to construct using either analogue or digital components. Figure 3.3 below illustrates the block diagram of PID controller.

![Block diagram of PID controller](image)

Figure 3.3: Block diagram of the closed loop servo motor with PID controller

The PID controller can be defined as equation (3.10) by the following relationship between controller input $e(t)$ and the controller output $V(t)$ that is applied to the motor armature [16].

$$V(t) = K_p e(t) + K_i \int_0^t e(t) \, dt + K_d \frac{de(t)}{dt}$$  \hspace{1cm} (3.10)
By using the Laplace transform, the transfer function of PID controller as following in equation (3.11).

\[
\frac{V(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s
\]  

(3.11)

Assumed that the  
\[ K_p = K, \quad K_i = \frac{K}{t_i} \quad \text{and} \quad K_d = Kt_d \]

Then, the transfer function of PID controller is depend by the following equation (3.12).

\[
K(s) = \frac{Kt_d}{s} \left( s^2 + \frac{1}{t_d} s + \frac{K}{td t_i} \right)
\]

(3.12)

The values of \( K_p, K_i \) and \( K_d \) are obtained by using Ziegler Nichols tuning algorithm. This method gives automatic oscillation of the process to compute the proportional, integral and derivative gains. The PID controller has been simulated based on simulink.

\[ K_p = 15 \]
\[ K_i = 5 \]
\[ K_d = 0.5 \]
3.4 Fuzzy Logic Controller Design

A basic structure of fuzzy logic controller system block diagram for position control is clearly shown in Figure 3.4.

![Fuzzy Logic Controller System Block Diagram](image)

Figure 3.4: Structure of fuzzy logic controller system

A fuzzy logic controller input variables involves receiving the error signal and change of error. These variables evaluate the fuzzy control rules using the compositional rules of inference and the appropriately computed control action is determined by using the defuzzification [17]. The essential steps in designing the fuzzy logic controller of this system are illustrated in Figure 3.5.
Figure 3.5: Design procedure of the fuzzy logic controller
3.4.1 Fuzzification

Fuzzification is a process of producing a fuzzy input on the base of a crisp one. It involves the conversion of the input and output signals into a number of fuzzy represented values (fuzzy sets). Figure 3.6 below describes the input and output variables that are used in this system.

![Diagram of Fuzzy Logic Controller](image)

Figure 3.6: Membership function for input and output of fuzzy logic controller

(i) Input variables:
- Error(e)
  Quantized into 3, 5 and 7 membership function: Negative N(e), Negative Small NS(e), Negative Medium NM(e), Negative Big NB(e), Zero Z(e), Positive P(e), Positive Small PS(e), Positive Medium PM(e) and Positive Big PB(e).
• Rate(de)
  Quantized into 3, 5 and 7 membership function: Negative N(de), Negative Small NS(de), Negative Medium NM(de), Negative Big NB(de), Zero Z(de), Positive P(de), Positive Small PS(de), Positive Medium PM(de) and Positive Big PB(de).

(ii) Output variables:
• Output
  Quantized into 5, 7 and 9 membership function: Negative Small (NS), Negative Medium (NM), Negative Big (NB), Zero (Z), Positive Small (PS), Positive Medium (PM) and Positive Big (PB).

3.4.2 Rule Base

The basic function of the rule based is to represent the expert knowledge in a form of if-then rule structure. The fuzzy logic can be derived into combination of input (3 × 3, 5 × 5 and 7 × 7). The rule table of fuzzy logic controller as listed in Table 3.1, Table 3.2 and Table 3.3.

Table 3.1: Rule table of fuzzy logic controller (3 × 3)

<table>
<thead>
<tr>
<th>Rate (de)</th>
<th>P(de)</th>
<th>Z(de)</th>
<th>N(de)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error (e)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P(e)</td>
<td>PB</td>
<td>PS</td>
<td>Z</td>
</tr>
<tr>
<td>Z(e)</td>
<td>PS</td>
<td>Z</td>
<td>NS</td>
</tr>
<tr>
<td>N(e)</td>
<td>Z</td>
<td>NS</td>
<td>NB</td>
</tr>
</tbody>
</table>
The fuzzy logic control rules based on Table 3.1 are:

1) IF Error(e) is N(e) AND Rate(de) is P(de) THEN Output(u) is Z(u)
2) IF Error(e) is Z(e) AND Rate(de) is P(de) THEN Output(u) is PS(u)
3) IF Error(e) is P(e) AND Rate(de) is P(de) THEN Output(u) is PB(u)
4) IF Error(e) is N(e) AND Rate(de) is Z(de) THEN Output(u) is NS(u)
5) IF Error(e) is Z(e) AND Rate(de) is Z(de) THEN Output(u) is Z(u)
6) IF Error(e) is P(e) AND Rate(de) is Z(de) THEN Output(u) is PS(u)
7) IF Error(e) is N(e) AND Rate(de) is N(de) THEN Output(u) is NB(u)
8) IF Error(e) is Z(e) AND Rate(de) is N(de) THEN Output(u) is NS(u)
9) IF Error(e) is P(e) AND Rate(de) is N(de) THEN Output(u) is Z(u)

Table 3.2: Rule table of fuzzy logic controller (5 × 5)

<table>
<thead>
<tr>
<th>Error (e)</th>
<th>Rate (de)</th>
<th>PB(de)</th>
<th>PS(de)</th>
<th>Z(de)</th>
<th>NS(de)</th>
<th>NB(de)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PB(e)</td>
<td>Z</td>
<td>NS</td>
<td>NM</td>
<td>NM</td>
<td>NB</td>
<td></td>
</tr>
<tr>
<td>PS(e)</td>
<td>PS</td>
<td>Z</td>
<td>NS</td>
<td>NS</td>
<td>NM</td>
<td></td>
</tr>
<tr>
<td>Z(e)</td>
<td>PM</td>
<td>PS</td>
<td>Z</td>
<td>NS</td>
<td>NM</td>
<td></td>
</tr>
<tr>
<td>NS(e)</td>
<td>PM</td>
<td>PS</td>
<td>PS</td>
<td>Z</td>
<td>NS</td>
<td></td>
</tr>
<tr>
<td>NB(e)</td>
<td>PB</td>
<td>PM</td>
<td>PM</td>
<td>PS</td>
<td>Z</td>
<td></td>
</tr>
</tbody>
</table>

The fuzzy logic control rules based on Table 3.2 are:

1) IF Error(e) is NB(e) AND Rate(de) is PB(de) THEN Output(u) is PB(u)
2) IF Error(e) is NS(e) AND Rate(de) is PB(de) THEN Output(u) is PM(u)
3) IF Error(e) is Z(e) AND Rate(de) is PB(de) THEN Output(u) is PM(u)
4) IF Error(e) is PS(e) AND Rate(de) is PB(de) THEN Output(u) is PS(u)
5) IF Error(e) is PB(e) AND Rate(de) is PB(de) THEN Output(u) is Z(u)
6) IF Error(e) is NB(e) AND Rate(de) is PS(de) THEN Output(u) is PM(u)
7) IF Error(e) is NS(e) AND Rate(de) is PS(de) THEN Output(u) is PS(u)
8) IF Error(e) is Z(e) AND Rate(de) is PS(de) THEN Output(u) is PS(u)
9) IF Error(e) is PS(e) AND Rate(de) is PS(de) THEN Output(u) is Z(u)
10) IF Error(e) is PB(e) AND Rate(de) is PS(de) THEN Output(u) is NS(u)
11) IF Error(e) is NB(e) AND Rate(de) is Z(de) THEN Output(u) is PM(u)
12) IF Error(e) is NS(e) AND Rate(de) is Z(de) THEN Output(u) is PS(u)
13) IF Error(e) is Z(e) AND Rate(de) is Z(de) THEN Output(u) is Z(u)
14) IF Error(e) is PS(e) AND Rate(de) is Z(de) THEN Output(u) is NS(u)
15) IF Error(e) is PB(e) AND Rate(de) is Z(de) THEN Output(u) is NM(u)
16) IF Error(e) is NB(e) AND Rate(de) is NS(de) THEN Output(u) is PS(u)
17) IF Error(e) is NS(e) AND Rate(de) is NS(de) THEN Output(u) is Z(u)
18) IF Error(e) is Z(e) AND Rate(de) is NS(de) THEN Output(u) is NS(u)
19) IF Error(e) is PS(e) AND Rate(de) is NS(de) THEN Output(u) is NS(u)
20) IF Error(e) is PB(e) AND Rate(de) is NS(de) THEN Output(u) is NM(u)
21) IF Error(e) is NB(e) AND Rate(de) is NB(de) THEN Output(u) is Z(u)
22) IF Error(e) is NS(e) AND Rate(de) is NB(de) THEN Output(u) is NS(u)
23) IF Error(e) is Z(e) AND Rate(de) is NB(de) THEN Output(u) is NM(u)
24) IF Error(e) is PS(e) AND Rate(de) is NB(de) THEN Output(u) is NM(u)
25) IF Error(e) is PB(e) AND Rate(de) is NB(de) THEN Output(u) is NB(u)
The fuzzy logic control rules based on Table 3.3 are:

1) IF Error(e) is NB(e) AND Rate(de) is PB(de) THEN Output(u) is NB(u)
2) IF Error(e) is NM(e) AND Rate(de) is PB(de) THEN Output(u) is NB(u)
3) IF Error(e) is NS(e) AND Rate(de) is PB(de) THEN Output(u) is PS(u)
4) IF Error(e) is Z(e) AND Rate(de) is PB(de) THEN Output(u) is PS(u)
5) IF Error(e) is PS(e) AND Rate(de) is PB(de) THEN Output(u) is PM(u)
6) IF Error(e) is PM(e) AND Rate(de) is PB(de) THEN Output(u) is PM(u)
7) IF Error(e) is PB(e) AND Rate(de) is PB(de) THEN Output(u) is PB(u)
8) IF Error(e) is NB(e) AND Rate(de) is PM(de) THEN Output(u) is NB(u)
9) IF Error(e) is NM(e) AND Rate(de) is PM(de) THEN Output(u) is NB(u)
10) IF Error(e) is NS(e) AND Rate(de) is PM(de) THEN Output(u) is NS(u)
11) IF Error(e) is Z(e) AND Rate(de) is PM(de) THEN Output(u) is Z(u)
12) IF Error(e) is PS(e) AND Rate(de) is PM(de) THEN Output(u) is PM(u)
13) IF Error(e) is PM(e) AND Rate(de) is PM(de) THEN Output(u) is PM(u)

Table 3.3: Rule table of fuzzy logic controller (7 × 7)

<table>
<thead>
<tr>
<th>Error (e)</th>
<th>Rate (de)</th>
<th>PB(de)</th>
<th>PM(de)</th>
<th>PS(de)</th>
<th>Z(de)</th>
<th>NS(de)</th>
<th>NM(de)</th>
<th>NB(de)</th>
</tr>
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<tbody>
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<td>PB(e)</td>
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<td>PB</td>
<td>PB</td>
<td>PB</td>
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<td>PM</td>
<td>PM</td>
<td>PS</td>
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<td>PS</td>
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<td>PS(e)</td>
<td>PM</td>
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<td>Z</td>
<td>Z</td>
<td>Z</td>
<td>NS</td>
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REFERENCES


