

PATH PLANNING FOR UNMANNED AERIAL VEHICLES
USING VISIBILITY LINE-BASED METHODS

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Abstract

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This thesis concerns the development of path planning algorithms for unmanned aerial vehicles (UAVs) to avoid obstacles in two- (2D) and three-dimensional (3D) urban environments based on the visibility graph (VG) method. As VG uses all nodes (vertices) in the environments, it is computationally expensive. The proposed 2D path planning algorithms, on the contrary, select a relatively smaller number of vertices using the so-called base line (BL), thus they are computationally efficient. The computational efficiency of the proposed algorithms is further improved by limiting the BL's length, which results in an even smaller number of vertices. Simulation results have proven that the proposed 2D path planning algorithms are much faster in comparison with the VG and hence are suitable for real time path planning applications. While vertices can be explicitly defined in 2D environments using VG, it is difficult to determine them in 3D as they are infinite in number at each obstacle's border edge. This issue is tackled by using the so-called plane rotation approach in the proposed 3D path planning algorithms where the vertices are the intersection points between a plane rotated by certain angles and obstacles edges. In order to ensure that the 3D path planning algorithms are computationally efficient, the proposed 2D path planning algorithms are applied into them. In addition, a software package using Matlab for 2D and 3D path planning has also been developed. The package is designed to be easy to use as well as user-friendly with step-by-step instructions.



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Chapter 1

Introduction

1.1 Motivation

Unmanned Aerial Vehicles (UAVs) are a vital means of performing hazardous missions in adversarial environments without endangering human life. They have been used for peaceful purposes in civilian applications such as weather forecasting, environmental research, search and rescue missions, observation during wildfire incidents and traffic control [3]. Fig. 1.1 illustrates a Pathfinder UAV used for environmental research. On the other hand, UAVs have also been used for warfare such as carrying out aerial reconnaissance and surveillance over the opponent's area or attacking strategic facilities in enemy territory. Fig. 1.2 shows an RQ-1 predator which is armed with missiles for combat purposes [1].



Figure 1.1: Pathfinder UAV used for environmental research.



Figure 1.2: A UAV, RQ-1 predator is equipped with missiles

Since UAV requires no human pilot, there is no loss to human life if it crashes or gets attacked during a mission. Besides, UAV also reduce operating costs because it does not require a highly trained pilot onboard as a manned aircraft does. The latter is cost-ineffective often caused by expensive investment needed as part of the pilot's training to cover advanced facilities such as buildings, flight simulators and support equipment including instrumentation, the cockpit and ejection systems. Therefore UAVs are by far the best way forward. In addition, with no human pilot, a UAV can be designed to achieve higher gravitational forces i.e. 50g [2], which results in relatively higher manoeuvrability (a human can sustain up to only 9g). A UAV with higher manoeuvrability may have better performance such as faster speed, smaller minimum turning radius and larger maximum roll angle and hold a higher probability of escaping from enemy's missile attack.

However, many current UAVs still involve a human-in-the-loop to oversee and control the UAVs' operation [4, 31]. This in turn requires a communication link through radio signals between the human operator and the UAV to transmit/receive the command/sensory signals over a frequency spectrum, which is often limited. Furthermore, the radio signal is vulnerable and might be jammed by opponents. In the event of a lost or interrupted signal, as the UAV is dependent on human operators' decisions, it would not be able to execute a mission as desired and to some extent, it

may crash. Thus, the dependency on human instructions through a communication link needs to be minimised or eliminated if possible. This requires the UAV to have the capability of making its own decisions based on the current state and circumstances of its surrounding environments. The capability of doing so will greatly enhance the autonomy of UAVs.

1.2 Autonomy in UAV

Current technologies are capable of operating a UAV in a relatively structured and known environment. However, in a dynamic environment where uncertainties exist such as obstacles that might pop-up during a mission, the technologies are insufficient due to the UAV's inability to make decisions by itself [32]. This requires a new concept called autonomy.

Autonomy means the capability of a UAV to make its own decision based on the information presently available captured by sensors, and potentially covers the whole range of the vehicle's operations with minimal human intervention [5]. Autonomy increases system efficiency because all decisions are executed onboard except for critical decisions such as launching a missile that have to be made by humans [30]. A UAV with autonomy would be able to execute a mission in environments with uncertainties. Furthermore, with autonomy, the UAV can perform a long duration mission, which is beyond the capability of human (operators). Autonomy covers the following areas [6]:

- i. sensor fusion
- ii. communications
- iii. path planning
- iv. trajectory generation
- v. task allocation and scheduling
- vi. cooperative tactics

Additionally, as introduced in [33], there are ten UAV autonomy levels known as Autonomous Control Level (ACL). The ACL and trends in UAV autonomy are illustrated in Fig. 1.3. The concept of ACL as a metric to describe the autonomy in

UAVs is widely accepted [31]. Readers are referred to [33] for a detailed description of ACL.

However, autonomy technology is still in its early stage, fairly undeveloped [5] and is the bottleneck for UAV development in the future [6]. The RQ-1 Predator as shown in Fig. 1.1 for example, at present, can perform up to only level 3 of ACL.

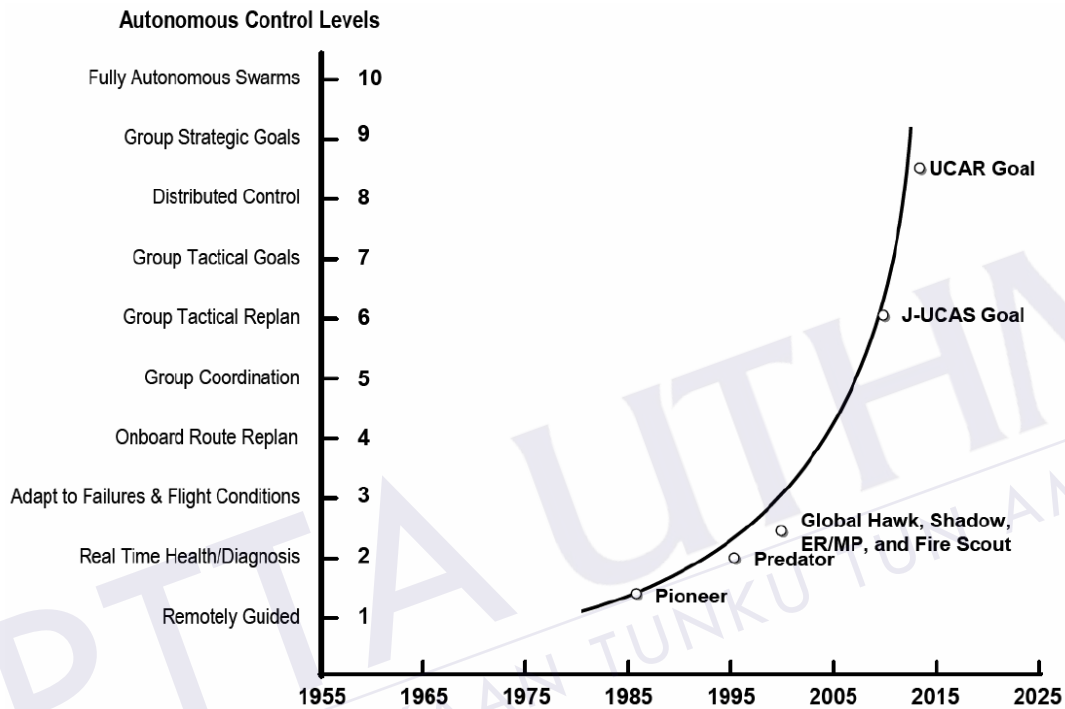


Figure 1.3: UAV autonomy levels and trend (adapted from [33])

The list of autonomy areas included previously, as well as the ACL (Fig. 1.3), have shown that onboard path planning and re-planning, which deals with traversing a vehicle through obstacles is one of the keys components of autonomy.

Research on UAV autonomy including path planning have progressed steadily since the beginning of this century. For example, [31] has designed and conceptually developed a simple UAV path planning mission that is used to reduce the UAV's dependency on human operators, and hence increases the UAV's autonomy level. The so-called Mission Management System (MMS) has been designed, developed and flight-tested in [31]. From sensory data, MMS makes decisions and issues high level commands which are then executed by the Flight Control Systems (FCS).

As path planning plays an important role in enhancing UAV's autonomy level, it has to be considered in the design of a UAV.

1.3 Path Planning Overview and Issues

From a technical perspective, path planning is a problem of determining a path for a vehicle in a properly defined environment from a starting point to a target point such that the vehicle is free from collisions with surrounding obstacles and its planned motion satisfies the vehicle's physical/kinematic constraints [25]. In a report by [12], path planning is associated with a number of terms as follows:

- ***Motion planning***

This term is frequently associated with manipulator robotics. It involves deliberative high level and low level planning of a way to move a robotic manipulator.

- ***Trajectory planning***

It is about planning the next movement of a robot. Trajectory planning is similar to motion planning.

- ***Navigation***

It is a very general term which has several meanings. In general it means "getting there from here". It is also part of path planning, motion planning, obstacle avoidance and localisation.

- ***Global path planning***

The planning is done prior to vehicle movement. It uses the information from the surrounding world to reach a target point from a starting point. As the information contains global data, the process is slow, but the planned path may be optimal.

- **Local navigation**

It is a process of avoiding obstacles by using only acquired data of the current surrounding environment. It is also a process of ensuring the vehicle's stability and safety and runs in real time using a reactive path planning approach.

1.3.1 Criteria of Path Planning

Path planning related problems have been extensively investigated and solved by many researchers [7-10], mostly focusing on ground robotics and manipulators. Important criteria for path planning that are commonly taken into account are the computational time, path length and completeness. A path planning algorithm with less computational time is vital in real time application, which is desirable in dynamic environments. The generated optimal path in terms of path length by a path planning technique will minimise UAV flight time and hence prolongs the UAV's endurance and life cycle, minimises fuel/energy consumption and reduces exposure to possible risks. On the other hand, a path planning approach satisfies the completeness criterion if it is able to find a path if one exists.

However, sometimes, there are trade-offs between such criteria. For example, a path planning method has to disregard the path's optimality in order to increase the computational efficiency. It means that finding a slightly longer path with less computational time may be preferable. On the other hand, higher computational complexity is necessary if an optimal path is required for some reasons. These criteria have to be considered before any path planning technique/algorithm design process takes place.

1.3.2 Path Planning Steps

Typically, path planning of a vehicle \mathcal{A} consists of two phases. The first phase is called the pre-processing phase in which nodes and edges (lines) are built in the environment/workspace \mathcal{W} with \mathcal{A} and obstacles O . In this phase, it is common to apply the concept of a configuration space (C -space) to represent \mathcal{A} and O in \mathcal{W} [9, 12]. In C -space, the vehicle's size is reduced to a point, and accordingly the obstacles' sizes are enlarged according to the size of \mathcal{A} . Next, representation techniques are used

to generate maps of graphs. Each technique differs in the way it defines the nodes and edges.

The second phase of path planning is termed the query phase in which a search for a path from a starting point to a target point is performed using (graph) search algorithms.

However there are path planning methods that can find solutions without graph search algorithm such as Mixed Integer Linear Programming (MILP) [4, 105, 116-117] and Evolutionary Algorithm (EA) [118-120].

1.3.3 C-space Representation

In path planning for an object, there are a number of methods that are commonly used to represent the environment including potential field (PF) [21-24], cell decomposition (CD) [13-16] and roadmap (RM) [17-20], to name a few. A PF represents the environment by modelling the object as a particle, moving under the influence of potential fields throughout the C -space. The field's magnitude at a particular point in C -space is determined by the fields generated by starting point p_{start} , target point p_{target} and the obstacles O in the C -space. The p_{start} and O are repulsive surfaces (which generate repulsive forces), while the p_{target} is the attractive pole which generates attractive forces [21]. The path is then calculated based on the resulting potential fields from a point with the highest magnitude of the resultant potential field, i.e. p_{start} , to a point with the minimum potential, i.e. p_{target} . The PF has several advantages such as the planning process is done as the vehicle moves and thus is suitable for real time application and the generated path is also smooth. However, conventional PF methods suffer from local minima causing the vehicle to become stuck before it reaches p_{target} , hence it might not satisfy the completeness criterion.

CD-based are among the most popular methods to represent the environment especially for outdoor scenarios [12] as it is the most straightforward technique [29]. This is due to the fact that the cells can represent anything such as free space or obstacles. The first step in CD is to divide the C -space into simple, connected regions termed cells [35]. The cells are regions that might be square, rectangular or polygonal in shape. They are discrete, non-overlapping but adjacent to each other. If the cell

contains obstacle (or part of obstacle), it is marked as occupied, otherwise it is marked as obstacle free. A connectivity graph is then constructed and a graph search algorithm is used to find a path throughout the cells from the starting point to the target point. In order to increase the quality of the path, the size of the cells has to be made smaller, which in turn increases the grid's resolution, and hence computational time. In the literature, there are several variants of CD. These include Approximate Cell Decomposition, Adaptive Cell Decomposition and Exact Cell Decomposition.

Path planning using RM-based methods on the other hand represent the environment by constructing graphs or maps from sets of nodes and edges. Path planning methods which are specific cases of RM are Voronoi diagrams (VD) and Visibility Graphs (VG). The nodes and edges to build a roadmap are defined differently for each method. VD defines nodes that are equidistant from all the points' surrounding obstacles. The paths generated from a graph by VD are relatively highly safe due to the fact that the edges of the paths are positioned as far as possible from the obstacles. However, the paths are inefficient [12] and not optimal in terms of path length. On the other hand, VG uses the vertices of the obstacles including the starting and target points in the C -space as the nodes. A VG (or visibility lines, VL) network is then formed by connecting pairs of mutually-visible nodes by a set of lines E . A pair of mutually-visible nodes means that those nodes can be linked by a line/edge $e \in E$ that does not intersect with any edge of obstacles in the C -space. Additionally, there is a cost associated with each E , possibly in terms of Euclidean distance. One advantage of VL is the capability of finding a path with the shortest length if one exists. A standard VL's computational complexity is $O(N^3)$ to find a path in a C -space with N nodes therefore VL is computationally intractable in the C -space with many obstacles.

1.3.4 Graph Search Algorithms

It has previously been stated that the second step of path planning is to calculate a path using (graph) search algorithms. Two basic search algorithms are Breadth-First Search (BFS) and Depth-First Search (DFS). BFS searches paths in a systematic way which guarantees that the first solution found will utilise the smallest number of iterations [34]. Like BFS, DFS is also systematic but it focuses on one direction and completely misses large portions of the C -space as the number of iterations become very large.

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