INTEGRATED OPTIMAL CONTROL AND PARAMETER ESTIMATION ALGORITHMS FOR DISCRETE-TIME NONLINEAR STOCHASTIC DYNAMICAL SYSTEMS

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A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy (Mathematics)

Faculty of Science
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JUNE 2011

ABSTRACT

This thesis describes the development of an efficient algorithm for solving nonlinear stochastic optimal control problems in discrete-time based on the principle of model-reality differences. The main idea is the integration of optimal control and parameter estimation. In this work, a simplified model-based optimal control model with adjustable parameters is constructed. As such, the optimal state estimate is applied to design the optimal control law. The output is measured from the model and used to adapt the adjustable parameters. During the iterative procedure, the differences between the real plant and the model used are captured by the adjustable parameters. The values of these adjustable parameters are updated repeatedly. In this way, the optimal solution of the model will approach to the true optimum of the original optimal control problem. Instead of solving the original optimal control problem, the model-based optimal control problem is solved. The algorithm developed in this thesis contains three sub-algorithms. In the first sub-algorithm, the state mean propagation removes the Gaussian white noise to obtain the expected solution. Furthermore, the accuracy of the state estimate with the smallest state error covariance is enhanced by using the Kalman filtering theory. This enhancement produces the filtering solution by using the second sub-algorithm. In addition, an improvement is made in the third sub-algorithm where the minimum output residual is combined with the cost function. In this way, the real solution is closely approximated. Through the practical examples, the applicability, efficiency and effectiveness of these integrated sub-algorithms have been demonstrated through solving several practical real world examples. In conclusion, the principle of model-reality differences has been generalized to cover a range of discrete-time nonlinear optimal control problems, both for deterministic and stochastic cases, based on the proposed modified linear optimal control theory.

ABSTRAK

Tesis ini membincangkan pembangunan algoritma yang berkesan untuk menyelesaikan masalah kawalan optimum berstokastik tak linear dalam masa diskrit berasaskan prinsip bezaan model-realiti. Integrasi kawalan optimum dan penganggaran parameter merupakan idea utama. Dalam usaha ini, model kawalan optimum berasaskan model ringkas dibina dengan parameter boleh dilaraskan. Dengan ini, anggaran keadaan optimum digunakan untuk merekabentuk hukum kawalan optimum. Keluaran dari model pula digunakan untuk membetulkan parameter boleh dilaraskan. Dalam prosedur lelaran, nilai bezaan antara loji nyata dan model diguna diambilkira oleh parameter boleh dilaraskan. Kemudian, nilai-nilai penyelarasan tersebut dikemaskini secara berulangan. Dengan cara sebegini, penyelesaian optimum model tersebut akan menghampiri penyelesaian optimum sebenar bagi masalah kawalan optimum asal. Daripada menyelesai masalah kawalan optimum yang asal, masalah kawalan optimum berasaskan model diselesaikan. Algoritma yang dibangunkan dalam tesis ini mengandungi tiga sub-algoritma. Dalam sub-algoritma pertama, perambatan purata keadaan menyingkirkan hingar putih Gaussan supaya penyelesaian jangkaan dapat diperolehi. Selanjutnya, ketepatan anggaran keadaan berkovarian ralat terkecil diatasi dalam sub-algoritma kedua dengan mengguna teori penurasan Kalman. Peningkatan ini menghasilkan penyelesaian turasan. Di samping itu, penambahbaikan yang menggabungkan reja keluaran minimum dengan fungsi kos dilakukan dalam sub-algoritma ketiga. Dengan cara ini, penyelesaian sebenar dihampiri secara rapat. Akhirnya, melalui kajian contoh praktik, ciri-ciri kebolehgunaan, kecekapan dan keberkesanan terhadap sub-algoritma yang dibangun dibuktikan melalui penyelesaian beberapa contoh praktik dunia nyata. Secara kesimpulan, kegunaan prinsip bezaan model-realiti telah diperluaskan untuk meliputi sejumlah penyelesaian masalah kawalan optimum tak linear masa diskrit, termasuk kes berketentuan dan berstokastik, berdasarkan teori kawalan optimum terubahsuai.

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LIST OF ABBREVIATIONS

2-D	-	two dimensional
BF	-	Bryson-Frazier
IOCPE	-	integrated optimal control and parameter estimation
ICEES	-	integrated optimal control and estimation for expectation solution
ICEFS	-	integrated optimal control and estimation for filtering solution
ICERS	-	integrated optimal control and estimation for real solution
ISOPE	-	integrated system optimization and parameter estimation
DISOPE	-	dynamic integrated system optimization and parameter estimation
EOP	-	expanded optimal control problem
KF	-	Kalman filtering
LMI	-	linear matrix inequality
LQ	-	linear quadratic
LQG	-	linear quadratic Gaussian
LQR	-	linear quadratic regulator
MMOP	-	modified model-based optimal control problem
MOP	-	model-based optimal control problem
ROP	-	real optimal control problem
PDF	-	probability density function
RTS	-	Rauch-Tung-Striebel
TPBVP	-	two-point boundary-value problem

LIST OF SYMBOLS

i	-	Iteration step
j	-	Discrete time step
k	-	Discrete time step
m	-	Order of control input
n	-	Order of state
p	-	Order of output
q	-	Order of process noise
τ	-	Time delay step
z^{-1}	-	Time-delay unit
A	-	$n \times n$ state transition matrix
B	-	$n \times m$ control coefficient matrix
C	-	$p \times n$ output coefficient matrix
G	-	$n \times q$ process noise coefficient matrix
$I_{2\times 2}$	-	2×2 identity matrix
$K(k)$	-	$m \times n$ feedback gain
$K_f(k)$	-	$n \times p$ filter gain
$K_p(k)$	-	$n \times p$ predictor gain
$K_s(k)$	-	$n \times n$ smoother gain
$M_x(k)$	-	$n \times n$ state error covariance / *a priori* covariance
$M_y(k)$	-	$p \times p$ output error covariance
N	-	Final fixed time
$P(k)$	-	$n \times n$ *a posteriori* covariance
Q	-	$n \times n$ cost weighting matrix
Q_a	-	Augmented Q

Q_ω	-	$q \times q$ process noise covariance
Q_y	-	$p \times p$ least-square weighting matrix
R	-	$m \times m$ cost weighting matrix
R_a	-	Augmented R
R_η	-	$p \times p$ measurement noise covariance
$S(N)$	-	$n \times n$ terminal cost weighting matrix
$p(k)$	-	n-co-state vector
$\hat{p}(k)$	-	n-separable co-state vector
$s(k)$	-	n-vector
$u(k)$	-	m-control vector
$v(k)$	-	m-separable control vector
$x(k)$	-	n-state vector
$\bar{x}(k)$	-	State mean / expected state sequence
$\hat{x}(k)$	-	State estimate / filtered state sequence
$y(k)$	-	p-output vector
$\bar{y}(k)$	-	Output mean / expected output sequence
$\hat{y}(k)$	-	Output estimate / filtered output sequence
$z(k)$	-	n-separable state vector
$f(\cdot)$	-	Real plant dynamic function $\Re^n \times \Re^m \times \Re \to \Re^n$
$h(\cdot)$	-	Real measurement output function $\Re^n \times \Re \to \Re^p$
$p(\cdot)$	-	Probability density function
$H(\cdot)$	-	Hamiltonian function $\Re^n \times \Re^m \times \Re^n \times \Re \to \Re$
$H_e(\cdot)$	-	Hamiltonian function for expanded optimal control problem
$J(\cdot)$	-	Scalar cost
$J_0(\cdot)$	-	Scalar cost for stochastic optimal control problem
$J_e(\cdot)$	-	Scalar cost for expanded optimal control problem
$J'_e(\cdot)$	-	Augmented scalar cost
$J_m(\cdot)$	-	Scalar cost for model-based optimal control problem
$J_{mm}(\cdot)$	-	Scalar cost for modified model-based optimal control problem

$J_p(\cdot)$	-	Scalar cost for stochastic optimal control problem
$L(\cdot)$	-	Cost measure under summation $\Re^n \times \Re^m \times \Re \to \Re$
$\alpha(k)$	-	Model adjusted parameter
$\alpha_1(k)$	-	Model adjusted parameter
$\alpha_2(k)$	-	Output adjusted parameter
$\beta(k)$	-	State multiplier
$\beta_a(k)$	-	Augmented $\beta(k)$
$\eta(k)$	-	p-measurement noise vector
$\gamma(k)$	-	Cost functional adjusted parameter
$\lambda(k)$	-	Control multiplier
$\lambda_a(k)$	-	Augmented $\lambda(k)$
$\mu(k)$	-	n-multiplier vector
$\varphi(\cdot)$	-	Terminal cost $\Re^n \times \Re \to \Re$
$\pi(k)$	-	p-multiplier vector
$\omega(k)$	-	q-process noise vector
$\xi(k)$	-	Scalar multiplier
Γ	-	Terminal state multiplier
ε	-	Tolerance
σ	-	Constant parameter for standard deviation
♦	-	End of a proof
$diag(\cdot)$	-	Diagonal matrix with respective dimension
xb(k)	-	Expected state sequence
xh(k)	-	Estimated state sequence
xme(k)	-	State margin of error
x_e(k)	-	State error sequence
yh(k)	-	Estimated output sequence
yme(k)	-	Output margin of error
y_e(k)	-	Output error sequence

LIST OF PUBLICATIONS

The following papers (which have been published or accepted for publication) were completed during Ph.D. candidature.

Refereed Journals*: *International or National

Kek, S.L. (2008). Discrete-Time Linear Optimal Control with a Random Input Study. *MATEMATIKA*, Special Edition, Part II, December, 621-629. (ISSN 0127-8274).

Kek, S.L., Teo, K.L. and Mohd Ismail, A.A. (2010). An Integrated Optimal Control Algorithm for Discrete-Time Nonlinear Stochastic System. *International Journal of Control*. Vol. 83, No. 12, 2536-2545. (2009 Impact Factor 1.124)

Refereed Proceedings*: *Conference or Symposium Presentations

Kek, S.L. and Mohd Ismail, A.A. (2008). Solution of Discrete Linear Stochastic Optimal Control Problem *via* Dynamic Programming. *Proceeding of 16th National Symposium of Mathematical Sciences*. 03-05 June, UMT, 73-82. (ISBN 978-983-2888-91-8).

Kek, S.L. and Mohd Ismail, A.A. (2009). Optimal Control of Discrete-Time Linear Stochastic Dynamic System with Model-reality Differences. *Proceeding of 2009 International Conference on Computer Research and Development* (ICCRD09). July 10-12, Perth, Australia, 573-578.

Mohd Ismail, A.A. and Kek, S.L. (2009). Optimal Control of Nonlinear Discrete-Time Stochastic System with Model-Reality Differences. *2009 IEEE International Conference on Control and Automation* (ICCA09). 9-11 December, Christchurch, New Zealand, 722-726.

Kek, S.L., Mohd Ismail, A.A. and Rohanin, A. (2010). An Expectation Solution of Discrete-Time Nonlinear Stochastic Optimal Control Problem Using DISOPE Algorithm. Faculty of Science: Post-Graduate Conference. 5-7 October, Ibnu Sina, UTM, Book of Abstracts, 14.

Mohd Ismail, A.A., Rohanin, A., Kek, S.L. and Teo, K.L. (2010). Computational Integrated Optimal Control and Estimation with Model Information for Discrete-Time Nonlinear Stochastic Dynamic System. *Proceeding of the 2010 IRAST International Congress on Computer Applications and Computational Science* (CACS 2010). 4-6 December, Singapore, 899-902.

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The following papers were completed during Ph.D. candidature and are currently under review.

Refereed Journals*: *International or National

Kek, S.L., Teo, K.L. and Mohd Ismail, A.A. (2011). Filtering Solution of Nonlinear Stochastic Optimal Control Problem in Discrete-Time with Model-Reality Differences. *Numerical Algebra, Control and Optimization*.

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Refereed Proceedings*: *Conference or Symposium Presentations

Kek, S.L., Mohd Ismail, A.A., Teo, K.L. and Rohanin, A. (2011). Integrated Optimal Control and Parameter Estimation Algorithm for Discrete-Time Nonlinear Stochastic Dynamic Systems. *The Australian Control Conference* 2011 (AUCC2011). 10-11 November 2011, Melbourne, Australia.

Kek, S.L., Mohd Ismail, A.A., Teo, K.L. and Rohanin, A. (2011). Discrete-Time Nonlinear Stochastic Optimal Control in Its Expectation. 2011 *International Conference on Control and Automation* (CAA2011), 28-30, October 2011, Shanghai, China.

CHAPTER 1

INTRODUCTION

1.1 Introduction

Many decision and control problems can be formulated as optimization problems, where a decision policy is to be designed such that some performance indexes are optimized. However, in real-world situations, the presence of disturbances is unavoidable. Because the disturbances are random in nature, they are unpredictable. Consequently, the solutions of a real system obtained during simulation are often corrupted by noise. Thus, the decision policy obtained based on these distorted solutions is unlikely to be acceptable if the disturbances are not taken into consideration. Therefore, the governing system of the real-world problem shall be formulated as a stochastic dynamical system. On this basis, the decision policy designed subject to this stochastic dynamical system is much more useful in practice. As clearly mentioned in (Ho and David, 2001), it is impossible to devise a single general-purpose approach which works the best for all kinds of problems. Different approaches are needed for different types of problems by exploring the specific nature of the problem concerned.

In this thesis, a class of nonlinear discrete-time stochastic control problems is considered. To obtain the optimal solution of this control problem, a simplified optimal control model is constructed. It is being applied to developing an efficient iterative algorithm. In this algorithm, the different structures among the real plant and the model employed are taking into account as stated in the principle of model-reality differences (Becerra, 1994; Roberts 1992). Details of the development of the algorithm will be elaborating in the following chapters.

Here, a comprehensive introduction is delivered. Initially, a brief introduction to the approach based on model-reality differences is given. Next, a general class of stochastic dynamical systems is discussed. Then, it is followed by the statements of the objectives of the study. Furthermore, the significances of study are discussed and highlighted. Finally, the overview of each chapter is given.

1.2 The Model-Reality Differences Approach: A Brief

The principle of model-reality differences provides a new attractive computational methodology in tackling the integrated problem formulation of the control problem, where the system optimization is coupled with the parameter estimation. The integrated system optimization and parameter estimation (ISOPE) and the dynamic-ISOPE (DISOPE) algorithms are well-known in the literature. Over the past 30 years, these integrated algorithms have matured and they have been used in developing algorithms for solving optimal control problems, where the random disturbances are, however, ignored.

1.2.1 Background

The integrated system optimization and parameter estimation (ISOPE) algorithm, which was inspired by Haimes and Wismer (1972), was originally developed by Roberts (1979) and Roberts and Williams (1981) for on-line steady-state optimization of industrial processes. The main concept is that the optimization is achieved through the adjustment of regulatory controller set-points. This method is iterative in nature. It uses the repeated solution of system optimization and optimal parameter estimation to calculate the optimum. This iterative solution converges to the real optimum in spite of the differences that exist between the model used and the real plant. It is a well known fact that the measurement of the derivatives of the real process is difficult to carry out. However, this difficulty has been incorporated within the ISOPE iterative procedure by using the directional derivative, the methods of finite difference approximation, dual control optimization, Broydon's method and a dynamic identification method. For details, see (Brdyś and Tatjewski, 2005; Mansour and Ellis, 2003).

The ISOPE algorithm is used daily in an oil processing plant in Western Australia (Becerra, 1994). A number of ISOPE algorithms has been developed, which includes the centralized and hierarchical algorithms (Roberts *et al*, 1992). The conditions for the convergence of the algorithms are rigorously investigated in (Brdyś and Roberts, 1987). Since the appearance of the ISOPE algorithm, its extension to the dynamic optimization is suggested, leading to the development of the DISOPE algorithm by following similar principle and philosophy of the ISOPE algorithm.

In a dynamic optimization problem, a control law is to be determined such that a given cost functional is minimized subject to a dynamic system. It is more complicated when compared to the optimization of a steady-state control problem. Nonetheless, by using the principle of ISOPE algorithm, an algorithm, termed as DISOPE algorithm, is developed to solving a class of continuous-time optimal control problems in (Roberts, 1992). The DISOPE algorithm is developed for solving nonlinear optimal control problems *via* solving the modified linear quadratic regulator model iteratively, where the dynamic parameters are updated at each iteration step. It specifically takes into account the differences in the structures and the parameters between the model employed and the real plant during the process of computation.

On the other hand, many control schemes are implemented digitally in industry, where the control input is altered in discrete time steps and it is normally held constant between samples by a zero-order hold (Sage, 1977; Vaccaro, 1995; Leigh, 1992; Ogata 1994; Kirk, 1970). Such control inputs are designed for the discretized version of a continuous plant. However, there are also processes that are in discrete nature and can only admit discrete time controllers. For these types of problems, a discrete DISOPE algorithm has been developed, analysed and implemented. For details, see, for example, (Becerra, 1994; Becerra and Roberts, 1996; Li *et al*, 1999).

1.2.2 Evolutionary: Hierarchical, Predictive and Bilinear

The DISOPE algorithm has been introduced to a large-scale system (Becerra and Robert, 1995; Roberts and Becerra, 2001; Mohd Ismail, 1999), where it decomposes the optimal control problem into an interconnected-subsystem problem. It is then solved in parallel based on the hierarchical structure, where each sub-process is controlled by a separate decision unit in a parallel processing fashion.

The DISOPE algorithm can also be used to compute the receding horizon optimal control in a nonlinear predictive control problem (Becerra *et al*, 1998), where an estimated optimal control sequence that minimizes a given performance index is computed based on the prediction of the future output response. The extended Kalman filtering approach is used for the state and parameter estimation of the real plant in the presence of disturbances (Becerra, 1994). In this way, the nonlinear predictive control problem can be solved by using the DISOPE algorithm.

However, many practical processes in industry are modelled as bilinear models. To apply the DISOPE algorithm to solve these problems, it requires much more numbers of iterations and it may even lead to divergence. In view of these shortcomings, Li and Wan (1999), Li *et al* (2000) and Li *et al* (2002) proposed a new version of the DISOPE algorithm based on a bilinear model. They concluded that the bilinear DISOPE algorithm can also produces the optimal solution in spite of model-reality differences, where the number of iterations is reduced and the convergence is improved when compared with the DISOPE algorithm.

1.2.3 Efficiency, Convergence and Stability

The efficiency of the DISOPE algorithm has been well documented. To improve the rate of convergence, the DISOPE method based on neural network is introduced in (Kong and Wan, 1999), while the intelligent DISOPE method based on the optimal selection of algorithm parameter, model and model parameter is introduced in (Kong and Wan 2000). Furthermore, a DISOPE interaction balance coordination algorithm is developed in (Kong and Wan, 2001), where the stability of the algorithm is studied. These works have enhanced the efficiency of the DISOPE algorithm.

The convergence analysis of the algorithm, which guarantees the real optimum to be achieved, is studied based on a 2-D analysis by Li and Ma (2004), Roberts (2002; 2003), and Mohd Ismail and Rohanin (2007). Furthermore, the methods of momentum and gradient parallel tangent have been introduced to the DISOPE algorithm so that the slow convergence is overcome, leading to a more efficient DISOPE algorithm after some modifications (Rohanin and Mohd Ismail, 2002; Rohanin, 2005).

1.2.4 Applications to Optimal Control

The principle of model-reality differences has attracted the researchers to apply the DISOPE algorithm to solving real optimal control problems. These applications include fed-batch fermentation process (Becerra and Roberts, 1998), and robot manipulator control problem (Li *et al*, 1999; Rohanin and Mohd Ismail, 2003). In addition, Zhang and Li (2005) proposed a novel distributed model predictive control scheme based on the DISOPE algorithm for nonlinear cascaded systems under network environment. The study of the DISOPE algorithm for complex system under network environment was also carried out by Kong and Wan (2003).

Recently, the DISOPE algorithm has been employed to solve nonlinear discrete-continuous hybrid systems for co-state prediction (Hu *et al*, 2006; Hu *et al*, 2008). Furthermore, the study of the linear and nonlinear discrete-time stochastic optimal control problems is being carried out by using the approach based on the model-reality differences in (Kek and Mohd Ismail, 2009; Mohd Ismail and Kek 2009; Mohd Ismail *et al*, 2010; Kek *et al* 2010a; Kek *at al*, 2010b).

1.3 Stochastic Dynamical Systems

A stochastic dynamical system is a dynamic system which is affected by some kinds of noise. The fluctuation caused by the noise is commonly referred to as a noisy or stochastic phenomenon (Spall, 2003). In this circumstance, the deterministic trajectories of the system are corrupted. Clearly, the disturbances will cause errors in system behaviour, sensor errors and other measurement errors. These errors are highly undesirable, but they are unavoidable. Thus, the noise characteristic shall be taken into consideration in the methods of analysis and design.

Essentially, a dynamical system can be formulated as a system of differential equations or difference equations (Socha, 2008; Grewal and Andrews, 2001; Bar-Shalom *et al*, 2001). In the presence of noise, these dynamical systems shall be modelled by stochastic differential equations or stochastic difference equations. An optimization problem involving a stochastic dynamical system is called a stochastic optimal control problem.

1.3.1 Random Noise Disturbances

Noise is a random variable that fluctuates aperiodically. This variable takes on different sets of values during different sampling processes. Thus, modelling a dynamical system, which is perturbed by one or more sources of noises, is a difficult and challenging task. However, the assumptions about the nature of the noise can be made (Bryson 2002). As such, the accuracy of the assumptions is assessed by comparing the results obtained from the prediction model and the experimental data measured.

Mathematically, it is assumed that the noise that is appeared in the dynamical system is categorized as follows (Grewal and Andrews, 2001; Bar-Shalom *et al*, 2001):

(a) Observation noise – it is an additive noise appeared only in the observation model, and

(b) Parametric noise – it is an additive noise or a multiplicative noise or both appeared in the system dynamic.

1.3.2 State-Space Models

Consider a system model given below.

$$x(k+1) = f(x(k),k) + \omega(k) \tag{1.1}$$

where k denotes the discrete time step, the vector $x(k)$ denotes the current state, and $x(k+1)$ denotes the one-step ahead future state, $f(x(k),k)$ is a vector-valued continuously differential function, and $\omega(k)$ denotes the process noise. It is assumed that $\omega(k)$ is the zero-mean white noise sequence, and that $f(x(k),k)$ does not depend on the previous values of $x(k-\tau)$, τ = 1, …, k. Thus, this process is a Markov process (Bryson, 2002). The vector $x(k)$ is a state vector and the conditional probability density function (PDF) for $x(k)$, denoted by $p(x(k+1)\,|\,x(k))$, is sometimes called the hyperstate of the process. It is characterized by the mean and the covariance (Torsten, 1994) of the process.

Additional, an observation model is given below.

$$y(k) = h(x(k),k) + \eta(k) \tag{1.2}$$

where the vector $y(k)$ denotes a set of observables, $h(x(k),k)$ is a vector-valued continuously differentiable function and the vector $\eta(k)$ denotes the measurement noise. It is assumed that $\eta(k)$ is a zero-mean measurement white noise sequence.

From (Bryson, 2002), it is also assumed that

(a) $\eta(k)$ and $\eta(j)$ are independent if $k \neq j$, and

(b) $x(k)$, $\omega(k)$ and $\eta(j)$ are statistically independent for all k and j.

These assumptions assert that

$$p(x(k+1) \mid x(k), y(k)) = p(x(k+1) \mid x(k)) .$$

The coupling of the system model and the observation model is the state-space model of the stochastic dynamical system. The signal-flow graph representation (Lewis, 1992) of this state-space model is expressed in Figure 1.1.

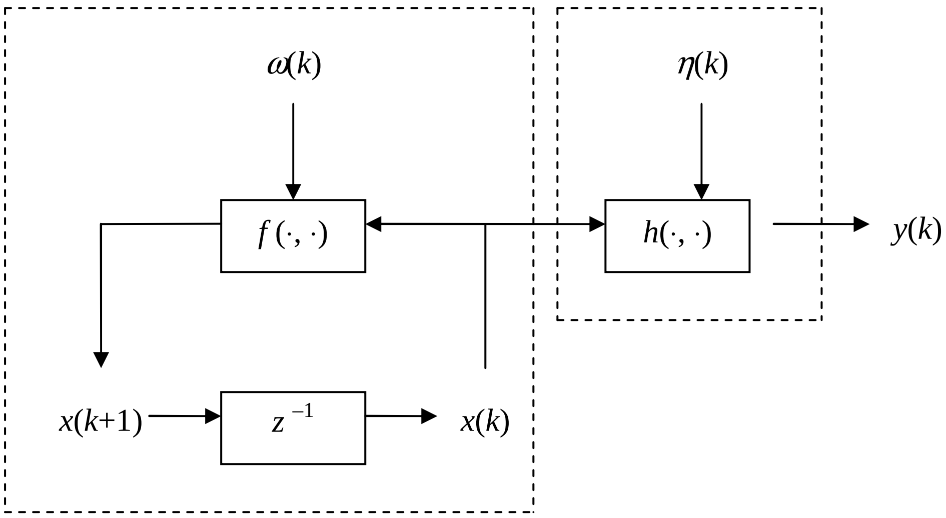

Figure 1.1 State-space model, where z^{-1} denotes time-delay unit

1.3.3 Optimal Control of Stochastic System

Consider the optimal control of a stochastic system given below.

$$x(k+1) = f(x(k), u(k), k) + \omega(k) \tag{1.3}$$

$$y(k) = h(x(k), k) + \eta(k) \tag{1.4}$$

where the vector $u(k)$ is the control variable to be determined such that the following cost functional

$$J(u) = E[\varphi(x(N), N) + \sum_{k=0}^{N-1} L(x(k), u(k), k)] \tag{1.5}$$

is minimized, where E denotes the mathematical expectation, J is a scalar expected value, φ is the terminal cost and L is the cost measure under summation. Note that L is a general time-varying scalar function in terms of the state and control variables at each k for k = 0 to k = N – 1. It is assumed that φ and L are continuously differentiable functions with respect to their respective arguments.

This is a general class of nonlinear stochastic optimal control problems in discrete-time, where the probability density function (PDF) of the state variables is unknown, but their error covariance can be calculated (Bryson, 2002).

1.4 Motivation and Background of the Study

The integrated system optimization and parameter estimation algorithm is a novel approach. Its efficiency and applicability have been well documented, as described in Section 1.2. However, in real-world problems, the presence of random disturbances is unavoidable. These disturbances could occur due to measurement errors from the sensors, instruments, data transmission channels, or human error. In most cases, these errors are random in nature. Thus, the original model-reality differences approach cannot be implemented in this noisy environment.

Furthermore, real-world problems are often modelled as nonlinear dynamical systems subject to stochastic disturbances. In this thesis, only the Gaussian white noise is considered. It is known that a complex dynamical system is difficult to be solved analytically. For mathematical tractability, it is assumed that the corresponding functions in the nonlinear system are continuously differentiable functions. As such, the solution, which is in the forms of expectation and filtering, is to be calculated by using appropriate numerical schemes.

By virtue of the efficiency of the model-reality differences approach to the deterministic nonlinear control system, the goal of this thesis is to develop an efficient and effective computational approach based on the principle of model-reality differences to a class of nonlinear discrete-time stochastic optimal control problems. The principle of model-reality differences is not applicable to continuous-time nonlinear stochastic control systems. It is a future research topic.

1.5 Objectives and Scope of the Study

The objective of this study is to develop an efficient algorithm for solving the nonlinear stochastic optimal control problems in discrete-time based on the principle of model-reality differences. For this, some results on linear system theory are revealed and applied to construct the optimal solution of the discrete-time nonlinear stochastic control system. These results include integrated optimal control and parameter estimation. They are obtained based on the principle of model-reality differences, where the reality is referred to as the real plant dynamic of the nonlinear optimal control problem (also called the real optimal control problem), while the model is the linear model involving the linear quadratic (LQ) optimal control problem. Clearly, solving only the LQ optimal control problem would not provide the optimal solution of the real optimal control problem. Thus, this system optimization is equipped with the iterative optimal parameter estimates so as to capture and to be adapted to the behaviour of the reality. Then, the optimum of the system optimization is updated iteratively in spite of the model-reality differences.

Here, we summarize our research objectives as follows.

- To review the existing approaches for solving the nonlinear stochastic optimal control problems.
- To develop an efficient and effective computational approach based on the principle of model-reality differences for a class of nonlinear discrete-time stochastic optimal control problems.
- To apply the optimal state estimate for the nonlinear state estimation.
- To design a feedforward-feedback optimal control law such that the dynamic of discrete-time stochastic system can be stabilized.
- To propose an effective computational methodology for discrete-time stochastic dynamic optimization.

In addition, the scope of our research covers

- Linear and nonlinear stochastic optimal control in discrete-time;
- Linear and nonlinear state estimation using Kalman filtering theory;
- Minimum output residual with the concept of the weighted least-square approach;
- Convergence, optimality and stability for the discrete-time stochastic dynamic optimization; and
- Stochastic modelling with the Gaussian white noises.

1.6 Significance of the Study

The computational algorithm derived based on the idea of the integrated optimal control and parameter estimation developed in this thesis provides a novel scheme to the control and optimization of the nonlinear stochastic optimal control problem in discrete-time. This novel scheme also generalizes the model-reality differences approach to cover a range of discrete-time nonlinear optimal control problems, both for deterministic and stochastic cases, based on the proposed modified linear optimal control theory.

The significance of this study includes

(a) *Comprehensive review of literature*

Various computational methods are reviewed. Most of the computational schemes are based on approximation. Others are based on the probability density function. This literature review leads us to the study in the thesis.

(b) *Algorithm development*

A class of integrated optimal control and parameter estimation (IOCPE) algorithms is developed based on the principle of model-reality differences. There are three sub-algorithms listed below.

(i) The ICEES algorithm, which looks for the expectation solution;

(ii) The ICEFS algorithm, which computes the filtering solution; and

(iii) The ICERS algorithm, which generates the real output solution.

They are coded in MATLAB version 7.0 (R14) and implemented in Microsoft Window XP, Pentium M, 496 MB.

(c) *Application of optimal state estimate*

In the presence of the random disturbances, the best state estimates are (i) the expected state when there is no observation; and (ii) the filtered state when the observation is available. The optimal state estimate generated from Kalman filtering theory is applied instead of using the extended Kalman filter. The online calculation of the filter gain is avoided during the iterative procedure. This will save the computation time of the state estimation.

(d) *Design of optimal control law*

A specific optimal control law, known as the feedforward-feedback optimal control law, is designed. The feedforward control corrects the differences between the real plant and the model employed, while the feedback control takes into consideration of the entire optimal state estimate. This combined optimal control law stabilizes the dynamic of the discrete-time stochastic system in closed-loop form.

(e) *Computational methodology for optimization*

The computational methodology integrates the computation of the adjustable parameters and the optimization of the modified model-based optimal control problem interactively in a unified framework. Hence, the model-based optimal control problem is solved instead of solving the original optimal control problem. As a result, the true optimal solution of the original optimal control problem could be obtained in spite of the model-reality differences. This methodology is effective for the discrete-time stochastic optimal control problems.

1.7 Overview of Thesis

In the previous section, comprehensive introductions to the model-reality differences approach and the optimal control of stochastic dynamical systems were given. In particular, the evolution of the model-reality differences approach was reported and the discrete-time nonlinear stochastic optimal control problem was described. The purpose of this thesis is to present new algorithms based on the principle of model-reality differences for solving the discrete-time nonlinear stochastic optimal control problems. The literature review, the development of these algorithms and the theoretical analysis are briefly described below.

In Chapter 2, a brief introduction of the principle of model-reality differences is given. Various types of random processes are described and dynamic estimation, which is carried out from Kalman filtering theory, is discussed. In addition, the relation of smoothing and filtering is revealed. Furthermore, the optimal control of a nonlinear stochastic system is considered, where the stochastic value function and the stochastic Hamiltonian function are presented. For a linear stochastic control system, the dynamic regulator and its optimal control are derived. Finally, some recent results on stochastic control strategies are reviewed.

In Chapter 3, the principle of model-reality differences is explained through introducing an expanded optimal control problem. Applying the mean propagation equation to the state dynamic of the stochastic system, the stochastic control system is transformed into a nonlinear deterministic control system. On this basis, the **ICEES** algorithm with the linear quadratic regulator (LQR) optimal control model, similar to the DISOPE algorithm, is developed. It can then be used to obtain the true expected solution of the original optimal control problem.

In Chapter 4, the observation is considered available. The optimal state estimate is derived from Kalman filtering theory. The off-line computation of the filter gain and the corresponding state error covariance is performed before the iterative procedure begins. The extension of the principle of model-reality differences is carried out such that the **ICEFS** algorithm is developed. By using the linear quadratic Gaussian (LQG) optimal control model in the algorithm proposed, the true filtering solution of the original optimal control problem is obtained.

In Chapter 5, an improvement on the methodology discussed in Chapter 4 is made. The weighted least-square output residual is introduced and is combined with the cost functional of the model-based optimal control problem. Again, applying the principle of model-reality differences, the corresponding version of the **ICERS** algorithm is derived. It is important to note that the weighting matrix with the smallest value shall be determined. As such, the true real output solution of the original optimal control problem can be tracked. Eventually, the minimum output residual reduces the noisy level of the problem.

In Chapter 6, a theoretical analysis is carried out. Firstly, the optimal state estimator, which is a modification from Kalman filtering theory, is analysed. The stability, consistency and efficiency of the Kalman filter are discussed. Secondly, an analysis of the algorithm implementation is made. The optimality conditions and their algorithmic mapping are shown. Convergence and stability properties are also presented. Thirdly, the minimum output residual is analysed. This explains the advantages of the algorithm developed. At last, a confidence interval for the actual state and the actual output is constructed.

In Chapter 7, the main contributions of this thesis are summarized. The limitations of the algorithms developed are mentioned. Some interesting directions of the integrated optimal control and parameter estimation algorithms for future research are discussed.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The aim of this chapter is to give a detailed overview of the model-reality differences approach and the computational issues of stochastic optimal control problems. On this basis, efficient algorithms are developed based on the principle of model-references for solving nonlinear stochastic optimal control problems in discrete-time.

Firstly, this chapter begins with a discussion on the model-reality differences approach and its application to solve a class of nonlinear deterministic optimal control problems, where the true optimal solution of the nonlinear optimal control problem is obtained without having to solve the original optimal control problem. The resulting algorithm aims to satisfy the necessary optimality conditions, while solving the parameter estimation problem and optimizing the model-based optimal control problem iteratively. In this way, the parameters are iteratively adapted in response to the differences between the real plant and the model used. Then, the optimal solution of the model-based optimal control problem is updated iteratively.

Next, the chapter gives a brief description of various random processes. The optimal state estimation is carried out on the linear stochastic system *via* Kalman filtering theory. Furthermore, the optimal control of a nonlinear stochastic system is presented, where the stochastic value function and the stochastic Hamiltonian function are discussed. For a linear stochastic control system, the control law is derived using the linear quadratic Gaussian technique and the dynamic programming approach. The duality of the estimation and the control is given through the separation principle. In addition, some recent results on stochastic control strategies are reviewed. Finally, some concluding remarks are made and a summary showing the study direction of the thesis is given.

2.2 Nonlinear Optimal Control with Model-Reality Differences

Generally, applying the linear optimal control model to solve a nonlinear optimal control problem is a challenging task. However, if the function of the plant dynamic (also called as reality) is continuously differentiable, then a mathematical model can be formulated from the linearization of the plant dynamic. The solution of the linear optimal control model can be obtained readily. However, solving the linear optimal control problem does not provide the optimal solution to the nonlinear optimal control problem. It is important to fill the gap by reducing the differences between the reality and the model employed.

2.2.1 General Nonlinear Optimal Control Problem

Consider a general optimal control problem, referred to as the real optimal control problem (ROP), given below.

$$\min_{u(k)} J_p(u) = \varphi(x(N), N) + \sum_{k=0}^{N-1} L(x(k), u(k), k) \tag{2.1}$$

subject to

$$x(k+1) = f(x(k), u(k), k) \tag{2.2}$$

$$x(0) = x_0 \tag{2.3}$$

where $x(k) \in \Re^n$, $k = 0, \cdots, N$, is the real state sequence, $u(k) \in \Re^m$, $k = 0, \cdots, N-1$, is the real control sequence, and $x_0 \in \Re^n$ is a given vector.

$f:\Re^n \times \Re^m \times \Re \to \Re^n$ represents the real plant (also called reality). $J_p \in \Re$ is the real cost function, where $\varphi:\Re^n \times \Re \to \Re$ is the terminal cost and $L:\Re^n \times \Re^m \times \Re \to \Re$ is the cost under summation. Note that L is a general time-varying scalar function in terms of the state and control variables at each k for k = 0 to k = N – 1. It is assumed that all functions in (2.1) and (2.2) are continuously differentiable with respect to their respective arguments.

The Problem (ROP) is a general deterministic nonlinear optimal control problem. It satisfies the following necessary conditions for optimality (Young, 1969; Pontryagin *et al*, 1962; Hestenes, 1966; Leitmann, 1981).

$$\frac{\partial H(k)}{\partial u(k)} = \frac{\partial L(x(k),u(k),k)}{\partial u(k)} + \left(\frac{\partial f(x(k),u(k),k)}{\partial u(k)}\right)^T p(k+1) = 0 \tag{2.4}$$

$$p(k) = \frac{\partial H(k)}{\partial x(k)} = \frac{\partial L(x(k),u(k),k)}{\partial x(k)} + \left(\frac{\partial f(x(k),u(k),k)}{\partial x(k)}\right)^T p(k+1) \tag{2.5}$$

$$x(k+1) = \frac{\partial H(k)}{\partial p(k+1)} = f(x(k),u(k),k) \tag{2.6}$$

for $k = 0,\cdots,N-1$, with the boundary conditions

$$p(N) = \frac{\partial \varphi(x(N),N)}{\partial x(N)} \text{ and } x(0) = x_0.$$

Here, $p(N)$ is the final co-state and $x(0)$ is the initial state, and the function $H:\Re^n \times \Re^m \times \Re^n \times \Re \to \Re$ is the Hamiltonian function defined by

$$\begin{aligned} H(k) &= H(x(k),u(k),p(k+1),k) \\ &= L(x(k),u(k),k) + p(k+1)^T f(x(k),u(k),k) \end{aligned} \tag{2.7}$$

Equation (2.4) is called the stationary condition. The co-state equation (2.5) and the state equation (2.6) are coupled difference equations that define a two-point boundary-value problem (TPBVP) with the given initial state x_0 and the final co-state $p(N)$. These problems are, in general, very difficult to solve (Lewis and Syrmos, 1995). Consequently, numerical methods for solving Problem (ROP) are indispensable (Bryson and Ho, 1975; Lewis, 1986; Chapra, 2008; Kirk, 1970; Ascher *et al*, 1988; Bryson, 1999; Butcher 1964; Nocedal and Wright, 1999; Bryne, 2008;

Bazaraa *et al*, 2006; Hull, 2003). For example, the finite difference method and the shooting method are common approaches to solve TPBVP defined by (2.5) and (2.6). The approach proposed in (Teo *et al*, 1991) solves the problem as an optimization problem. In fact, this approach works for a much general class of problems, where constraints on the state and the control variables are allowed to appear in the problem formulation. The approach proposed in (Hargraves and Paris, 1991) is another approach to solve Problem (ROP) numerically. Others interested approaches are multiparametric quadratic programming (Tøndel, 2003) and Gauss pseudospectral transcription (Benson, 2005).

2.2.2 Linear Model-Based Optimal Control Problem

Because the structure of the real plant is complex, a linear optimal control problem is constructed and is solved instead of solving Problem (ROP). This linear optimal control problem is a linear quadratic regulator (LQR) optimal control model, which is constructed as a simplified model-based optimal control problem (MOP) given below.

$$\min_{u(k)} J_m(u) = \tfrac{1}{2} x(N)^T S(N) x(N) + \gamma(N)$$

$$+ \sum_{k=0}^{N-1} \tfrac{1}{2} (x(k)^T Q x(k) + u(k)^T R u(k)) + \gamma(k) \tag{2.8}$$

subject to

$$x(k+1) = Ax(k) + Bu(k) + \alpha(k) \tag{2.9}$$

$$x(0) = x_0 \tag{2.10}$$

where $\alpha(k) \in \Re^n$, $k = 0, \cdots, N-1$, and $\gamma(k) \in \Re$, $k = 0, \cdots, N$, are the adjustable parameters, while $A \in \Re^{n \times n}$ is a state transition matrix and $B \in \Re^{n \times m}$ is a control coefficient matrix. $J_m \in \Re$ is the model cost function, where $S(N) \in \Re^{n \times n}$ and $Q \in \Re^{n \times n}$ are positive semi-definite matrices, and $R \in \Re^{m \times m}$ is a positive definite matrix.

Notice that the adjusted parameters $\alpha(k)$ and $\gamma(k)$ are introduced to capture the nonlinear behavior of the reality and they are used to reduced the differences between the reality and the model used. When their values are zero, Problem (MOP) is actually a standard linear quadratic regulator (LQR) optimal control problem. In the literature, this linear optimal control problem is well studied. See, for example, (Slotine and Li, 1991; Speyer, 1986; Lee and Marckus, 1986; Chen, 1984; Kirk, 1970; Walsh, 1975; Bryson and Ho, 1975; Lewis, 1986).

The linear optimal control problem is much easier to be solved. It is because the corresponding TPBVP is a system of linear homogeneous differential equations. Unlike the general nonlinear TPBVP, it can be solved by using the transition matrix method or the backward sweep method (Bryson and Ho, 1975; Lewis, 1986). The transition matrix method is conceptually easy but the difficulty may arise during the computation of the inverse of the transition matrices. The backward sweep method is more popular (Teo *et al*, 1991) due to the assumption of a linear relationship between the state and the costate for the computational efficiency.

2.2.3 Principle of Model-Reality Differences

The structure of the reality in Problem (ROP) is nonlinear, while the structure of the model used in Problem (MOP) is linear. The methodology which is proposed to reduce the differences between the reality and the model used is known as the principle of model-reality differences. In this principle, Problem (MOP), instead of Problem (ROP), is solved in such a way that the true solution of Problem (ROP) is obtained despite model-reality differences by updating the adjustable parameters iteratively.

To be more specific, an expanded optimal control problem (EOP), which integrates the system optimization with the parameter estimation, is introduced as follows.

$$\min_{u(k)} J_e(u) = \tfrac{1}{2} x(N)S(N)x(N) + \gamma(N)$$

$$+ \sum_{k=0}^{N-1} \tfrac{1}{2} (x(k)^T Qx(k) + u(k)^T Ru(k)) + \gamma(k)$$

$$+ \tfrac{1}{2} r_1 \| u(k) - v(k) \|^2 + \tfrac{1}{2} r_2 \| x(k) - z(k) \|^2 \tag{2.11}$$

subject to

$$x(k+1) = Ax(k) + Bu(k) + \alpha(k) \tag{2.12}$$

$$x(0) = x_0 \tag{2.13}$$

$$\tfrac{1}{2} z(N)^T S(N)z(N) + \gamma(N) = \varphi(z(N), N) \tag{2.14}$$

$$\tfrac{1}{2} (z(k)^T Qz(k) + v(k)^T Rv(k)) + \gamma(k) = L(z(k), v(k), k) \tag{2.15}$$

$$Az(k) + Bv(k) + \alpha(k) = f(z(k), v(k), k) \tag{2.16}$$

$$z(k) = x(k) \tag{2.17}$$

$$v(k) = u(k) \tag{2.18}$$

where $v(k) \in \Re^m$, $k = 0, \cdots, N-1,$ and $z(k) \in \Re^n$, $k = 0, \cdots, N,$ are introduced to separate the control sequence and the state sequence in the optimization problem from the respective signals in the parameter estimation. The terms appearing with $r_1 \in \Re$ and $r_2 \in \Re$ are introduced to improve convexity and to aid convergence of the resulting iterative algorithm. It is important to note that the algorithm is to be designed such that the constraints $v(k) = u(k)$ and $z(k) = x(k)$ will be satisfied at the end of the iterations, assuming that the convergence is achieved. In this situation, the state $z(k)$ and the control $v(k)$ will be used in the computations related to the reality, which include parameter estimation and matching schemes. The corresponding state sequence $x(k)$ and the control sequence $u(k)$ will be reserved for optimizing the model-based optimal control problem.

2.2.4 Optimality Conditions

Now, let us define the Hamiltonian function of Problem (EOP) as follows.

$$\begin{aligned} H_e(k) &= \tfrac{1}{2}(x(k)^T Qx(k) + u(k)^T Ru(k)) + \gamma(k) \\ &\quad + \tfrac{1}{2} r_1 \| u(k) - v(k) \|^2 + \tfrac{1}{2} r_2 \| x(k) - z(k) \|^2 \\ &\quad + p(k+1)^T (Ax(k) + Bu(k) + \alpha(k)) \\ &\quad - \lambda(k)^T u(k) - \beta(k)^T x(k) \end{aligned} \tag{2.19}$$

Then, we append the system (2.12) and the additional constraints (2.14) – (2.18) to the cost function (2.11), in terms of $H_e(k)$, to define the augmented cost function as follows.

$$\begin{aligned} J'_e(u) &= \tfrac{1}{2} x(N)S(N)x(N) + \gamma(N) + p(0)^T x(0) - p(N)^T x(N) \\ &\quad + \xi(N)(\varphi(z(N), N) - \tfrac{1}{2} z(N)^T S(N)z(N) - \gamma(N)) \\ &\quad + \Gamma^T (x(N) - z(N)) \\ &\quad + \sum_{k=0}^{N-1} H_e(k) - p(k)^T x(k) + \lambda(k)^T v(k) + \beta(k)^T z(k) \\ &\quad + \xi(k)(L(z(k), v(k), k) - \tfrac{1}{2}(z(k)^T Qz(k) + v(k)^T Rv(k)) - \gamma(k)) \\ &\quad + \mu(k)^T (f(z(k), v(k), k) - Az(k) - Bv(k) - \alpha(k)) \end{aligned} \tag{2.20}$$

where $\lambda(k) \in \Re^m$, $k = 0, \cdots, N-1$, $\beta(k) \in \Re^n$, $k = 0, \cdots, N-1$, $\Gamma \in \Re^n$, $\xi(k) \in \Re$, $k = 0, \cdots, N$, and $\mu(k) \in \Re^n$, $k = 0, \cdots, N-1$, are the appropriate Lagrange multipliers.

According to the Lagrange multiplier theory, the first-order variation $\delta J'_e$ of the augmented cost function J'_e with respect to all variables shall be zero at a constrained minimum (Bryson and Ho, 1975; Lewis, 1986; Becerra, 1994). That is, $\delta J'_e = 0$. Hence, the variational calculus technique is applied to the augmented cost function (2.20) for deriving the necessary optimality conditions and these conditions are given below.

(a) Stationary condition

$\nabla_{u(k)} H_e(k) = 0$:

$$Ru(k) + B^T p(k+1) - \lambda(k) + r_1(u(k) - v(k)) = 0 \tag{2.21}$$

(b) Co-state equation

$p(k) = \nabla_{x(k)} H_e(k)$:

$$p(k) = Qx(k) + A^T p(k+1) - \beta(k) + r_2(x(k) - z(k)) \tag{2.22}$$

(c) State equation

$x(k+1) = \nabla_{p(k+1)} H_e(k)$:

$$x(k+1) = Ax(k) + Bu(k) + \alpha(k) \tag{2.23}$$

(d) Boundary conditions

$$p(N) = S(N)x(N) + \Gamma \text{ and } x(0) = x_0$$

(e) Parameter estimation equations

$$\varphi(z(N), N) - \tfrac{1}{2} z(N)^T S(N) z(N) - \gamma(N) = 0 \tag{2.24a}$$

$$L(z(k), v(k), k) - \tfrac{1}{2}(z(k)^T Q z(k) + v(k)^T R v(k)) - \gamma(k) = 0 \tag{2.24b}$$

$$f(z(k), v(k), k) - Az(k) - Bv(k) - \alpha(k) = 0 \tag{2.24c}$$

(f) Modifier equations

$$\nabla_{z(N)} \varphi - S(N) z(N) - \Gamma = 0 \tag{2.25a}$$

$$\lambda(k) + (\nabla_{v(k)} L - Rv(k)) + \left(\frac{\partial f}{\partial v(k)} - B \right)^T \hat{p}(k+1) = 0 \tag{2.25b}$$

$$\beta(k) + (\nabla_{z(k)} L - Qz(k)) + \left(\frac{\partial f}{\partial z(k)} - A \right)^T \hat{p}(k+1) = 0 \tag{2.25c}$$

with $\xi(k) = 1$ and $\mu(k) = \hat{p}(k+1)$.

(g) Separation of variables

$$v(k) = u(k),\ z(k) = x(k),\ \hat{p}(k) = p(k) \tag{2.26}$$

2.2.5 Relevant Problems from Integration

After satisfying the optimality conditions of Problem (EOP) defined by (2.21) – (2.26), the modified model-based optimal control problem (MMOP) is obtained as follows.

$$\min_{u(k)} J_{mm}(u) = \tfrac{1}{2} x(N)^T S(N) x(N) + \gamma(N) + \Gamma^T x(N)$$
$$+ \sum_{k=0}^{N-1} \tfrac{1}{2} (x(k)^T Q x(k) + u(k)^T R u(k)) + \gamma(k)$$
$$+ \tfrac{1}{2} r_1 \| u(k) - v(k) \|^2 + \tfrac{1}{2} r_2 \| x(k) - z(k) \|^2$$
$$- \lambda(k)^T u(k) - \beta(k)^T x(k) \tag{2.27}$$

subject to

$$x(k+1) = Ax(k) + Bu(k) + \alpha(k) \tag{2.28}$$

$$x(0) = x_0 \tag{2.29}$$

with the specified $\alpha(k)$, $\gamma(k)$, $\lambda(k)$, $\beta(k)$, Γ, $v(k)$ and $z(k)$ that are being calculated.

In addition, (2.24) defines the parameter estimation problem. From this problem, the adjustable parameters are determined by

$$\gamma(N) = \varphi(z(N), N) - \tfrac{1}{2} z(N)^T S(N) z(N) \tag{2.30a}$$

$$\gamma(k) = L(z(k), v(k), k) - \tfrac{1}{2} (z(k)^T Q z(k) + u(k)^T R u(k)) \tag{2.30b}$$

$$\alpha(k) = f(z(k), v(k), k) - Az(k) - Bv(k) \tag{2.30c}$$

and from the multipliers computation defined by (2.25), the multipliers are calculated from

$$\Gamma = \nabla_{z(N)} \varphi - S(N) z(N) \tag{2.31a}$$

$$\lambda(k) = -(\nabla_{v(k)} L - Rv(k)) - \left(\frac{\partial f}{\partial v(k)} - B \right)^T \hat{p}(k+1) \tag{2.31b}$$

$$\beta(k) = -(\nabla_{z(k)} L - Qz(k)) - \left(\frac{\partial f}{\partial z(k)} - A \right)^T \hat{p}(k+1) \tag{2.31c}$$

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