PATH PLANNING FOR UNMANNED AERIAL VEHICLES USING VISIBILITY LINE-BASED METHODS

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Abstract

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This thesis concerns the development of path planning algorithms for unmanned aerial vehicles (UAVs) to avoid obstacles in two- (2D) and three-dimensional (3D) urban environments based on the visibility graph (VG) method. As VG uses all nodes (vertices) in the environments, it is computationally expensive. The proposed 2D path planning algorithms, on the contrary, select a relatively smaller number of vertices using the so-called base line (BL), thus they are computationally efficient. The computational efficiency of the proposed algorithms is further improved by limiting the BL's length, which results in an even smaller number of vertices. Simulation results have proven that the proposed 2D path planning algorithms are much faster in comparison with the VG and hence are suitable for real time path planning applications. While vertices can be explicitly defined in 2D environments using VG, it is difficult to determine them in 3D as they are infinite in number at each obstacle's border edge. This issue is tackled by using the so-called plane rotation approach in the proposed 3D path planning algorithms where the vertices are the intersection points between a plane rotated by certain angles and obstacles edges. In order to ensure that the 3D path planning algorithms are computationally efficient, the proposed 2D path planning algorithms are applied into them. In addition, a software package using Matlab for 2D and 3D path planning has also been developed. The package is designed to be easy to use as well as user-friendly with step-by-step instructions.

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Chapter 1

Introduction

1.1 Motivation

Unmanned Aerial Vehicles (UAVs) are a vital means of performing hazardous missions in adversarial environments without endangering human life. They have been used for peaceful purposes in civilian applications such as weather forecasting, environmental research, search and rescue missions, observation during wildfire incidents and traffic control [3]. Fig. 1.1 illustrates a Pathfinder UAV used for environmental research. On the other hand, UAVs have also been used for warfare such as carrying out aerial reconnaissance and surveillance over the opponent's area or attacking strategic facilities in enemy territory. Fig. 1.2 shows an RQ-1 predator which is armed with missiles for combat purposes [1].

Figure 1.1: Pathfinder UAV used for environmental research.

Figure 1.2: A UAV, RQ-1 predator is equipped with missiles

Since UAV requires no human pilot, there is no loss to human life if it crashes or gets attacked during a mission. Besides, UAV also reduce operating costs because it does not require a highly trained pilot onboard as a manned aircraft does. The latter is cost-ineffective often caused by expensive investment needed as part of the pilot's training to cover advanced facilities such as buildings, flight simulators and support equipment including instrumentation, the cockpit and ejection systems. Therefore UAVs are by far the best way forward. In addition, with no human pilot, a UAV can be designed to achieve higher gravitational forces i.e. $50g$ [2], which results in relatively higher manoeuvrability (a human can sustain up to only $9g$). A UAV with higher manoeuvrability may have better performance such as faster speed, smaller minimum turning radius and larger maximum roll angle and hold a higher probability of escaping from enemy's missile attack.

However, many current UAVs still involve a human-in-the-loop to oversee and control the UAVs' operation [4, 31]. This in turn requires a communication link through radio signals between the human operator and the UAV to transmit/receive the command/sensory signals over a frequency spectrum, which is often limited. Furthermore, the radio signal is vulnerable and might be jammed by opponents. In the event of a lost or interrupted signal, as the UAV is dependent on human operators' decisions, it would not be able to execute a mission as desired and to some extent, it may crash. Thus, the dependency on human instructions through a communication link needs to be minimised or eliminated if possible. This requires the UAV to have the capability of making its own decisions based on the current state and circumstances of its surrounding environments. The capability of doing so will greatly enhance the autonomy of UAVs.

1.2 Autonomy in UAV

Current technologies are capable of operating a UAV in a relatively structured and known environment. However, in a dynamic environment where uncertainties exist such as obstacles that might pop-up during a mission, the technologies are insufficient due to the UAV's inability to make decisions by itself [32]. This requires a new concept called autonomy.

Autonomy means the capability of a UAV to make its own decision based on the information presently available captured by sensors, and potentially covers the whole range of the vehicle's operations with minimal human intervention [5]. Autonomy increases system efficiency because all decisions are executed onboard except for critical decisions such as launching a missile that have to be made by humans [30]. A UAV with autonomy would be able to execute a mission in environments with uncertainties. Furthermore, with autonomy, the UAV can perform a long duration mission, which is beyond the capability of human (operators). Autonomy covers the following areas [6]:

i. sensor fusion
ii. communications
iii. path planning
iv. trajectory generation
v. task allocation and scheduling
vi. cooperative tactics

Additionally, as introduced in [33], there are ten UAV autonomy levels known as Autonomous Control Level (ACL). The ACL and trends in UAV autonomy are illustrated in Fig. 1.3. The concept of ACL as a metric to describe the autonomy in

UAVs is widely accepted [31]. Readers are referred to [33] for a detailed description of ACL.

However, autonomy technology is still in its early stage, fairly undeveloped [5] and is the bottleneck for UAV development in the future [6]. The RQ-1 Predator as shown in Fig. 1.1 for example, at present, can perform up to only level 3 of ACL.

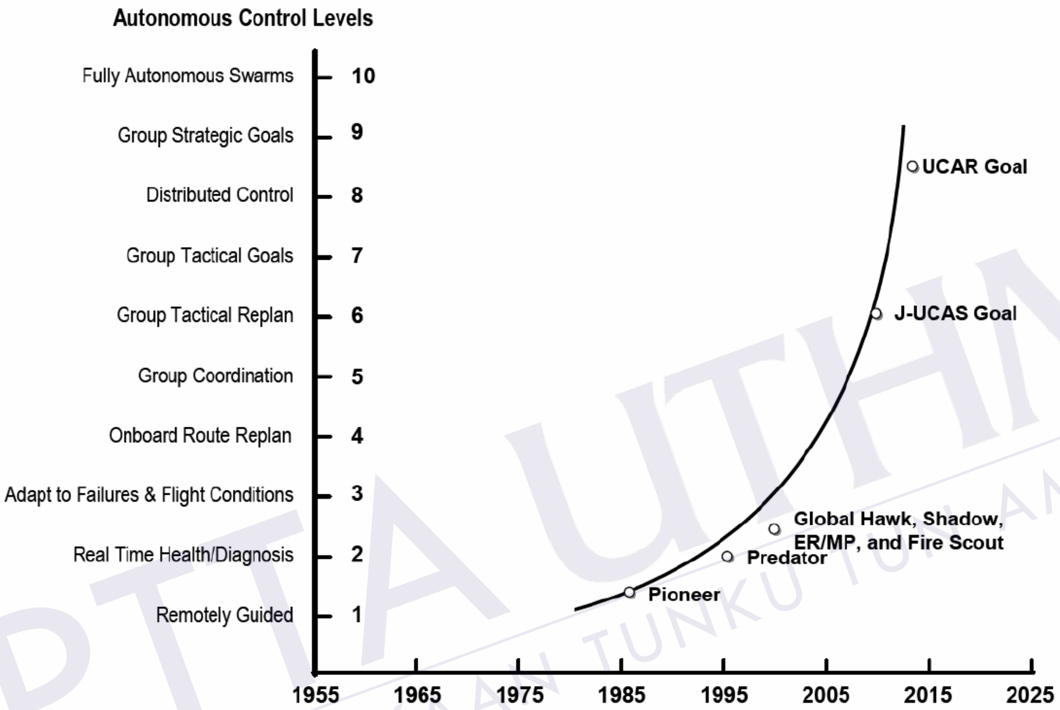

Figure 1.3: UAV autonomy levels and trend (adapted from [33])

The list of autonomy areas included previously, as well as the ACL (Fig. 1.3), have shown that onboard path planning and re-planning, which deals with traversing a vehicle through obstacles is one of the keys components of autonomy.

Research on UAV autonomy including path planning have progressed steadily since the beginning of this century. For example, [31] has designed and conceptually developed a simple UAV path planning mission that is used to reduce the UAV’s dependency on human operators, and hence increases the UAV’s autonomy level. The so-called Mission Management System (MMS) has been designed, developed and flight-tested in [31]. From sensory data, MMS makes decisions and issues high level commands which are then executed by the Flight Control Systems (FCS).

As path planning plays an important role in enhancing UAV's autonomy level, it has to be considered in the design of a UAV.

1.3 Path Planning Overview and Issues

From a technical perspective, path planning is a problem of determining a path for a vehicle in a properly defined environment from a starting point to a target point such that the vehicle is free from collisions with surrounding obstacles and its planned motion satisfies the vehicle's physical/kinematic constraints [25]. In a report by [12], path planning is associated with a number of terms as follows:

- ***Motion planning***
  This term is frequently associated with manipulator robotics. It involves deliberative high level and low level planning of a way to move a robotic manipulator.

- ***Trajectory planning***
  It is about planning the next movement of a robot. Trajectory planning is similar to motion planning.

- ***Navigation***
  It is a very general term which has several meanings. In general it means "getting there from here". It is also part of path planning, motion planning, obstacle avoidance and localisation.

- ***Global path planning***
  The planning is done prior to vehicle movement. It uses the information from the surrounding world to reach a target point from a starting point. As the information contains global data, the process is slow, but the planned path may be optimal.

- ***Local navigation***

  It is a process of avoiding obstacles by using only acquired data of the current surrounding environment. It is also a process of ensuring the vehicle's stability and safety and runs in real time using a reactive path planning approach.

1.3.1 Criteria of Path Planning

Path planning related problems have been extensively investigated and solved by many researchers [7-10], mostly focusing on ground robotics and manipulators. Important criteria for path planning that are commonly taken into account are the computational time, path length and completeness. A path planning algorithm with less computational time is vital in real time application, which is desirable in dynamic environments. The generated optimal path in terms of path length by a path planning technique will minimise UAV flight time and hence prolongs the UAV's endurance and life cycle, minimises fuel/energy consumption and reduces exposure to possible risks. On the other hand, a path planning approach satisfies the completeness criterion if it is able to find a path if one exists.

However, sometimes, there are trade-offs between such criteria. For example, a path planning method has to disregard the path's optimality in order to increase the computational efficiency. It means that finding a slightly longer path with less computational time may be preferable. On the other hand, higher computational complexity is necessary if an optimal path is required for some reasons. These criteria have to be considered before any path planning technique/algorithm design process takes place.

1.3.2 Path Planning Steps

Typically, path planning of a vehicle $\mathcal{A}$ consists of two phases. The first phase is called the pre-processing phase in which nodes and edges (lines) are built in the environment/workspace $\mathcal{W}$ with $\mathcal{A}$ and obstacles $\mathcal{O}$. In this phase, it is common to apply the concept of a configuration space ($\mathcal{C}$-space) to represent $\mathcal{A}$ and $\mathcal{O}$ in $\mathcal{W}$ [9, 12]. In $\mathcal{C}$-space, the vehicle's size is reduced to a point, and accordingly the obstacles' sizes are enlarged according to the size of $\mathcal{A}$. Next, representation techniques are used to generate maps of graphs. Each technique differs in the way it defines the nodes and edges.

The second phase of path planning is termed the query phase in which a search for a path from a starting point to a target point is performed using (graph) search algorithms.

However there are path planning methods that can find solutions without graph search algorithm such as Mixed Integer Linear Programming (MILP) [4, 105, 116-117] and Evolutionary Algorithm (EA) [118-120].

1.3.3 *C*-space Representation

In path planning for an object, there are a number of methods that are commonly used to represent the environment including potential field (PF) [21-24], cell decomposition (CD) [13-16] and roadmap (RM) [17-20], to name a few. A PF represents the environment by modelling the object as a particle, moving under the influence of potential fields throughout the *C*-space. The field's magnitude at a particular point in *C*-space is determined by the fields generated by starting point p_{start}, target point p_{target} and the obstacles O in the *C*-space. The p_{start} and O are repulsive surfaces (which generate repulsive forces), while the p_{target} is the attractive pole which generates attractive forces [21]. The path is then calculated based on the resulting potential fields from a point with the highest magnitude of the resultant potential field, i.e. p_{start}, to a point with the minimum potential, i.e. p_{target}. The PF has several advantages such as the planning process is done as the vehicle moves and thus is suitable for real time application and the generated path is also smooth. However, conventional PF methods suffer from local minima causing the vehicle to become stuck before it reaches p_{target}, hence it might not satisfy the completeness criterion.

CD-based are among the most popular methods to represent the environment especially for outdoor scenarios [12] as it is the most straightforward technique [29]. This is due to the fact that the cells can represent anything such as free space or obstacles. The first step in CD is to divide the *C*-space into simple, connected regions termed cells [35]. The cells are regions that might be square, rectangular or polygonal in shape. They are discrete, non-overlapping but adjacent to each other. If the cell contains obstacle (or part of obstacle), it is marked as occupied, otherwise it is marked as obstacle free. A connectivity graph is then constructed and a graph search algorithm is used to find a path throughout the cells from the starting point to the target point. In order to increase the quality of the path, the size of the cells has to be made smaller, which in turn increases the grid's resolution, and hence computational time. In the literature, there are several variants of CD. These include Approximate Cell Decomposition, Adaptive Cell Decomposition and Exact Cell Decomposition.

Path planning using RM-based methods on the other hand represent the environment by constructing graphs or maps from sets of nodes and edges. Path planning methods which are specific cases of RM are Voronoi diagrams (VD) and Visibility Graphs (VG). The nodes and edges to build a roadmap are defined differently for each method. VD defines nodes that are equidistant from all the points' surrounding obstacles. The paths generated from a graph by VD are relatively highly safe due to the fact that the edges of the paths are positioned as far as possible from the obstacles. However, the paths are inefficient [12] and not optimal in terms of path length. On the other hand, VG uses the vertices of the obstacles including the starting and target points in the C-space as the nodes. A VG (or visibility lines, VL) network is then formed by connecting pairs of mutually-visible nodes by a set of lines E. A pair of mutually-visible nodes means that those nodes can be linked by a line/edge $e \in E$ that does not intersect with any edge of obstacles in the C-space. Additionally, there is a cost associated with each E, possibly in terms of Euclidean distance. One advantage of VL is the capability of finding a path with the shortest length if one exists. A standard VL's computational complexity is $O(N^3)$ to find a path in a C-space with N nodes therefore VL is computationally intractable in the C-space with many obstacles.

1.3.4 Graph Search Algorithms

It has previously been stated that the second step of path planning is to calculate a path using (graph) search algorithms. Two basic search algorithms are Breadth-First Search (BFS) and Depth-First Search (DFS). BFS searches paths in a systematic way which guarantees that the first solution found will utilise the smallest number of iterations [34]. Like BFS, DFS is also systematic but it focuses on one direction and completely misses large portions of the C-space as the number of iterations become very large.

References

[1] http://www.richard-seaman.com
/Aircraft/AirShows/Nellis2006/Highlights/Predator2006.jpg

[2] W. A. Kamal. Safe trajectory planning techniques for autonomous air vehicles. PhD Thesis, 2005.

[3] http://www.theregister.co.uk/2006/08/30/uav_project

[4] J. S. Bellingham, M. Tillerson, A. G. Richards and J. P. How. Multi-Task Allocation and Path Planning for Cooperating UAVs. In *Proceedings of Conference on Cooperative Control and Optimization*, 2001.

[5] R. Frampton, UAV autonomy, In *MOD Codex Journal*, Issue 1 Summer, 2008, [Online]
www.science.mod.uk/codex/Issue1/Journals/documents/Issue1_2Journals_UAV_autonomy.pdf .

[6] http://www.theuav.com/

[7] Nilsson, N.J. A mobile automaton: An application of artificial intelligence techniques. In *Proceedings of International Joint Conference on Artificial Intelligence*, pages 509-520, 1969.

[8] Thompson,A.M., The navigation of the JPL robot. In *Proceedings of the 5th International Joint Conference on Artificial Intelligence*, pages 749- 57, 1977.

[9] T. Lozano- Perez and M. A. Wesley, An algorithm for planning collision-free paths among polyhedral obstacles, *Contmum*. *ACM*, 22, pages 560-570, 1979.

[10] A. Tokuta. Extending the VGRAPH algorithm for robot path planning. In *International Conference in Central Europe on Computer Graphics and Visualization*. 1998.

[11] A. Louchene, N. E. Bouguechal, A. Dahmani, S. Yahiaoui, and S. Merrouchi. Automated guided vehicle path planning without obstacles expansion. In *Proceedings*

of the IEEE International Conference on Control Applications, pages 1333-1337, 1998.

[12] J. Giesbrecht and Defence R&D Canada. Path planning for unmanned ground vehicles. *Technical Memorandum DRDC Suffield TM 2004-272*, 2004.

[13] R. Brooks and T. L.-Perez. A subdivision algorithm in configuration space for findpath with rotation. In *Proceedings of the 8th International Conference on AI*, pages 799-806, 1983.

[14] D. Zhu and J. Latombe. Constraint reformulation in hierarchical path planning. In *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 1918-1923, 1995.

[15]. H. Samet. An overview of quadtrees, octrees and related hierarchical data structures. *NATO ASI Series*, F40, pages 187-260. 1988.

[16]. H. N. T. Naniwa and S. Arimoto. A quadtree-based path-planning algorithm for a mobile robot. In *Robotic Systems*, pages 668-681, 1990.

[17] D. M. Coleman and J. T. Wunderlich. O3: An optimal and opportunistic path planner (with obstacle avoidance) using voronoi polygons. In *IEEE International Workshop on Advanced Motion Control*, pages 371-376. 2008

[18] Y. Qu, Q. Pan and J. Yan. Flight path planning of UAV based on heuristically search and genetic algorithms. In P*roceedings of the 31st Annual Conference of Industrial Electronics Society of IEEE*, pages 45-49, 2005.

[19] Q. Xiao, X. Gao, X. Fu and H. Wang. New local path re-planning algorithm for unmanned combat air vehicle. In *Proceedings of the 6th World Congress on Intelligent Control and Automation*, pages 4033-4037, 2006.

[20] A. Sud, E. Andersen, S. Curtis, M. C. Lin, and D. Manocha. Real-time path planning in dynamic virtual environments using multi-agent navigation graphs. In *IEEE Transactions On Visualization And Computer Graphics*, pages 526-538, 2008.

[21] O. Khatib. Real-time obstacle avoidance for manipulators and mobile robots. In *Proceedings of the IEEE International Conference on Robotics and Automation*, pages 500-505, 1985.

[22] Y. Kitamura, T. Tanaka, F. Kishino and M. Yachida. 3-D Path planning in a dynamic environment using an octree and an artificial potential field. In *Proceedings of the IEEE International Conference on Intelligent Robots and Systems*, pages 474-481, 1995.

[23] K. H. Yong and N. Ahuja. A potential field approach to path planning. In *IEEE Transactions on Robotics and Automation*, pages 23-32, 1992.

[24] A. Poty, M. Melchior and A. Oustaloup. Dynamic path planning for mobile robots using fractional potential field. In *First International Symposium on Control, Communications and Signal Processing*, pages 557-561, 2004.

[25] I. Hasircioglu, H. R. Topcuoglu and M. Ermis. 3-D path planning for the navigation of unmanned aerial vehicles by using evolutionary algorithms. In *Proceedings of the Conference on Genetic and Evolutionary Computation*, pages 1499-1506, 2008.

[26] P. B Sujit and R. Beard. Multiple UAV path planning using anytime algorithms. In *American Control Conference*, pages 2978-2983, 2009.

[27] Y. Kuwata and J. How. Three dimensional receding horizon control for UAVs. In *Proceedings of the AIAA Guidance, Navigation and Control Conference*, 2004.

[28] N. Vandapel, J. Kuffner and O. Amidi. Planning 3-D path networks in unstructured environments. In *Proceedings of the IEEE International Conference on Robotics and Automation*, pages 4624-4629, 2005.

[29] G. Dudek and M. Jenkin. Computational principles of mobile robotics. Cambridge University Press, Cambridge, UK, 2000.

[30] F. Mitch, Z. Tu, L. Stephens and G. Prickett. Towards true UAV autonomy. In *Proceedings of the IEEE International Conference on Information, Decision and Control*, pages 170-175, 2007.

[31] S. Karim, C. Heinze, and S. Dunn. Agent-based mission management for a UAV. In *Proceedings of the IEEE International Conference on Intelligent Sensors, Sensor Networks & Information Processing*, pages 481-486, 2004.

[32] C. Hai. A survey of autonomous control for UAV. In *Proceedings of the IEEE International Conference on Artificial Intelligence and Computational Intelligence*, pages 267-271. 2009.

[33] S. Eric. Evolution of a UAV autonomy classification taxonomy. In *Proceedings of the IEEE International Conference on Aerospace*, 2007.

[34] S. M. LaValle. Planning Algorithms, Cambridge University Press, 2006.

[35] S. J. Russell and P. Norvig. Artificial intelligence: A modern approach, 2nd Edition, Prentice Hall, Upper Saddle River, N.J., 2003.

[36] K. Yang and S. Sukkarieh. An enhanced optimization approach for generating smooth robot trajectories in the presence of obstacles. In *Proceedings of the 1995 IEEE European Chinese Automation Conference*, pages 263–268, 2008.

[37] J. Zhang and A. Knoll. Real time continuous curvature path planning of UAVs in cluttered environments. In *Proceedings of the 5th IEEE International Symposium on Mechatronics and its Applications*, pages 1-6. 2008.

[38] S. Aydin and H. Temeltas. A novel approach to smooth trajectory planning of mobile robot. In *IEEE International Workshop on Advanced Motion Control*, pages 472–477, 2008.

[39] H. K. Sung and R. Bhattacharya. Multi-layer approach for motion planning in obstacle rich environments. In *Conference and Exhibit of AIAA Guidance, Navigation and Control*, 2007.

[40] M. Shanmugavel, A. Tsourdos, B. White and R. Z Bikowski. Co-operative path planning of multiple UAVs using dubin paths with clothoid arcs. In *Control Engineering Practice*, Elsevier, 2009.

[41] H. Choset, G. Kantor, W. Burgard, L. Kavraki and S. Thrun. Principles of robot motion: Theory, algorithms, and implementations, The MIT Press, 2005.

[42] R. Siegwart and I. R. Nourbakhsh. Introduction to autonomous mobile robots, Bradford Company, Scituate, MA, USA, 2004.

[43] A. B. Doyle. Algorithm and computational techniques for robot path planning. PhD Thesis, 1995.

[44] B. Faverjon. Object level programming using an octree in the configuration space of a manipulator. In *Proceedings of the IEEE International Conference* on *Robotics and Automation San Francisco*, pages 1406-1412, 1986.

[45] J- C. Latombe. Robot motion planning, Kluwer Academic Publisher, 1991.

[46] H. Samet. Neighbor finding techniques for images represented by quadtrees. In *Computer Graphics and Image Processing*, pages 35-57, 1982.

[47] J. Kim and D. Kim. Visibility graph path planning using a quadtree, In *Proceedings of The 9th POSTECH-KYUTECH Joint Workshop On Neuroinformation*, pages 37-38, 2009.

[48] D. Chen, R. Szczerba, and J. Uhran. Planning conditional shortest paths through an unknown environment: A framed-quadtree approach. In *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 33-38, 1995.

[49] Y. Alex, S. Anthony, S. Singh and B. L. Barry. Framed-quadtree path planning for mobile robots operating in sparse environments. In *Proceedings, IEEE Conference on Robotics and Automation*, 1998.

[50] T. K. Priva and K. Sridharan. An efficient algorithm to construct reduced visibility graph and its FPGA implementation. In *Proceedings of I7th International Conference on VLSI Design*, pages 1057-1062, 2004.

[51] Y. Wang, G. S. Chirikjian. A new potential field method for robot path planning. In *Proceedings of the IEEE International Conference on Robotics and Automation*, pages 977-982, 2000.

[52] D. Chen, L. Zhan, X. Chen, Mobile robot path planning based on behaviour information potential field in unknown environments. In *Proceedings of the IEEE International Conference on Robotics and Biomimetics*, pages 683–687, 2004.

[53] D. Parsons and J. Canny. A motion planner for multiple mobile robots. In *Proceedings of the IEEE International Conference on Robotics and Automation*, pages 8-13, 1990.

[54] M. Jun and R. D'Andrea. Path planning for unmanned aerial vehicles in uncertain and adversarial environments, cooperative control: Models, applications and algorithms, Kluwer, 2002.

[55] T. Arney. An efficient solution to autonomous path planning by approximate cell decomposition. In *Proceedings of the IEEE International Conference on Information and Automation for Sustainability*, pages 88-93, 2007.

[56] N. S. V. Rao. Algorithmic framework for learned robot navigation in unknown terrains. In *Journal of Computer*, 22(6), pages 37-43, 1989.

[57] T. Guang, M. Bertier and A-M Kermarrec. Visibility graph-based shortest-path geographic routing in sensor networks. In *Proceeding of the IEEE International Conference on Computer Communications*, pages 1719-1727, 2009.

[58] S. M. LaValle. Rapidly-exploring random trees: A new tool for path planning. TR 98-11, Computer Science Dept., Iowa State University, October 1998.

[59] J. T. Schwartz and M. Sharir. A survey of motion planning and related Geometric algorithms. *Artificial Intelligence*, MIT Press, pages 157-169, 1988.

[60] S. Akishita, T. Hisanobu and S. Kawamura. Fast path planning available for moving obstacle avoidance by use of laplace potential. In *Proceedings of the IEEE International Conference on Intelligent Robots and Systems*, pages 673-678, 1993.

[61] F. Lingelbach. Path planning using probabilistic cell decomposition. In *Proceedings of the IEEE International Conference on Robotics and Automation*, pages 467-472, 2004.

[62] L. Zhang, J. Y. Kim and D. Manocha. A hybrid approach for complete motion planning. In *Proceedings of the 2007 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 7-14, 2007.

[63] J. O. Kim and P. K. Khosla. Real-time obstacles avoidance using harmonic potential functions. *IEEE Transactions on Robotics and Automation*, 8(3), pages 338-349, 1992.

[64] S. A. Bortoff. Path planning for UAVs. In *Proceedings of the American Control Conference Chicago Illinois*, pages 364-368, 2000.

[65] S. Garrido, L. Moreno, M. Abderrahim and F. Martin. Path planning for mobile robot navigation using voronoi diagram and fast marching. In *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 2376-2381, 2006.

[66] P. Khosla and R. Volpe. Superquadratic artificial potentials for obstacle avoidance and approach. In *Proceedings of the IEEE Conference on Robotics and Automation,*1988.

[67] L. E Kavraki, P. Svestka, J-C. Latombe and M. H. Overmars. Probabilistic roadmaps for path planning in high-dimensional configuration spaces. In *IEEE Transactions on Robotics and Automation*, 12(4) pages 566-580, 1996.

[68] B. J. Oommen, S. Iyengar, N. S. V. Rao and R. L. Kashyap. Robot navigation in unknown terrains using learned visibility graphs. Part I: The disjoint convex obstacle case. In *IEEE Journal of Robotics and Automation*, RA-3(6), pages 672-681, 1987.

[69] C. Godsil and G. Royle. Algebraic Graph Theory, Springer, 2001.

[70] A. Louchene, N-E. Bouguechal, A. Dahmani, S. Yahiaoui and S. Merrouchi. Automated guided vehicle path planning without obstacles expansion, In *Proceedings of the IEEE International Conference on Control Applications*, pages 1333-1337, 1998,

[71] K. Jiang, L .D. Seneviratne and S.W. E. Earle. Finding the 3D shortest path with visibility graph and minimum potential energy. In *Proceedings of the IEE/RSJ International Conference on Intelligent Robots and Systems*, pages 679-684, 1993.

[72] D. Wooden and M. Egerstedt. Oriented visibility graphs: low-complexity planning in real-time environments. In *Proceedings of the IEEE International Conference on Robotics and Automation*, pages 2354-2359, 2006.

[73] E. F. Moore. The shortest path through a maze. *Proceedings of an International Symposium on the Theory of Switching*. Cambridge: Harvard University Press, pages 285-292, 1957.

[74] H. P. Huang and S. Y. Chung. Dynamic visibility graph for path planning. In *Proceedings of 2004 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 2813-2818, 2004.

[75] J.A Janet, R. C. Luo and M. G. Kay. The essential visibility graph: An approach to global motion planning for autonomous mobile robots. In *Proceedings of the IEEE International Conference on Robotics and Automation,* pages 1958-1963, 1995.

[76] G. Song, S. Thomas and N. M. Amato. A general framework for PRM motion planning. In *Proceedings of the IEEE International Conference on Robotics & Automation,* pages 4445-4450, 2003.

[77] K. Belghith, F. Kabanza, Hartman and R. Nkambou. Anytime dynamic path-planning with flexible probabilistic roadmaps. In *Proceedings of the IEEE International Conference on Robotics and Automation*, pages 2372-2377, 2006.

[78] E. Plaku, K. E. Bekris, B. Y. Chen, A. M. Ladd, and L. E. Kavraki. Sampling-based roadmap of trees for parallel motion planning, In *IEEE Transactions on Robotics*, 21(4), pages 597-608, 2005.

[79] T. Siméon, J-P. Laumond and C. Nissoux. Visibility-based probabilistic roadmaps for motion planning. In *Advanced Robotics* 14(6). Pages 477-494, 2000.

[80] P. O. Pettersson and P. Doherty. Probabilistic roadmap based path planning for an autonomous unmanned aerial vehicle. In *Proceedings of the Workshop on Connecting Planning and Theory with Practice. 14th International Conference on Automated Planning and Scheduling*, 2004.

[81] Y. Tian, L. Yan, G. Y. Park, S. H. Yang, Y. S. Kim and S. R. Lee, C-Y. Lee. Application of RRT-based local path planning algorithm in unknown environment. In *International Symposium on Computational Intelligence in Robotics and Automation,* pages 456-460, 2007.

[82] J. Kim and J. P. Ostrowski. Motion planning of aerial robot using rapidly-exploring random trees with dynamic constraints. In *Proceedings of the IEEE International Conference on Robotics &Automation*, pages 2200-2205, 2003.

[83] N. A. Melchior and R. Simmons. Particle RRT for path planning with uncertainty. In *Proceedings of the IEEE International Conference on Robotics and Automation*, pages 1617-1624. 2007.

[84] S. Kamio and H. Iba. Random sampling algorithm for multi-agent cooperation planning. In *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 1265-1270, 2005.

[85] R. Pepy and A. Lambert. Safe Path Planning in an Uncertain-Configuration Space using RRT. In *Proceedings of the 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 5376-5381, 2006.

[86] M. Shao and J. Y. Lee. Development of autonomous navigation method for non-holonomic mobile robots based on the generalized voronoi diagram. In *Proceedings of the IEEE International Conference on Control Automation and Systems*, pages 309-313, 2010.

[87] Q. Zhang and X. Wang. Global path planning method in uncertain environment. In *Proceedings of the IEEE International Conference on Control Applications*, pages 2725-2730, 2006.

[88] P. Bhattacharya and M. L. Gavilova. Voronoi diagram in optimal path planning. In *Proceedings of the IEEE International Symposium on Voronoi Diagrams in Science and Engineering*, pages 38-47, 2007.

[89] R. Wein, J. P. Van and D. Halperin. The visibility-voronoi complex and its application. In *Computational Geometry: Theory Applications* 36(1), pages 66-87, 2007.

[90] R. Daily and D. M. Bevly. Harmonic potential field path planning for high speed vehicles. In *American Control Conference*, pages, 4609-4614, 2008.

[91] T. Ishida. Real-time search for autonomous agents and multiagent systems. In *Autonomous Agents and Multi-Agent Systems*. Kluwer Academic Publishers, pages 139-167, 1998.

[92] D. Glavaski, M. Volf and M. Bonkovic. Robot motion planning using exact cell decomposition and potential field methods. In *Proceedings of the WSEAS International Conference on Simulation, Modelling and Optimization*, pages 126-131, 2009.

[93] P. Broz. Path planning in combined 3D grid and graph environment. In *Proceedings of the 10th Central European Seminar on Computer Graphics*, 2006.

[94] M. N. Bygi and M. Ghodsi, 3D visibility graph. In *12th CSI Computer Conference*, 2006.

[95] C. H. Chung and G. N Saridis. Path planning for an intelligent robot by the extended VGraph algorithm. In *Proceedings of the IEEE International Symposium on Intelligent Control*, pages 544-549, 1989.

[96] P. Hart. A formal basis for the heuristic determination of minimum cost paths. In *IEEE Transactions on Systems Science and Cybernetics*, pages 100-107, 1968.

[97] E. W. Dijkstra. A note on two problems in connexion with graphs. In *Numerische Mathematik 1*, pages 269-271, 1959.

[98] R. J Szczerba, D. Z. Chen and K. S. Klenk. Minimum turns/shortest path problems: A framed-subspace approach. In *Proceedings of the IEEE International Conference on Systems, Man, and Cybernetics*, pages 398-403, 1997.

[99] J. L. Latombe. Motion planning: A journey of robots, molecules, digital actors, and other artefacts. In *International Journal of Robotics Research*, 18(11), pages 1119-1128, 1999.

[100] K. G. Joll, R. S. Kumar and R. Vijayakumar. A Bezier curve based path planning in a multi-agent robot soccer system without violating the acceleration limits. In *Elsevier Journal of Robotics and Autonomous Systems*, 57(2009), pages 23–33, 2009.

[101] K. Yang and S. Sukkarieh. An analytical continuous-curvature path-smoothing algorithm. In *IEEE Transactions on Robotic*, 26(3), pages 561-568, 2010.

[102] T. Kito, J. Ota, R. Katsuku, T. Mizuta, T. Arai, T. Ueyama and T. Nishiyama. Smooth path planning by using visibility graph-like method. In *Proceedings of the IEEE International Conference on Robotics and Automation*, pages 3770-3775, 2003

[103] G. Yang and V. Kapila. Optimal path planning for unmanned air vehicles with kinematic and tactical constraints. In *Proceedings of the IEEE Conference on Decision and Control*, pages 1301-1306, 2002.

[104] E. P. Anderson, R. W. Beard and T. W. McLain. Real-time dynamic trajectory smoothing for unmanned air vehicles. In *IEEE Transactions on Control Systems Technology*, 13(3), pages 471-477, 2005.

[105] A. Richards and J. P. How. Aircraft Trajectory Planning with Collision Avoidance Using Mixed Integer Linear Programming. In *Proceedings of the American Control Conference,* pages 1936-1941, 2002.

[106] L. Labakhua. Smooth trajectory planning for fully automated passengers vehicles–spline and clothoid based methods and its simulation. In *Proceedings of the International Conference on Informatics in Control, Automation and Robotics (ICINCO),* pages 89-96, 2006.

[107] K. Yang and S. Sukkarieh. Planning continuous curvature paths for UAVs amongst obstacles. *In Proceedings of the Australasian Conference on Robotics and Automation (ACRA)*, 2008.

[108] J-W. Choi, R. E. Curry and G. H. Elkaim. Continuous curvature path generation Based on bezier curve for autonomous vehicles. In *IAENG International Journal of Applied Mathematics* 40(2), 2010.

[109]. R. H. Bartels, J. C. Beatty and B. A. Barsky. An introduction to splines for use in computer graphics and geometry modeling. M. Kaufmann Publishers, 1987.

[110] M. Bak, N. K Poulsen and O. Ravn. Path following mobile robot in the presence of velocity constraints. In *Technical Report, Informatics and Mathematical Modelling, Technical University of Denmark*, Richard Petersens Plads, Building 321, DK-2800 Kgs. Lyngby, 2001.

[111] R. Omar and D-W Gu. Visibility line based methods for UAV path planning. In *Proceedings of the International Conference on Control, Automation and Systems (ICCAS-SICE)*, pages 3176-3181, 2009.

[112] R. Omar and D.-W. Gu. 3D path planning for unmanned aerial vehicles using visibility line based method. In *Proceedings of the International Control on Informatics in Control, Automation and Robotics*, pages 80-85, 2010.

[113] M. Kothari, I. Postlethwaite and D.-W. Gu. Multi-UAV path planning in obstacle rich environments using rapidly-exploring random trees. In *Proceedings of the IEEE Conference on Decision and Control, held jointly with the 28th Chinese Control Conference*, pages 3069-3074, 2009.

[114] P. Zarchan. Tactical and strategic missile guidance.In *Progress in Astronautics and Aeronautics (4th Ed.)*, 176, AIAA, 2002.

[115] Y. Baba and H. Takano. Robust flight trajectory tracking control using fuzzy logic. In *Proceedings of the 8th ISDG&A*, Maastricht, pages 68–75, 1998.

[116] A. G. Richards, J. P. How, T. Schouwenaars and E. Feron. Plume avoidance maneuver planning using mixed integer linear programming. In *Proceedings of Conference of AIAA Guidance, Navigation and Control*, 2001.

[117]Y. Kuwata and J. P. How. Robust cooperative decentralized trajectory optimization using receding horizon MILP. In *Proceedings of American Control Conference*, pages 522-527, 2007.

[118] I. K. Nikolos, N. C. Tsourveloudis and K. P. Valavanis. Evolutionary algorithm based off-line path planner for UAV navigation. *Automatika Journal*, 42(2001) 3-4, pages 143-150, 2001.

[119] D. Jia. Parallel evolutionary algorithms for UAV path planning. In *Proceedings of the AIAA 1st Intelligent Systems Technical Conference*, 2004.

[120] Y. Dadi, Z. Lei, R. Rong and X. Xiaofeng. A new evolutionary algorithm for the shortest path planning on curved surface. In *Proceedings of IEEE Conference on Computer-Aided Industrial Design and Conceptual Design*, pages 1-4, 2007.

[121] J. O'Rourke. Computational geometry in C (2nd Edition). Cambridge University Press, 1998.

[122] A. C. Kermode. Mechanics of flights (10th Edition). Prentice Hall, 1996.