SHEAR STRENGTH ANALYSIS OF CONCRETE BEAMS REINFORCED WITH GFRP BARS USING STRUT AND TIE MODEL

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ABSTRACT

This dissertation presents an experimental investigation on the behavior and ultimate shear strength of reinforced concrete beam. Sixteen reinforced concrete beams was design and tested to failure. This study consists of two series of beams, which are conventional steel reinforced beams (BSN) and reinforced concrete beams with Strut and Tie Model (STM) using StaadPro software and both result were compared in term of shear strength. The main test variables were shear span-to-depth ratio (2.1 and 2.9), percent of longitudinal reinforcement ratio (tension) steel and GFRP (0.6% and 0.9%), and shear reinforcement ratio (1.5% and 0.6%). The test results revealed that the mode of failure for all beam is flexural with shear reinforcement characteristics and longitudinal reinforcement ratio play a critical role in controlling the mode of failure. The experimental approved that the spacing between shear cracks for the specimens with larger shear span to depth ratio is greater than the smaller shear span to depth ratio and while the shear span to depth ratio (a/d) decreases, the shear strength increase. For longitudinal reinforcement ratio it can be inferred that the higher longitudinal reinforcement ratio brings the smaller diagonal crack. Also, greater stirrup spacing leads to the greater diagonal crack, confirming that there is a significant influence of the stirrup spacing on the spacing between shear cracks. The reason for this behavior is the decreasing effective concrete area, in which shear crack width is controlled by the stirrup, and hence the increasing bond effect between the stirrup and the surrounding concrete.

ABSTRAK

Disertasi ini mempersembahkan suatu kajian yang berkaitan sifat-sifat dan kekuatan ricih rasuk konkrit bertulang. Enam belas unit konkrit bertulang telah direkabentuk dan diuji hingga gagal. Kajian ini terdiri daripada dua siri kaedah rekabentuk rasuk iaitu rasuk yang direka menngunakan kaedah konvensional dan rasuk yang direka menggunakan perisian komputer dan keputusan kedua-duanya dibandingkan dari aspek kekuatan ricih. Pembolehubah utama yang dianalisis adalah nisbah a/d iaitu(2.1 dan 2.9), peratus dari nisbah tetulang memanjang (tegangan) (0.6% dan 0.9%), dan nisbah tetulang ricih (1.5% dan 0,6%). Keputusan ujian menunjukkan bahawa jenis kegagalan untuk semua rasuk adalah '*flexural*' dengan nisbah a/d dan nisbah tetulang memanjang berperanan sebagai faktor kritikal kepada penentuan jenis kegagalan. Kajian telah menunjukkan bahawa nilai a/d mempengaruhi kepada perbezaan jarak keretakan. In terbukti apabila nilai a/d besar maka jarak keretakan juga besar berbanding nilai a/d yang kecil dan apabila a/d mempunyai nilai yang kecil nilai kekuatan ricih akan meningkat. Manakala nilai nisbah tetulang memanjang yang besar menyumbang kepadda jarak keretakan yang kecil.. Selain itu, jarak tetulang ricih yang besar akan menyebabkan keretakan yang besar. Hal ini adalah kerana penurunan keluasan efektif konkrit yang mana kelebaran keretakan ricih adalah di kawal oleh tetulang ricih.

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LIST OF ABBREVIATIONS

ACI	-	American Concrete Institution
BS	-	British Standard
CSA	-	Canadian Standard Association
FRP	-	Fibre-reinforced polymer
GFRP	-	Glass Fibre-reinforced polymer
STM	-	Strut and Tie Model

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CHAPTER I

INTRODUCTION

1.1   Background of study

Reinforced concrete is concrete in which reinforcement bars, reinforcement grids, plates or fibers have been incorporated to strengthen the concrete in tension. Ferro Concrete is the common term that we usually known which refer only to concrete that is reinforced with iron or steel. But there are other materials can be used to reinforce concrete that can be organic and inorganic fibres as well as composites in different forms such as Glass Fiber-reinforced polymer (GFRP).

The use of concrete structures reinforced with Glass fiber-reinforced polymer (GFRP) composite materials has been growing to overcome the common problems caused by corrosion of steel reinforcement. The climatic conditions where large amounts of salts are used for ice removal during winter months may contribute to accelerating the corrosion process. These conditions normally accelerate the need for costly repairs and may lead to catastrophic failure. Therefore, replacing the steel reinforcement with the noncorrosive FRP reinforcement eliminates the potential of corrosion and the associated deterioration. The direct replacement of steel with Glass Fiber-reinforced polymer (GFRP) bars, however, is not possible due to various

differences in the mechanical properties of the Glass Fiber-reinforced polymer (GFRP) materials compared to steel, especially the higher tensile strength, the lower modulus of elasticity, bond characteristics, and the absence of a yielding plateau in their characteristic stress-strain relationships.

Reinforced concrete can encompass many types of structures and components, including slabs, walls, beams, columns, foundations, frames and more. In constructions most cases reinforced concrete uses steel rebars that have been inserted to add strength, therefore in this study the steel bars will be replace with glass fibre-reinforced polymer and will focus on beam structure. Sixteen unit of beam structure will be cast with certain dimension by using steel rebar and Glass Fibre-reinforced Polymer bars as reinforced complete with stirrup to study the shear behavior and finally the shear strength will be analyze and compare by using the strut and tie model.

The strut and tie models have been widely used as effective tools for designing reinforced concrete structures. The idea of a Strut-and-Tie Model came from the truss analogy method introduced independently by Ritter and Morsch in the early 1900s for shear design. This method employs also called Truss Models as its design basis. The model was used to idealize the flow of forced in a cracked concrete beam. In parallel with the increasing availability of the experimental results and the development of limit analysis in the plasticity theory, the truss analogy method has been validated and improved considerably in the form of full member or sectional design procedures.

Strut-and-Tie modeling is an analysis and design tool for reinforced concrete elements in which it may be assumed that flexural and shearing stresses are transferred internally in a truss type member comprised of concrete compressive struts and steel reinforcing tension ties. The strut and tie is always worthwhile because it can often reveal weak points in a structure which otherwise could remain hidden to the design engineer if he approaches them by standard procedures. As

reinforced concrete beams have become an important structural element, their behavior and ultimate shear strength has been the subject of many researchers devoted to determine the influence of effective parameters. Several different modes of failure can predict well from the experimental studies, due to the variability in the failure, the determination of their shear capacity and identification of failure mechanisms are very complicated. The existing methods for analysis and design of deep beams consist of rational and semi rational approaches as sectional approach or strut and tie Model (STM).

Beginning in 2002, the ACI building code stated that beams should be designed using the strut and tie model. The strut and tie provisions in ACI 318 were developed for the design of all forms of discontinuity regions. The proposed compatibility based strut and tie method, which considers the effects of compression softening, is shown to provide accurate estimates of the measured load carrying capacities of reinforced concrete beams. The strut and tie model compressive force are carried by a compressive field or concrete struts and tensile force by main longitudinal reinforcement, the concrete compressive softening effect was usually applied to diagonal struts. Strut and tie models was laid by Ritter (1899). The strut and tie method is gaining rapid popularity for beams which some approaches applicable in D-regions. These approaches help design a complex structure maximally safe. Most recently has included strut and tie method approach in 2008 edition of Building Code Requirements For Structural Concrete (ACI 318).

1.2   Objectives of Study

The objectives of this study are:-

i.   To study shear behavior of reinforced concrete beams with steel and Glass Fibre-reinforced Polymer bars with stirrups.

ii.   To investigate the shear strength of reinforced concrete beam with shear reinforcement and reinforced concrete beam using the Strut and Tie Model (STM).

1.3   Problem Statement

There are many parameters affecting on the shear strength of reinforced concrete beams, where the most important of them consist of concrete compressive strength, shear span-depth ratio and the amount and arrangement of vertical and web reinforcements. Therefore the some analysis are needed to identify and encounter this problem.

1.4   **SCOPE OF STUDY**

The scope of this research will cover on:

i.   The application of reinforced concrete beam using Strut and Tie Model and conventional beams which Glass Fibre-reinforced polymer as shear reinforcement.

ii.   Each specimen will be constructed with identical amount and arrangement of main reinforcement and shear reinforcement and the specimens were designed and constructed in accordance to ACI 318.

iii.   The behavior of the specimens will be compared in term of shear capacity and shear strength.

1.4 Research Significance

The strut and tie method is today considered by researchers and practitioners to be the rational and appropriate basis for the design of cracked reinforced concrete beam loaded in bending, shear and torsion. For design of structural concrete, the truss analogy in order to apply it in the form of strut and tie model to every part of any structure. This propose is justified by the fact that reinforced concrete structures carry load through a set of compressive stress fields which are distributed and interconnect by tensile ties. The ties may be reinforcing bars, prestressing tendons, or concrete tensile stress fields.

For analytical purposes, the strut and tie models condense all stresses in compression and tension members and join them by nodes. Strut and tie models could lead to a clearer understanding of the behavior of structural concrete, and codes based on such an approach would lead to improved structures.

CHAPTER II

LITERATURE REVIEW

2.1 Glass Fiber-reinforced polymer (GFRP)

Extensive research in recent years has been undertaken to investigate the performance of Glass Fiber-reinforced polymer (GFRP) as primary reinforcement for concrete members. Glass Fiber-reinforced polymer (GFRP) bars are currently available as a substitute for conventional steel bars in concrete structures exposed to de-icing salts and marine environments. In addition to superior durability, Glass Fiber-reinforced polymer (GFRP) reinforcing bars have a high strength-to-weight ratio, which makes them attractive as reinforcement for concrete structures. However, the material properties of Glass Fiber-reinforced polymer (GFRP) differ significantly from those of steel reinforcement, especially the modulus of elasticity. The modulus of elasticity is 20 to 25 % that of steel compared to 60 to 75 % for carbon Fiber-reinforced polymer (FRP) bars.

Due to the relatively low modulus of elasticity of Glass Fiber-reinforced polymer (GFRP) bars, concrete members reinforced longitudinally with Glass Fiber-reinforced polymer (GFRP) bars experience reduced shear strength compared to the shear strength of those reinforced with the same amounts of steel reinforcement. This fact is supported by the findings from the experimental investigations on concrete beams without stirrups and reinforced longitudinally with carbon and Glass Fiber-reinforced polymer (GFRP)

bars (El-Sayed, et al., 2004, 2005b). The investigation also revealed that the axial stiffness of the reinforcing bars is a key parameter in evaluating the concrete shear strength of flexural members reinforced with Glass Fiber-reinforced polymer (GFRP) bars.

The current ACI 440.1R-03 guide has proposed a design approach for calculating the concrete shear strength of Glass Fiber-reinforced polymer (GFRP)-reinforced concrete beams accounting for the axial stiffness of Glass Fiber-reinforced polymer (GFRP) reinforcing bars. Recent research has indicated that the ACI 440 shear design method provides very conservative predictions, particularly for beams reinforced with Glass Fiber-reinforced polymer (GFRP) bars (El-Sayed et al., 2004, 2005a, b, c; Razaqpur et al., 2004; Gross et al., 2004; Tureyen and Frosch, 2002). Furthermm, the research has indicated that the level of conservatism of the shear strength predicted by ACI 440 method is neither consistent nor proportioned to the axial stiffness of Glass Fiber-reinforced polymer (GFRP) reinforcing bars (El-Sayed, et al., 2005a).

Some Extensive research programs have been conducted to investigate the flexural behavior of concrete members reinforced with Glass Fiber-reinforced polymer (GFRP) reinforcement (Benmokrane et al., 1996; El-Salakawy et al., 2004; Gravina et al., 2008). The shear behavior of Glass Fiber-reinforced polymer (GFRP) reinforced concrete (RC) beams without shear reinforcement has also been studied (El-Sayed et al., 2006). Due to the unidirectional characteristics of Glass Fiber-reinforced polymer (GFRP) materials, bending of Glass Fiber-reinforced polymer (GFRP) bars into stirrup configuration significantly reduces the strength at the bend portions (Maruyama et al., 1993; Shehata et al., 2000; El-Sayed et al., 2007). The reduced strength of the Glass Fiber-reinforced polymer (GFRP) stirrup at the bend is attributed to local stress concentration at the bend due to curvature and the intrinsic weakness of fibers perpendicular to their axis. The bend capacity of Glass Fiber-reinforced polymer (GFRP) bars is influenced by the bending process, the ratio of bend radius to bar diameter (rb/db), and type of reinforcing fibers (ACI Committee 440. 2006). The recent editions of the ACI 440.1R-069 guidelines and the CAN/CSA S6-06 (Canadian Standard Association (CSA),

2006,2009) code, along with the commercially available Glass Fiber-reinforced polymer (GFRP) bent bars, encouraged the use of Glass Fiber-reinforced polymer (GFRP) stirrups.

Through a collaboration project between the University of Sherbrooke, the Ministry of Transportation of Quebec (MTQ), and an FRP manufacturer, new FRP (carbon and glass) stirrups have been recently developed and characterized according to B.5 and B.12 test methods of ACI 440.3R-04 (ACI Committee 440., 2004). The behavior of these stirrups in large-scale beam specimens, however, had not been investigated. To achieve this, an experimental program was conducted to investigate the shear performance of FRP stirrups in large-scale beam specimens. The first phase evaluated the structural performance of carbon FRP (CFRP) stirrups in beam specimens. There is a recent increase in demand for glass FRP (GFRP) bars because of its many successful applications, including bridge deck slabs,(Benmokrane et al., 2006,2007) barrier walls, (El-Salakawy et al., 2003; El-Gamal et al.,2008) parking garages (Benmokrane et al.,2006), continuous pavement (Benmokrane et al.,2008), and other concrete structures. Furthermore, considering the lower costs of GFRP bars in comparison to CFRP and aramid FRP (AFRP), GFRP reinforcement is becoming more attractive for the construction industry.

2.2   Strut and Tie Model Basis

When Ritter and Morsch introduced the truss analogy. This method was later refined and expanded by Leonhardt,Rusch, Kupfer and others until Thurlimann's Zurich school, with Marti and Mueller, created its scientific basis for a rational application in tracing the concept back to the theory of plasticity. Collins and Mitchell further considered the deformations of the truss model and derived a rational design method for shear and torsion. In various applications, Bay, Franz, Leonhardt and Thurlimann had

shown that strut and tie models could be usefully applied to deep beams and corbels. From that point, the present authors began their efforts to systematically expand such models to entire structures and all structures. The approaches of the various authors cited above differ in the treatment of the prediction of ultimate load and the satisfaction of serviceability requirement. Form a practical viewpoint, true simplicity can only be achieved if solutions are accepted with sufficient accuracy. Therefore, it is proposed here to treat in general the ultimate limit state and serviceability in the cracked state by using one and the same model for both.

ACI 318 for the design of R.C beams is Strut and Tie Model. STM comprise compression struts and tension ties that transfer the forces through the member, through the joints referred to as nodes, and to the supports; which transfers the force through shear reinforcement and an internal moment couple with flexural reinforcement. Both design processes have benefits and should be considered when designing beams.

Before cracking has occurred in a reinforced concrete beam, an elastic stress field exists. Cracking disturbs the stress field causing the internal forces to alter their path. These reoriented forces can be modeled as an STM (MacGregor, et al., 2005). The STM analysis evaluates stresses as either compression (struts) or tension members (steel ties) and joins the struts and ties through nodes and nodal regions (Schlaich,et al., 1987). After inclined cracks have formed in deep beams, the beam takes on a "tied arch" behavior allowing the forces to transfer directly to the supports, not vertically through the member until being transferred by the web and flexural reinforcement. This behavior provides some reserve shear capacity in beams and generally fails shortly after inclined cracks form unless flexural reinforcement is provided (Rogowsky, et al., 1983). Figure 2.1 represents a beam with a point load applied on the compression face.

Figure 2.1: Represents a beam with a point load applied on the compression face.

In testing, the stresses in the tension chord reinforcement decreased much less at the ends of the girder, indicating that the steel acts as a tension tie that carries a relatively constant force from one end of the girder to the other, thus confirming the methodology of the STM (Rogowsky, et al.,1983). The STM was developed as a practical way to design for discontinuity regions where non-linear, elastic behavior occurs (commonly referred to as D-Regions). ACI 318 allows the use of STM for the design of R.C beams.

The Figure 2.2 shows a typical R.C beam and its Strut and Tie Model, this beam is loaded on top face by two vertical point loads and supported at the opposite face. The longitudinal main reinforcements are located at *a* distance *d* from top. This beam is not detailed with any web reinforcement. Assuming that the flexure strength is sufficient, the failure of beam is governed by the compressive stress at the strut and its diagonal crushing. The shear strength is predicted by STM due to the diagonal struts and shear force flows along the strut from loaded point to the support. The equilibrium of the applied forces leads to the following expressions.

$$T_s = C_c \cos\theta$$
$$V_c = C_c \sin\theta$$

Figure 2.2: Geometry of concrete beam

Where Cc is the compression force in the diagonal strut, is the angle between strut and longitudinal reinforcements, is the tension force on longitudinal reinforcements (or ties) and is the applied load on top of the deep beam. The inclined angle of the diagonal strut is given by

$$\theta = \tan^{-1}(\frac{jd}{a})$$

where: *a* is the shear span measured center-to center from load to support and is the distance of lever arm from the resultant compressive force to the center of the main tensile longitudinal reinforcements. Using the assumption of Hwang et al, this term can be estimated as

$$jd = d - \frac{kd}{3} = (1 - \frac{k}{3})d$$

Where

Kd is the depth of the compression zone or horizontal prismatic strut.

Figure 2.3: Equilibrium of strut in absence of web reinforcement

2.2.1 Element of Strut and Tie Models

2.2.1.1 *Struts*

Most research and design specifications specify the limiting compressive stress of a strut as the product of the concrete compressive strength, *f'c,* and a reduction factor. The reduction factor is often a function of the geometric shape (or type) of the strut. The shape of a strut is highly dependent upon the force path from which the strut arises and the reinforcement details of any reinforcement connected to the tie. As discussed by Schlaich and Schäfer, there are three major geometric shape classes for struts: prismatic, bottle-shaped, and compression fan (1991).

I. Prismatic struts are the most basic type of strut. Prismatic struts have uniform cross-sections. Typically, prismatic struts are used to model the compressive stress block of a beam element as shown in Fig. 2.4(a).

II. Bottle-shaped struts are formed when the geometric conditions at the end of the struts are well-defined, but the rest of the strut is not confined to a specific portion of the structural element. The geometric conditions at the ends of bottle-shaped

struts are typically determined by the details of bearing pads and/or the reinforcement details of any adjoined steel. The best way to visualize a bottle-shaped strut is to imagine forces dispersing as they move away from the ends of the strut as shown in Fig. 2.4 (b). The bulging stress trajectories cause transverse tensile stresses to form in the strut which can lead to longitudinal cracking of the strut. Appropriate crack control reinforcement should always be placed across bottle-shaped struts to avoid premature failure. For this reason, most design specifications require minimum amounts of crack control reinforcement in regions designed with STMs.

III. The last major type of strut is the compression fan. Compression fans are formed when stresses flow from a large area to a much smaller area. Compression fans are assumed to have negligible curvature and, therefore, do not develop transverse tensile stresses. The simplest example of a compression fan is a strut that carries a uniformly distributed load to a support reaction in a deep beam as shown in Fig. 2.4 (c).

Once the general location of the nodes has been determined, the effective compressive strength of the concrete for both the struts and the nodal regions is determined. According to ACI 318-08, Equation A-2 given here as Equation 5.2, the nominal compressive strength of a strut without longitudinal reinforcement, Fns, shall be taken as the smaller value at the two ends of the strut.

$$F_{ns} = f_{ce}A_{cs} \quad \text{(EQ'N 5.2)}$$

where:

A_{cs} = cross sectional area of one end of the strut;

f_{ce} = effective compressive strength.

The effective compressive strength of the strut shall be taken as the smaller of the effective compressive strength of the concrete in the strut or the concrete in the nodal zone according to ACI 318. The compressive strength of the concrete in the strut is

determined using ACI 318, and the strength in the nodal zone is determined using both equation which given respectively below.

$$f_{ce} = 0.85\beta_s f_c' \text{ (EQ'N 5.3)}$$
$$f_{ce} = 0.85\beta_n f_c' \text{ (EQ'N 5.4)}$$

Where:

β_s = factor to account for the effect of cracking and confining reinforcement on the effective compression strength of the concrete in a strut;

β_n = factor to account for the effect of the anchorage of ties on the effective compressive strength of a nodal zone.

The area of steel is multiplied by the angle of the strut to vertical and horizontal reinforcement to get the perpendicular steel area crossing through the strut axis which is divided by the area of concrete to achieve the steel ratio.

Figure 2.4: Geometric shapes of struts

(a) Struts in a beam web with inclined cracks parallel to struts - Section A.3.2.4

(b) Struts crossed by skew cracks - Section A.3.2.4

Figure 2.5: Types of Struts; courtesy of (Committee 318, 2008)

2.1.1.2 *Ties*

As previously stated, ties are STM members that are subjected to tensile forces. Although, concrete is known to have tensile capacity, its contribution to the tie resistance is normally neglected for strength considerations. Therefore, only reinforcing or prestressing steel are used to satisfy the calculated tie requirements. Because only reinforcing or prestressing steel are attributed to the ties resistance, the geometry and the capacity of the tie are much easier to determine.

Ties consist of reinforcement in the tension regions of the element being designed as well as in the surrounding concrete. The concrete does not contribute to the resistance of forces but does increase the axial stiffness of the tie through tension stiffening. The nominal strength of the tie is determined using ACI 318-08 Equation A-6, given as

$$F_{nt} = A_{ts} f_y + A_{tp} (f_{se} + \Delta f_p) \quad \dots\dots\dots\dots\dots\dots\dots Eq.1$$

Where

$$(f_{se} + \Delta f_p) \leq f_{py} \text{ and Atp is 0 for nonprestressed members.}$$

According to ACI 318, Section A.4.2, the axis of the reinforcement in a tie shall coincide with the axis of the tie, and the effective tie width, wt, is limited depending on the reinforcement geometry and distribution. If the bars are in one layer, wt can be taken as the diameter of the bar plus twice the cover, which is the lower limit of wt. The upper limit is determined in accordance with equation given below:

$$w_{t.\max} = \frac{F_{nt}}{f_{cu}b}$$

2.1.1.3 *Nodes*

The nodes are idealized pinned joints where the forces meet from the struts and ties. The nodal zone is the surrounding body of concrete that transfers the load from the struts to the ties or supports. Because these joints are idealized as pinned joints, they must be at static equilibrium. This implies that the forces must pass through a common point, or the forces can be resolved around a certain point to remain in equilibrium. At nodal regions, at least three forces must keep the node at equilibrium because the forces come into the node at different angles. These nodal regions are classified as C-C-C for three compressive forces, C-C-T for two compressive forces and one tensile force, C-T-T for one compressive force and two tensile forces, or T-T-T for three tensile forces (MacGregor, et al.,2005). Figure 2.6 represents the four nodal regions in static equilibrium specified.

(a) C-C-C Node   (b) C-C-T Node

(c) C-T-T Node   (d) T-T-T Node

Figure 2.6: Classifications of Nodes; Mitchell et al. (2004)

2.2.2 D-region and B-region of Strut

Strut and tie models are an approach used to design discontinuity region (D-regions) in reinforced and prestressed concrete structures. A strut and tie reduces complex states of stress within a D-region of a reinforced or prestressed concrete member into a truss comprised of simple, uniaxial stress paths. Each uniaxial stress path is considered member of the strut and tie models.

The B-region design is still being disputed, it is only reasonable to expect that the more complex D-region design will need to be simplified with some loss of accuracy. In using the strut and tie model approach, it is helpful and informative to first subdivide the structures into its B and D-regions. The truss models and the design procedures for the B-regions are then readily available and only the strut and tie model for the D-regions remain to be developed and added.

For the majority of structures it would be unreasonable and too cumbersome to begin immediately to model the entire structure with struts and ties. Rather, it is more convenient to first carry out a general structural analysis; it is advantageous to subdivide

the given structure into its B and D-regions. The overall analysis will then include not only the B-regions but also the D-regions. If structure contains to a substantial part B-regions, it is represented by its statical system. The general analysis of linear structures (beams, frames and arches) results in the support reactions and sectional effects, the bending moments (M), normal forces (N), shear forces (V), and torsion moments (Mt).

A structure consists of one D-region, the analysis of sectional effects by a statically system may be omitted and the inner forces or stresses can be determined directly from the applied loads. However, for structures with redundant supports, the support reactions have to be determined by an overall analysis before strut and tie models can be properly developed.

Figure 2.7: Position for B-Region and D-Region of strut

2.2.3 Procedure for Strut and Tie Modeling

The process used in the development of a STM model is illustrated in Figure 2.8.

Figure 2.8: Flowchart illustrating STM steps. (Brown et al. 2006)

2.2.4   Shear strength of strut

The current American Concrete Institute code states that the nominal shear strength V_n of a reinforced concrete beam consists of the concrete contribution V_c and shear reinforcement contribution V_s, such as $V_n=V_c+V_s$. The method is based on strut-and-tie approach, with the effect of transverse tensile stresses on concrete compressive strength of the diagonal strut properly accounted for.

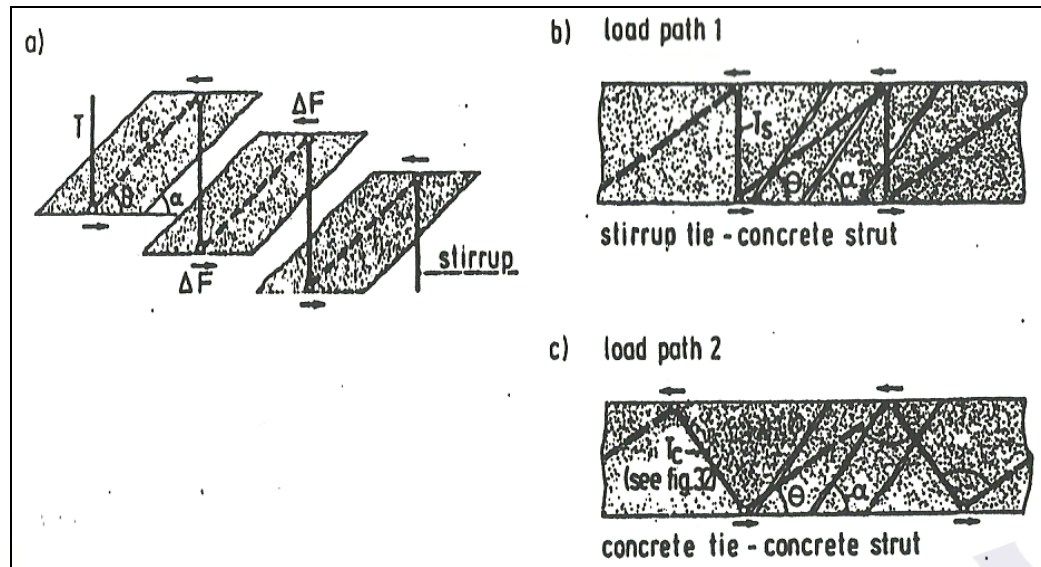Two common failure modes, namely, diagonal splitting and concrete crushing, are examined in the paper. Premature failures such as shear tension failure (due to insufficient anchorage of main longitudinal reinforcement) and bearing failures are not considered. The resistance to diagonal splitting is mainly provided by the main and shear reinforcement. Additional resistance from concrete tensile strength included in the analysis.

The resistance to crushing of concrete is contributed by the concrete compressive strength. Ultimate shear strengths of deep beams are governed by both the transverse tensile stresses perpendicular to the diagonal strut, and the compressive stresses in the diagonal strut, resulting in an interaction between the two failure modes. Predictions by the proposed modal are compared with experimental results and other established calculation methods. Generally, the predictions are not only accurate and consistent in each case study, but also conservative.

After the principal tensile stresses have reached the tensile strength of the concrete, following the direction of the load, individual pieces of the web, only controlled in their movement by the flanges, try to fall down. There, they are caught by the stirrups which hang up the load via T into the adjacent piece evoking C in the struts for vertical equilibrium. The chords provide horizontal equilibrium with additional tensile forces F. this is the principal load path 1, if the concrete tensile strength is disregarded.

Looking closer, it is recognized that the kinematics as described evoke an additional load path 2 which combines with the load path 1 but which is usually neglected; the vertical movement v has two component, the crack opening w perpendicular to the crack and a sliding A parallel to the crack. The sliding A is obviously resisted by aggregate interlock in the crack and it appears reasonable to assume, that the resisting force R acts in the direction of A. the force R has two components, a compressive force Cc with an inclination $\theta<\alpha$ and concrete tensile force Tc perpendicular to it.

Both load paths jointly carry the load and therefore their combined compressive struts together assume the inclination $\theta<\alpha$. As long as it can be sustained by the concrete, the concrete tensile force perpendicular to the struts is responsible for the fact that the stirrups needs to carry only part of the shear loads. However, it also causes the concrete of the struts to be biaxially loaded, thus either reducing their compressive strength or resulting in a second array of cracks with inclinations less than α, depending on the load cases. Only if $\theta=\alpha$ does load path 2 disappear. When this occurs the compressive struts are unaxially loaded and can therefore develop their maximum strength. Therefore, the maximum capacity of a beam for shear force is achieved if the struts are parallel to the cracks and if the corresponding large amount of stirrups provided.

Figure 2.9: Internal forces in the web due to shear (a) kinematic of load path 1 if acting alone ($\theta=\alpha$). (b) through (c) load paths 1 and 2 in the web if acting combined ($\theta<\alpha$).

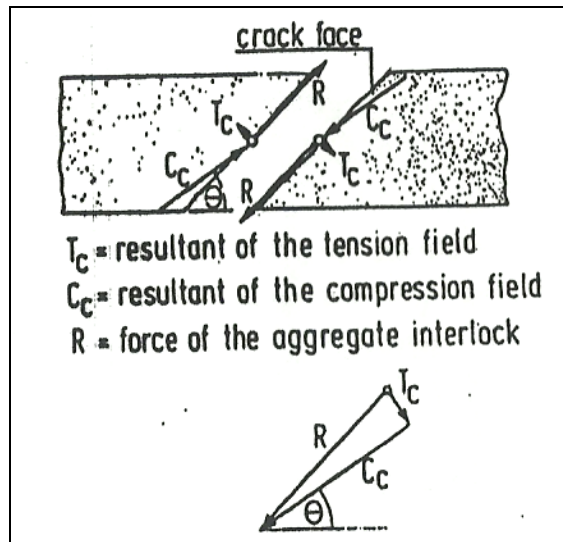**Figure 2.10: Displacements in the web because of the crack.**

Figure 2.11: Aggregate interlock force *R* corresponding compression *Cc* and tension *Tc* in the concrete.
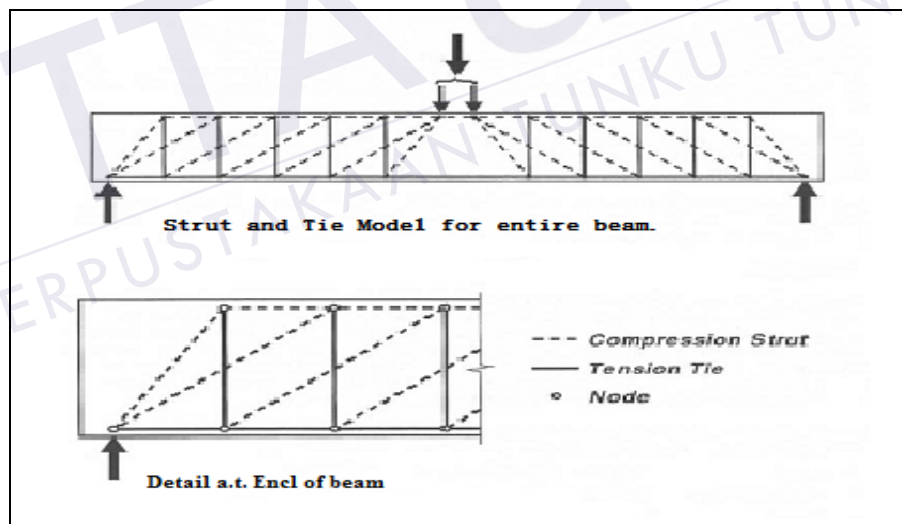

Figure 2.12: Shear strength of strut for R.C beams.

2.2.5   Shear Concerns in Strut and Tie Models

The strut and tie model pattern of parallel inclined crack forms in region of high shear, shown in Figure 2.12 and that the concrete in between adjacent inclined cracks can carry an inclined compressive force, and hence act like a diagonal strut. A feature of truss method is that the forces in the stirrups and the diagonal strut can be determined using simple statics. For example, in Figure 13 the strut is inclined at θ degrees while stirrup is vertical, so that the shear force acting in a cross-section is carried by the vertical component of the diagonal compressive force D: $D\sin\theta = V$

In common case, the inclined crack cut and stirrups and these together carry the applied shear force V. Figure 2.15 compares the experimentally determined shear strength of the series of beam tested using sectional design model and strut and tie models Collins and Mitchell. In these tests, the shear span to depth ratio a/d was varied from 1 to 7 and no web reinforcement was provided. At a/d values less than 2.5, the resistance is governed by strut and tie action, with the resistance dropping off rapidly as a/d increased.

The test showed that for span to depth ratios from 1 to 2.5 the shear is carried by strut-and-tie action; however, over the 2.5 ratio a sectional model transfers the shearing stress. The findings of Kani et al. would further support the ability of the truss model to transfer the shear in disturbed regions near supports and point loads. However, bridge designers are typically uncomfortable with the idea of not using shear reinforcement and therefore after a strut-and tie has been developed most engineers have then also conducted a sectional analysis to detail additional shear reinforcement.

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