

HYDROMAGNETIC RADIATIVE EFFECT ON WATER, KEROSENE BASED  
ON SINGLE WALLED CARBON NANOTUBES INDUCED BY A  
STRETCHING/SHRINKING SHEET WITH VISCOUS-  
OHMIC DISSIPATION

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*For my dearest father, Ilyasak bin Hussein,  
My lovely mother, Norfishah binti Othman & my family for their  
encouragement and blessing*



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*Mohd Qhushairi Bin Ilyasak*

## ABSTRACT

The flow and heat transfer characteristics of radiative magnetohydrodynamic convection boundary layer flow of nanofluids over a nonlinear stretching/shrinking sheet in the presence of viscous dissipation, thermal radiation and Ohmic heating is investigated numerically. The similarity transformation reduces the time-independent boundary layer equations for continuity, momentum and energy into a set of coupled ordinary differential equations. The obtained governing equations have been solved numerically by using similarity transformation method with shooting technique and the analysis for the problem are evaluated by using MAPLE 2016. The influence on the velocity and temperature are presented both in tabular and graphical forms for various parameter such as magnetic field parameter ( $M$ ), thermal radiation parameter ( $N$ ), nonlinear parameter ( $m$ ), Richardson number ( $RI$ ), heat source parameter ( $\lambda$ ), heat sink parameter ( $-\lambda$ ), suction parameter ( $S$ ) and injection parameter ( $-S$ ).



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## ABSTRAK

Ciri-ciri aliran dan pemindahan bagi aliran lapisan sempadan dalam magnetohidrodinamik radiasi terhadap helaian meregang/mengecut tak linear dengan kehadiran pelepasan likat, radiasi haba dan pemanasan ohmic telah diselidik secara berangka. Penjelmaan keserupaan menurunkan persamaan lapisan sempadan yang tidak bergantung pada masa untuk selanjut, momentum dan tenaga kepada set sepasang persamaan pembezaan biasa. Persamaan menakluk yang diperoleh telah diselesaikan secara berkala menggunakan kaedah penjelmaan keserupaan dengan teknik meluru dan analisis telah dinilai dengan menggunakan MAPLE 2016. Kesan terhadap halaju dan suhu telah dibentangkan dalam bentuk jadual dan graf untuk pelbagai parameter seperti parameter medan magnet ( $M$ ), parameter radiasi haba ( $N$ ), parameter tak linear ( $m$ ), nombor Richardson ( $RI$ ), parameter sumber haba ( $\lambda$ ), parameter penyedut haba ( $-\lambda$ ), parameter sedutan ( $S$ ) dan parameter suntikan ( $-S$ ).



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## LIST OF SYMBOLS AND ABBREVIATIONS

$c, b$	- constant
$C_f$	- skin friction coefficient
$C_p$	- specific heat at constant pressure
$B_o$	- strength of magnetic field
$Ec$	- Eckert number
$g$	- acceleration due to gravity
$M$	- magnetic field
$K^*$	- Rosseland mean spectral absorption coefficient
$m$	- power-law stretching/shrinking parameter
$N_r$	- Thermal radiation parameter
$Nu_x$	- local Nusselt number
$Pr$	- Prandtl number
$Pr_{eff}$	- Effective Prandtl number
$q_r$	- thermal radiative heat flux
$Q_o$	- dimensional heat generation/absorption coefficient
$Re_x$	- local Reynolds number
$Ri$	- Richardson number
$S$	- Suction/injection parameter
$T$	- temperature of the fluid
$T_\infty$	- free stream temperature
$T_w$	- temperature at the wall
$u$	- velocity component in $x$ -direction
$u_w$	- stretching/shrinking sheet velocity
$v$	- velocity component in $y$ -direction
$x, y$	- direction along and perpendicular to the plate, respectively
$\alpha_{nf}$	- effective thermal diffusivity of the nanofluid
$\beta_{nf}$	- thermal expansion coefficient of nanofluid
$\beta_f$	- thermal expansion coefficient of the fluid
$\beta_s$	- thermal expansion coefficient of the nanoparticles
$\phi$	- solid volume fraction of the nanoparticles
$\xi$	- non-dimensional parameter
$\sigma_{nf}$	- electrical conductivity of the nanofluids
$\sigma_s$	- electrical conductivity of the nanoparticles
$\sigma_f$	- electrical conductivity of the fluids
$\lambda$	- heat generation/absorption parameter
$\mu_{nf}$	- effective dynamic viscosity of the nanofluid

- $\mu_f$  - dynamic viscosity of the fluid
- $\nu_f$  - kinematic viscosity of the fluid
- $\rho_{n,f}$  - effective density of the nanofluid
- $\sigma^*$  - Stefan-Boltzmann constant
- $\theta$  - dimensionless temperature of the fluid
- $\psi$  - stream function
- $K_{n,f}$  - thermal conductivity of the nanofluid
- $K_f$  - thermal conductivity of the fluid



## CHAPTER 1

### INTRODUCTION

#### 1.1 Background

In recent decades, the rapid growth of nanotechnology is going to bring an unimaginable and could change our way of life. The investigation of nanofluid flow has been actively done by many researchers and engineers until recently the production of particles with the sizes of nanometers (nanoparticles) can be achieved with relative ease. The characteristics of fluids are the conventional heat transfer of fluids usually possess poor thermal conductivity compared to solid and fluids sized particles do not work well with the nanotechnology. To overcome the limitation of fluids, the modern technology developed an opportunity to produce nanoparticles.

Nanofluids are the new class of fluids engineered by immersing nanometer-sized particles in a base fluids. The term of nanofluid was discovered by Choi and Stephen (1995), to describe a heat transported in suspensions of submicronic solid particles known as nanoparticles in a base fluid. The size of solid nanoparticles typically of 1 to 100 nanometer(nm) usually used in high-tech applications in heat transfer, including electronics and microelectronics, engine cooling, domestic refrigerator, machine grinding, rocket fuels, cooling of nuclear reactors, glass fabrication and in boiler fuel gas temperature reduction. Water, oil, ethylene glycol and lubricants are the common base fluids while for aluminium, silver, titanium and copper are referred as nanoparticles. The reason why nanofluids are important because it provide high specific surface area, capable to stay suspended much longer

than micro-particles, high dispersion stability, adjustable thermal conductivity, able to reduce particle clogging and reduce pumping power but nanofluids also have limitation such as lower specific heat, higher production cost and difficulties in manufacturing process. There are three main causes that influence the thermal conductivity of nanofluids which are Brownian motion, interfacial layer and volume fraction of particles. These nanoparticles are metal with or without their oxides having higher thermal conductivity due to increases in conduction and convection coefficients as they are effectively as a bridge to connect between molecular structure and bulk materials. Figure 1.1 shows the cross section of nanofluid structure consisting of nanoparticles, bulk liquid and nanolayers at solid/liquid interface. From the previous studies that conducted by Xuan and Li (2003), it is shown that the nanoparticles having high enhancements thermal conductivity compared with the conventional fluids. Xuan and Lin claimed that it is possible to achieve thermal conductivity larger than 20% at a small volumetric fraction of nanoparticles which is smaller than 5%.

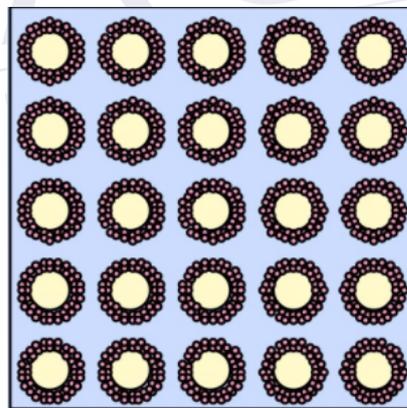


Figure 1.1: Cross section of nanofluid structure

Magnetohydrodynamic (MHD), also called as magneto fluid dynamics or hydromagnetics is a branch of the science which is the study of magnetic properties of electrically conducting fluids and the behaviour of a plasma. The word of magnetohydrodynamic is derived from magneto which means magnetic field, hydro meaning fluid and dynamics meaning movement. Magnetohydrodynamic was

initiated by Swedish physicist, Hannes (1942). The study of magnetohydrodynamic flow are becoming a great scientific interest for researchers as its many applications in modern technologies, including polymer industry, magnetic materials processing, purification of crude oil, solar physics and etc. The principal of MHD is based on Faradays law of electromagnetic induction where a conductor and a magnetic field moves relative to each other, then, the conductor will induce the voltage which results in flow of current across the terminal. The magnetohydrodynamic used a fluid for the energy conversion technique and magnetic nanofluids consist of colloidal magnetic nanoparticles suspended in a base fluid which possess magnetic, thermal and fluid properties. Plasmas, electrolytes, salt water and liquid metals are the examples of magnetofluids. The advantages of MHD is there is no working mechanical element, increased efficiency up to 50% – 60%, frictionless process, can be easily operated from different fluids, no air pollution and safe for environment. But magnetohydrodynamic also has limitations such as highly cost, it requires highly skilled labour, high operating temperature and short circuit may occur. The studies of MHD flow of an incompressible viscous fluid caused by deformation of a plane surface were investigated by Pavlov (1974). In order to solve the differential equations describing the convection flow of unsteady Magnetohydrodynamic (MHD) in a nanofluids past an accelerating vertical surface in the form of assisting flow, the Fourth order Range-Kutta method based shooting technique has been employed by Freidoonimehr *et al.* (2015) was used. Moreover, the studies of the flow of MHD of variable viscosity nanofluid over radially stretching convective surface were conducted by Makinde *et al.* (2016). The hydromagnetic rotating flow of third grade fluid was studied by Hayat *et al.* (2008) and Hayat *et al.* (1974) used a homotopy method on Magnetohydrodynamic (MHD) to analyze the three-dimensional flow of nanofluid over a convectively heated nonlinear stretching the surface.

In recent times, viscous and Ohmic heating effects have been the topic of extensive research. Ohmic heating, also known as Joule heating is the process of

generation of heat in the fluid by passing an electric current through a metal. Viscous heating also plays an important role in fluid dynamics due to strongly temperature-dependent viscosity and the irreversible process by mean of which the work done by a liquid due to the action of shear forces is changed into heat is known as viscous dissipation. Heat absorption is the heat transfer that occurs between two bodies that can occur by three mechanisms which is conduction, convection and radiation. The only way to transfer heat from one place to another without requiring a medium or via electromagnetic waves is known as thermal radiation heat transfer. In this case, heat generation travelling or propagating electromagnetic waves, while transferring energy from one body to another. The heat generation/absorption effects are used in many thermo-fluid applications including in nuclear reactors, transportation, cooling of electronic devices, biomedicine and etc. Keblinski *et al.* (2002) claimed that there are four explanations for thermal conductivity which are Brownian motion of the particles, nanoparticles clustering, molecular-level layering of particle interface and ballistic heat transfer in the nanoparticles. Selimefendigil and Oztop (2016a) stated that as the values of the solid volumetric fraction of nanoparticle increases, it will enhance the average heat transfer and more effective for Richardson number.

Single-walled carbon nanotubes are cylindrical nanostructures consist of atomic sheet of carbon atoms. These cylindrical nanostructures have the potential to revolutionize nanotechnology, nano-optics, nanoelectronics and other aspects of technology. These materials posses unique properties which are thermal, mechanical, and electrical. Carbon nanotubes are hexagonally shaped arrangements of carbon atoms that have been rolled into tube which is shown in figure 1.2 and typically having a diameter of 1 nanometer and sometimes microns or centimeters. These materials are known as the strongest and stiffness materials due to tensile strength and elastic modulus. On the other hand, carbon nanotubes also have a low density compared to carbon steel. All nanotubes are particularly promising to be good thermal conductors and good insulator along the tube. But, as with any material, single-walled carbon nanotubes also have weakness which

could affect the material properties such as crystallographic defect and lead to phonon scattering, reduces the thermal conductivity and it also can induce harmful effects such as inflammatory and fibrotic reactions. Due to nano-sized structure and high tensile strength of single-walled carbon nanotubes, it is commonly used as nanotube transistors, paper battery, bulletproof clothing and some of the researchers recommended it as a space elevator. According to Sinha *et al.* (2005), a single-walled carbon nanotube usually has a room temperature thermal conductivity of about  $1.52(Wm^{-1}K^{-1})$  which is almost similar with soil. Besides that, Savin *et al.* (2009) were investigating the thermal conductivity of single-walled carbon nanotubes while Ansari *et al.* (2017) analyzed the thermal effects on the vibration of embedded single-walled carbon nanotubes based on the nonlocal shell model.

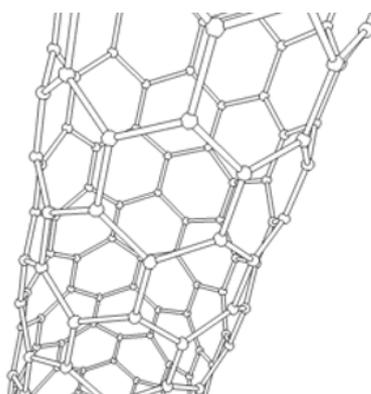


Figure 1.2: Structure of carbon nanotubes

The researchers and engineer are interested to investigate the transport phenomena from a stretching and shrinking sheet. The flow produced due to stretching of an elastic flat sheet which moves in its plane with velocity varying with the distance from a fixed point due to the application of a stress are known as stretching flow or the vice versa behaviour are known as shrinking flow. It seems that Sakiadis (1961) was the first who studied the boundary layer behaviour on continuous solid surfaces. Wang (2008), describing the stagnation flow as the fluid motion near the stagnation region, exists on all solid bodies moving in a fluid. Goldstein (1965) verified that the stretching sheet velocity formulation need not

necessarily be a linear one. Norfifah Bachok (2012) was investigating the boundary layer flow and heat and mass transfer over an exponentially stretching and shrinking sheet in a nanofluid. From the previous studies, it is shown that the linear stretching and shrinking sheet case solution is smaller compared with the similarity of the stagnation-point flow over an exponentially stretching and shrinking sheet.

## **1.2 Classification of Fluids**

Generally, fluids are classified in the following categories:

### **1.2.1 Ideal and Real Fluids**

Fluids that are incapable of sustaining any shearing stress (tangential force) or action in the form of shear but the normal pressure (force) acts between the adjoining layers of fluid are known as ideal fluids. Ideal fluids also known as perfect fluids offers no internal resistance to change its shape and usually have low viscosity such as water and air.

Whereas, real fluids are known as viscous fluids when normal as well as shearing stresses exist due to the shearing stress a viscous fluid offers resistance to the body moving through it as well as between the particles of the fluid itself. The example of real fluids are syrup and heavy oil which demonstrate the existence of a property of the fluid which controls the rate of fluid flow.

### **1.2.2 Newtonian and Non-Newtonian Fluids**

When the fluid continues to flow, regardless of the forces acting on it, then it is said to Newtonian fluid. A fluid in which the components of the stress tensor are linear functions of the first spatial derivatives of the velocity components and involve two material parameters taken as constants throughout the fluid, although depending on ambient temperature and pressure. The example of Newtonian fluid is water.

Non-Newtonian fluids are known as the fluid viscosity varies with the rate of deformation. It has a few characteristics such as viscosity and elasticity, for example, the viscosity of polymeric liquids changes with the shear rate.

### **1.3 Types of Flows**

#### **1.3.1 Steady and Unsteady Flows**

Steady flow is a flow that has the properties at every point in the flow do not depend upon time. While, unsteady or non-steady flow is one in which the properties do depend on time.

#### **1.3.2 Laminar and Turbulent Flows**

Laminar flow is known as the streamline or viscous flow where the layers of fluid flowing over one another at different speeds with virtually no mixing between layers and the fluid particles move in a definite path or streamlines. For turbulent flows, it shows the irregular movement of particles of the fluid. The particles travel in irregular paths with no observable pattern and no definite layers.

#### **1.3.3 Compressible and Incompressible Flows**

Compressible fluids are those one in which the fluid density changes when it is subjected to high pressure-gradients. The imposition of a force at one end of a system does not result in an immediate flow throughout the system. For an example, the compressed fluid expands against nearby fluid particles causing the other fluids itself to compress and setting in motion a wave pulse that travels throughout the system.

When the fluid density does not change with pressure, it is known as the incompressible fluid.

### 1.3.4 Viscosity of Fluids

When a fluid flowing in a pipe, it produces frictional force which results in friction within the fluid itself and converting some of its kinetic energy into thermal energy. Viscous forces is a frictional forces that try to prevent different layers of fluid from sliding past each other. Viscosity is a measure of a fluid resistance to relative motion within the fluid and depend strongly on the temperature. The viscosity of a liquid decreases with an increasing of the temperature.

### 1.4 Reynolds Number ( $Re$ )

Reynolds number is discovered by the British physicist, Osbourne Reynolds on 1883. Reynolds number is used to determine the type of flow patterns while flowing through a pipe, laminar, and turbulent. Reynolds number is expressed as the ratio of inertia forces to the viscous forces within a liquid due to different liquid velocities. The flow through the pipe is turbulent when the value of Reynolds number calculated is high and the flow is laminar if the value of Reynolds number calculated is low. Reynolds number is defined by the following relation,

$$Re = \frac{\text{Inertial force}}{\text{Viscous force}} = \frac{\rho v L}{\mu} = \frac{v L}{\nu}$$

Where,

1.  $v$  is the velocity of the object relative to the fluid
2.  $L$  is the characteristic linear dimension,
3.  $\mu$  is the dynamic viscosity of the fluid,
4.  $\rho$  is the density of the fluid.

### 1.5 Richardson number ( $Ri$ )

Richardson number is discovered by Lewis Fry Richardson. It is an important dimensionless quantity in fluid mechanics and it is expressed as the ratio the buoyancy term to the flow gradient term. In thermal convection, Richardson number is used to represent the relative magnitude of natural convection to forced convection for a given fluid. When the value of Richardson number is low, the forced convection will dominate. If the value of Richardson number is high, both forced and natural convection will dominate. Richardson number is defined as,

$$Ri = \frac{g\beta(T_{hot} - T_{ref})L}{V^2},$$

Where,

1.  $g$  is the gravitational acceleration,
2.  $\beta$  is the thermal expansion coefficient,
3.  $T_{hot}$  is the hot wall temperature,
4.  $T_{ref}$  is the reference temperature,
5.  $L$  is the characteristic length,
6.  $V$  is the characteristic velocity.

Furthermore, Richardson number can also be defined by using a combination of the Grashof number and Reynolds number as the following relation,

$$Ri = \frac{\text{Grashof number}}{\text{Reynolds}^2},$$

### 1.6 Eckert number ( $Ec$ )

Eckert number was named after Ernst R. G. Eckert and it is a dimensionless quantity. Eckert number represents the ratio of the kinetic energy of a flow to the boundary layer enthalpy difference. Eckert number is defined as,

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