

ELECTRIC FIELD STUDY OF SILICON RUBBER
INSULATOR USING FINITE ELEMENT METHOD
(SLIM)



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BORANG PENGESAHAN STATUS TESIS[◇]

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**ELECTRIC FIELD STUDY OF SILICON RUBBER INSULATOR USING
FINITE ELEMENT METHOD (SLIM)**

ROHAIZA BTE HAMDAN

**A project report submitted in partial fulfillments of the
requirements for the award of the degree of
Master of Engineering (Electrical – Power)**



**Faculty of Electrical Engineering
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APRIL 2006

I declare that my thesis entitled “*Electric Field Study of Silicon Rubber Insulator Using Finite Element Method (SLIM)*” is the result of my own research except as cited in references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.



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To my beloved parents, Hamdan Bin Ismail and Roziah Bte Khamis, and my lovely family members, thanks for your endless support and motivational inspiration

And also to my dearly fiance Mohd Hafiz Bin A Jalil @ Zainuddin, for your attention, devotion, cares and thanks for everything.



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ABSTRACT

Silicone rubber provides an alternative to porcelain and glass regarding to high voltage (HV) insulators and it has been widely used by power utilities since 1980's owing to their superior contaminant performances. Failure of outdoor high voltage (HV) insulator often involves the solid air interface insulation. As result, knowledge of the field distribution around high voltage (HV) insulators is very important to determine the electric field stress occurring on the insulator surface, particularly on the air side of the interface. Thus, concerning to this matter, this project would analyze the electric field distribution of energized silicone rubber high voltage (HV) insulator. For comparative purposes, the analysis is based on two conditions, which are silicon rubber insulators with clean surfaces and silicon rubber insulators with contamination layer taking place over its surfaces. In addition, the effect of water droplets on the insulator surface is also included. The electric field distribution computation is accomplished using SLIM software that performs two dimensions finite element method. The finding from this project shows that pollution layer distort the voltage distribution along the insulator surface while different pollution layer material and variation in zone of incidence would contribute different profile of electric field. Existence of water droplets would create field enhancement at the interface of the water droplet, air and silicon rubber material. Also, the intensification field created by water droplet is depending on the droplets size, number of droplets and the proximity of water droplets to each other.

ABSTRAK

Getah silikon memberikan alternatif kepada porselin serta kaca yang digunakan sebagai penebat voltan tinggi dan ia telah digunakan secara meluas oleh pembekal kuasa semenjak 1980-an memandangkan prestasinya yang baik semasa kehadiran bahan pencemar. Kegagalan penebat voltan tingi di kawasan terbuka pada kebiasaannya melibatkan bahagian di sempadan penebatan antara udara dan bahan penebat. Sehubungan dengan itu, informasi mengenai penyebaran medan disekitar penebat voltan tinggi adalah amat penting bagi menentukan tekanan medan elektrik yang terbentuk di atas permukaan penebat, terutamanya di bahagian udara pada sempadan antara penebat dan udara. Oleh yang demikian, merujuk kepada perkara tersebut, projek ini akan menganalisa penyebaran medan elektrik bagi penebat getah silikon voltan tinggi. Bagi tujuan perbandingan, analisa yang dilakukan adalah berdasarkan kepada dua situasi, getah silikon yang mempunyai permukaan yang bersih dan getah silikon yang mempunyai lapisan bahan pencemar di sepanjang bahagian permukaannya. Selain daripada itu, kesan titisan air yang terdapat di atas permukaan penebat juga dirangkumkan. Pengiraan bagi sebaran medan elektrik pada permukaan penebat disempurnakan menggunakan perisian SLIM yang melaksanakan kaedah elemen tak terhingga dua dimensi. Hasil daripada projek ini menunjukkan bahawa kehadiran lapisan pencemar memesongkan pengagihan voltan di sepanjang permukaan penebat sementara bahan pencemar yang berbeza serta variasi kepada zon yang terlibat akan menyumbang kepada profil medan elektrik yang berbeza. Kehadiran titisan air akan menghasilkan pertambahan medan di sempadan antara air, udara dan bahan getah silikon. Disamping itu, pertambahan tekanan medan yang dibentuk oleh titisan air adalah bergantung kepada saiz titisan, bilangan titisan dan jarak di antara satu titisan dengan titisan yang lain.

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LIST OF SYMBOLS / ABBREVIATIONS

D	-Distortion factor.
H	-Size of the element.
R	-Diameter of the largest circle in the element.
V	-Volt
m	-Meter
h_i	-Axial height
r_e	-Electrode radius
r_{ec}	-Electrode corner radius
r_i	-Core radius
r_{ic}	-Inner corner radius (the radius of curve fitting between shed and sheath)
r_o	-Shed radius
r_{oc}	-Outer corner radius (the radius of curve fitting between the upper and bottom shed)
E_{max}	-Maximum field at the surface
θ	-Shed slope angle (the slope angle of the upper shed)
ε	-Permittivity
$^\circ$	-Degree

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CHAPTER 1

INTRODUCTION

1.0 Introduction

This chapter would describe the overall overview of the project which includes the project objective, scope, project schedule and the outline of the thesis.

1.1 The Objective of the Project

The main objective of this project is to carry out a study on the electric field distribution of energized silicon rubber insulator under clean and contaminated condition using finite element method which is simulated by SLIM software.

1.2 The Scope of the Project

In order to limit this project under certain degree, the objectives of this project are assisted by certain scopes. Those scopes are as listed below:

- a) To appreciate the application of two dimensional linear finite element numerical method in electric field calculation.
- b) To observe and investigate the properties of silicon rubber.
- c) To implement the finite element method technique using SLIM.
- d) To model the contamination layer on the surface of silicon rubber insulator.
- e) To study the electric field pattern of silicon rubber insulator under clean and contaminated condition of energized silicon rubber insulator.

1.3 The Project Schedule

This project was accomplished in two consecutive phases which are Project I and Project II where Project II is the continuation from Project I. The theoretical part is being covered mostly within the Project I timeframe while Project II depict the simulation analysis of the project. Those project schedules are given separately by Appendix A.

1.4 Thesis Outline

This thesis is being divided into six consecutive chapters where each chapter review different issues regarding to the project objectives. Chapter 1 covers the introductory section of the project while Chapter 2 and Chapter 3 described the literature review and theoretical background that related to finite element method and silicon rubber respectively. The following chapter is Chapter 4 where this chapter provides the explanation on project methodology used throughout the operation of the project. Simulation results and analysis is explained individually in Chapter 5 and the last chapter, which is Chapter 6, considers the future recommendations in extending the project into a better prospect.



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CHAPTER 2

FINITE ELEMENT METHOD

2.0 Introduction

There are several methods for solving partial differential equation such as Laplaces and Poisson equation. The most widely used methods are Finite Difference Method (FDM), Finite Element Method (FEM), Boundary Element Method (BEM) and Charge Simulation Method (CSM). In contrast to other methods, the Finite Element Method (FEM) takes into accounts for the nonhomogeneity of the solution region. Also, the systematic generality of the methods makes it a versatile tool for a wide range of problems. The following topics in this chapter would describe briefly on the concept of Finite Element Method (FEM).



2.1 Historical Background of Finite Element Method

The ideas that gave birth to the Finite Element Method (FEM) evolved gradually from the independent contributions of many people in the fields of engineering, applied mathematics, and physics. Finite Element Analysis (FEA) was first termed by R.W. Clough in a paper published in 1960, but the roots of the theory relates back to the Ritz method of numerical analysis, first introduced in 1909.

The origins of the finite element method can be traced to two sources. A mathematician call Courant proposed the theoretical basis of the Finite Element Method (FEM), in the 1940s but his work was not followed up at that time. Later, practical Finite Element Analysis (FEA) was developed independently in the 1950s by Boeing engineers investigating structural dynamics problems in delta wing aircraft.

Hrenikoff (1941) found out that the elastic behavior of a physically continuous plate would be similar, under certain loading conditions, to a framework of physically separate one-dimensional rods and beams, connected together as discrete points. The problem then handled for trusses and frameworks with similar computational methods.

Courant's (1943) paper is a classic for finite element methods. To solve the torsion problem in elasticity, he defined piecewise linear polynomials over a triangularized region. Schoenberg's (1946) paper gave birth to the theory of splines, recommending the use of piecewise polynomials for approximation and interpolation. Synge (1957) used piecewise linear functions defined over triangularized region with a Reitz variational procedure.

With the introduction of high-speed digital computers, Langefors (1952) and Argyris (1954) took the framework analysis procedures and reformulated them into a matrix format suited for efficient automatic computation. McMahon (1953) solved a three-dimensional electrostatic problem using tetrahedral elements and linear trial functions. Polya (1954), Hersh (1955), and Weinberger (1956) used ideas similar to Courant's to estimate bounds for eigenvalues.

Turner *et al.* (1956) modeled the odd-shaped wing panels of high-speed aircraft as an assemblage of smaller panels of simple triangular shape. This was a breakthrough as it made it possible to model two- or three-dimensional structures as assemblages of similar two- or three-dimensional pieces rather than of one-dimensional bars. Greenstadt (1959) divided a domain into cells, assigned a different function to each cell, and applied a variational principle. White (1962) and Friedrichs (1962) used triangular elements to develop difference equations from variational principles. The name of the method, "*finite elements*", first appeared in Clough's (1960) paper. Melosh (1963), Besseling (1963), Jones (1964), and Fraeijs de Veubeke (1964) showed that the FEM could be identified as a form of the Ritz variational method using piecewise-defined trial functions. Zienkiewicz & Cheung (1965) showed that FEM is applicable to all field problems that could be placed in variational form.

By the early 70's, Finite Element Analysis (FEA) was limited to expensive mainframe computers generally owned by the aeronautics, automotive, defense, and nuclear industries. Since the rapid decline in the cost of computers and the phenomenal increase in computing power, Finite Element Analysis (FEA) has been developed to an incredible precision. Present day supercomputers are now able to produce accurate results for all kinds of parameters.

With the advent of micro-computers (personal computer and workstations) in the 1980's, however, the methods have become more widely used. During that time, a number of general purposes software packages have been developed. It is now

possible for engineers in virtually every industry to take advantage of this powerful tool.

In order to better conform to curve boundaries, curved finite elements have been widely used in recent years (Ertürk, 1995). Such elements are called the isoparametric elements (Zienkiewicz, 1971). Irregular computational grids have become increasingly popular for a wide variety of numerical modeling applications as they allow points to be situated on curved boundaries of irregularly shaped domains.

2.2 Finite Element Method (FEM) Application in Electrical Engineering

Numerical techniques have long been recognized as practical and accurate methods of field computation to aid in electrical design. Precursors to the Finite Element Method (FEM) are Finite Differences and Integral Equation techniques. Although all these methods have been used and continue to be used either directly or in combination with others for design, Finite Element Method (FEM) has emerged as appropriate techniques for low frequency applications. Recent literature also shows several applications in radio frequency (RF) scattering and propagation.

Since the late 1960's, when first applications of the so called "triangular finite differences" were made by A. Winslow to accelerator magnets, and the first real finite element solution of the scalar Helmholtz equation was presented by P. Silvester at the Alta Frequenza Conference, Finite Element Method (FEM) have advanced a great deal. Two-dimensional nonlinear magnetostatic techniques for electrical machines were first presented in the early seventies [1, 2, 3, 4].

These were followed by linear eddy current methods for telluric and magneto telluric geophysical prospecting and for evaluating eddy current losses in metallic slabs [5]. Various other applications of the steady state and transient eddy current methods soon followed [6], which included rotating machinery and power conditioners. Further applications included electronically commutated motors, electric furnaces and power lines. Coupled eddy current problems were modeled by finite elements [7]. Several papers have appeared on finite element modeling of solid state devices as shown in references. Applications in the high frequency area for radio frequency (RF) scattering and propagation problems have also been documented in the literature [8].

Alongside the above developments for practical engineering applications, techniques development had also taken place. To name a few: vector and scalar potential modeling, reduced scalar potential modeling with sources in the entire volume or only in permeable media, surface formulation for reduced scalar potentials; open boundary problems [9], 3D vector and scalar techniques for magneto static problem and 3D eddy current solutions [10]. With the increase in size of the geometry and its detailed modeling, the number of unknowns resulting from Finite Element modeling also increased by orders of magnitude. These have necessitated developments in equation solvers such as direct and iterative solvers, accelerated convergence methods, absorbing boundary conditions for RF problems to reduce problem size and others. Advances in computer aided engineering methods (CAE) include grid generation techniques, post processing methods and displays, and user interfaces for facilitating data input and file transfers. Another important development has been in the use of numerical integration methods for matrix assembly, and the use of isoparametric and sub parametric elements.

2.3 Definition of Finite Element Methods (FEM)

The Finite Element Method (FEM) is a numerical analysis technique used by engineers, scientists, and mathematicians to obtain solutions to the differential equations that describe, or approximately describe a wide variety of physical and non-physical problems. Physical problems range in diversity from solid, fluid and soil mechanics, to electromagnetism or dynamics.

The underlying premise of the method states that a complicated domain can be sub-divided into series of smaller regions in which the differential equations are approximately solved. By assembling the set of equations for each region, the behavior over the entire problem domain determined.

In other words, using the Finite Element Method (FEM), the solution domain is discretized into smaller regions called elements, and the solution is determined in terms of discrete values of some primary field variables ϕ (e.g. displacements in x, y z directions) at the nodes. The number of unknown primary field variables at a node is the degree of freedom at that node. For example, the discretized domain comprised of triangular shaped elements is shown below in Figure 2.1. In this example each node has one degree of freedom.

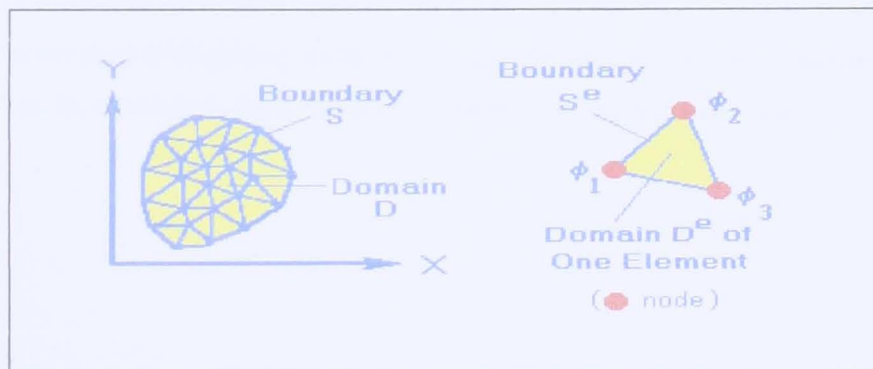


Figure 2.1 Typical finite element subdivisions of an irregular domain and typical triangular element

The governing differential equation is now applied to the domain of a single element (above right). At the element level, the solution to the governing equation is replaced by a continuous function approximating the distribution of ϕ over the element domain, expressed in terms of the unknown nodal values Φ_1 , Φ_2 , Φ_3 of the solution Φ . A system of equations in terms of Φ_1 , Φ_2 , and Φ_3 can then be formulated for the element.

Once the element equations have been determined, the elements are assembled to form the entire domain D . The solution $\Phi(x,y)$ to the problem becomes a piecewise approximation, expressed in terms of the nodal values of Φ . A system of linear algebraic equations results from the assembly procedure.

For practical engineering problems, it is not uncommon for the size of the system of equations to be in the thousands, making a digital computer a necessary tool for finding the solution. Furthermore, for most practical problems, it is impossible to find an explicit expression for the unknown, in terms of known functions, which exactly satisfies the governing equations and the boundary conditions. The purpose of the Finite Element Method (FEM) is to find an explicit expression for the unknown, in terms of known functions, which approximately satisfies the governing equations and the boundary conditions. However, the approximate solution may satisfy some of the boundary conditions exactly.

Each region is referred to as an element and the process of subdividing a domain into a finite number of elements is referred to as discretization. Elements are connected at specific points, called nodes, and the assembly process requires that the solution be continuous along common boundaries of adjacent elements.

2.4 Steps Included in Finite Element Method (FEM)

Theoretically, a Finite Element Method (FEM) analysis is composed by four different consequent steps. Those steps are as listed below:

- a) Discretizing the solution region into finite number of subregion or element.
- b) Deriving governing equation for a typical element.
- c) Assembling of all elements in the solution region.
- d) Solving the system of equations obtained.

In practical terms, such as using SLIM application, the Finite Element Method (FEM) analysis procedure consists of three steps. All of these steps basically, include four steps mentioned previously. These steps are:

- a) Pre-processing.
- b) Solution.
- c) Post-processing.

2.4.1 Pre-processing: Defining the Finite Element Model

Pre-processing, or model generation, is the most user-intensive part of the analysis. Perhaps up to 90% of the analyst's time is taken up creating the finite element mesh. In preprocessing, the analyst defines the geometry and material properties of the structure and the type of element to use. The finite element model, or mesh, is created by defining the shapes of element, the sizes of element and any variation of these throughout the model.

In most modern finite element programs, mesh generation is a two-stage process. The first stage is to create a solid model of the structural geometry in terms of geometrical entities such as points, lines, areas and volumes. Once the geometry is defined, the solid model is automatically discretized into a suitable finite element mesh using a variety of meshing tools. Usually, the mesh is created to give smaller elements in areas of stress concentration to enhance the accuracy of the solution.

Almost all finite element modeling is now done interactively. The analyst either types in commands from a keyboard or selects commands from a menu system using a mouse. The program executes these commands and stores the generated data in a file or database. Graphical representations of points, lines, nodes, elements, etc can then be viewed to ensure the model definition is correct.

2.4.2 Solution: Solving for Displacement, Stress, Strain etc.

In most of the types of analysis performed, the solution procedure is linear and straightforward. However, in non-linear problems or problems in which the boundary conditions vary with time (transient analysis) the user must define the loading history and define control parameters for the solution.

2.4.3 Post-processing: Reviewing Results in Text and Graphical Form

The results of the analysis are reviewed in the post-processing stage. Finite element programs (such as SLIM) generate huge amounts of data and it is convenient to use computer graphics to represent the results in graphical form, for example,

pictures of the deformed shape of a structure or contour plots of temperature variation in a thermal analysis.

2.5 Domain Discretization

An important step for the numerical simulation of physical phenomena is the transformation of the underlying differential equation into a finite dimensional space. In the considered domain the resulting partial differential equation is approximated using numerical methods on finite intervals. Determining the finite intervals requires a discretization of the domain. This discretization is in most cases obtained by generating a net. The nodes of the net consist of nodes on the domain boundary as well as nodes in the interior of the domain (Steiner points). The exact type of discretization is determined by the numerical solution method. While, especially in complex domains, finite element methods often use unstructured meshes (i.e. triangulations), difference methods require in any case block-structured nets, i.e. grids.

The solution quality of the numerical method is essentially influenced by the nature of the discretization, i.e. by the shape of the elements and the accurate approximation of the domain by the net.

For a continuous domain there is no natural subdivision, so the mesh pattern will appear somewhat arbitrary. The continuum would be replaced by a series of simple, interconnected elements whose force-displacement characteristics are relatively easy to compute.

In reality, these elements are connected to each other along their boundaries but in order to perform a theoretical approximation, the assumption is made that the elements are connected only at their nodes. Figure 2.2 below shows the deformation of two elements with nodal compatibility. Notice the excessive flexibility of the mesh.

This type of flexibility significantly reduces the accuracy of the approximation. However, the continuity requirement which forces the variation of the field variable along an element interface to be the same for adjacent elements is required within the finite element method. Finite elements of a continuum are special types of elements that are constrained to maintain overall continuity of the assemblage.



Figure 2.2 Deformation of two elements with nodal compatibility

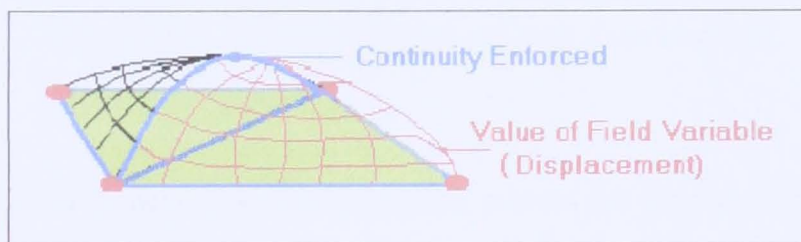


Figure 2.3 Deformation of two elements with Finite element Method (FEM)

2.5.1 Types of Elements

A wide variety of elements types in one, two, and three dimensions are well established and documented. It is up to the analyst to determine not only which types of elements are appropriate for the problem at hand, but also the density required to sufficiently approximating the solution. Engineering judgment is essential. In general, it is a geometrical shape (usually in solid color in modern programs) bounded by dots (nodes) connected by lines. Some solid and shell elements are illustrated in Figure 2.4 below.

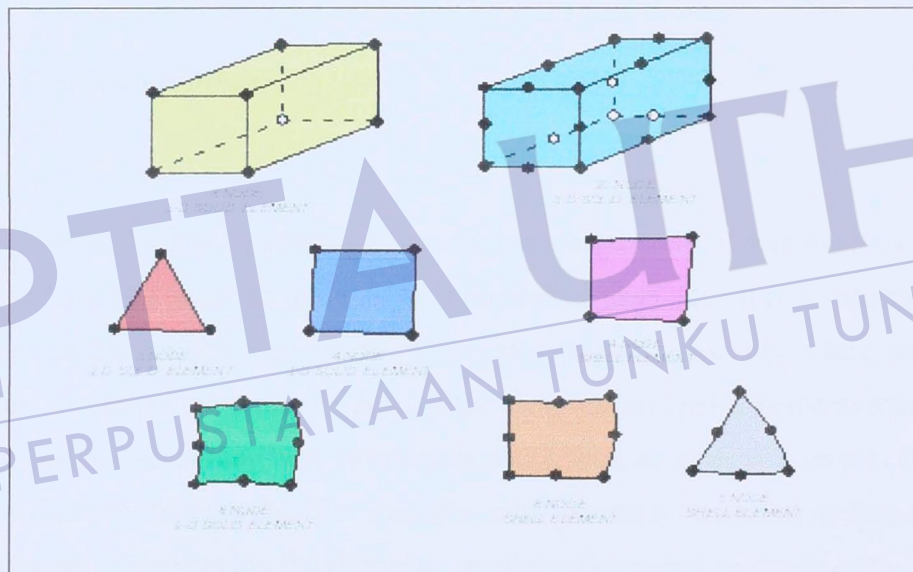


Figure 2.4 A variety of solid and shell finite elements.

All elements have nodes located at their vertices and some also have nodes along the edges, between the corner nodes. The number of nodes depends on the element order. Elements with corner nodes only must have straight edges. In the finite element theory of these elements, the edges are defined in terms of linear polynomial equations or linear shape functions. In reality, a real element of material may deform to a more complex shape and a number of elements must be used to capture the true deformation (a convergence problem).

Elements with nodes along the edges are based on higher order polynomial shape functions. Elements with one (mid-side) node between the corners are based on quadratic shape functions. Higher order elements can be used to define elements with straight or curved boundaries. When a quadratic element deforms, its sides must deform as quadratic curves in accordance with the quadratic polynomial form. Thus, a higher order element can capture more complex deformation behavior than a linear element and are consequently more accurate. This, however, is at the cost of more degrees of freedom and a bigger model to solve. And as with linear elements, a number of elements must be used to capture more complex deformations.

2.5.2 Continuous Mesh

The mesh must be continuous. A finite element mesh of a continuous region must not have any gaps due to improperly connected elements. If two volume elements are not disjoint then the set of the common points has to be a face, an edge or vertex of each of the element. Also, if the area, line and point elements touch any volume element, then they have to coincide with a face, an edge or a vertex of this volume element respectively. For example, suppose one is interested in discretizing a rectangular region having the following number of elements on the boundaries as shown in Figure 2.5.

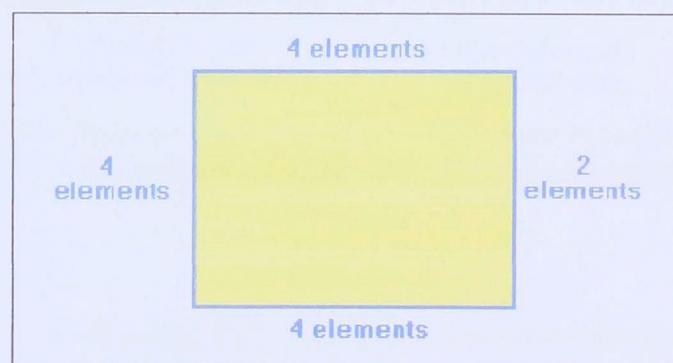


Figure 2.5 A rectangular region with the number of elements on the boundaries

Within the domain, a transition from four elements on the left to only two elements on the right is required. An incorrect mesh using four noded quadrilateral elements is shown below in Figure 2.6.

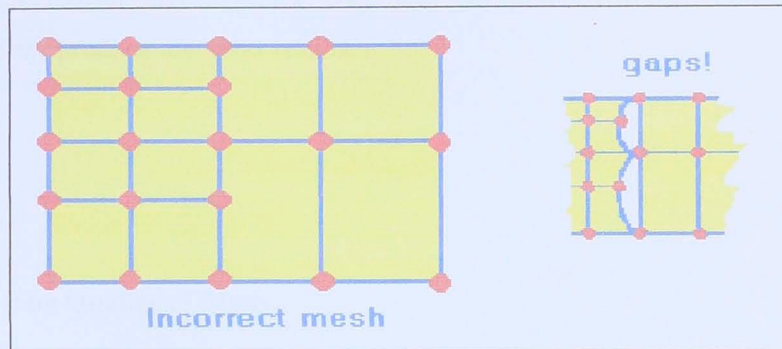


Figure 2.6 A discontinuous meshing within the rectangular region.

The gaps in the above mesh result from attempting to connect a node to an edge of an element, rather than to another element's node. In order to eliminate the gaps, elements must be connected node to node. Two correct possibilities are shown below in Figure 2.7.

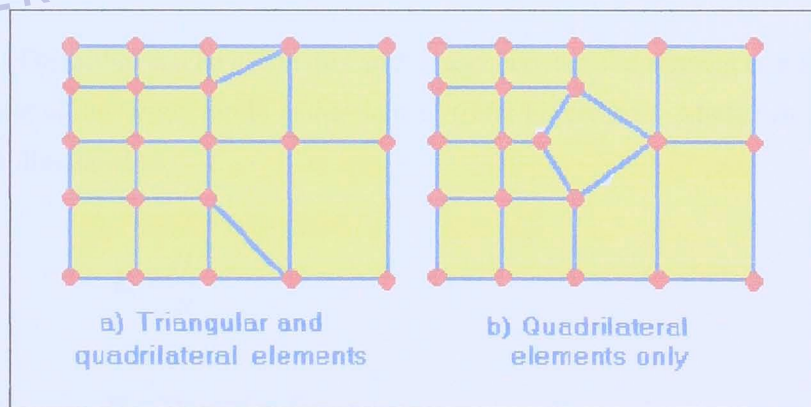


Figure 2.7 Examples of continuous meshing of a rectangular region.

Discretization of irregular shaped regions has traditionally been performed manually, from sketches. Today, software packages such as SLIM, automate the

meshing process. Caution is still advised, however, with any mesh-generation package, and the user's judgment and experience still remain important.

Once a finite element mesh has been created, it must be checked to ensure that each element satisfies certain criteria for acceptability, for example distortion, which may produce incorrect results.

2.5.3 The Quality of Mesh

The quality of mesh is defined as the level of distortion of the elements present in the mesh. The element quality is decided by many methods, out of which some are listed below:

- a) Skewness of the element.
- b) Aspect Ratio.
- c) Length and angle ratio.

Distortion can be calculated as the ratio between the element size and the diameter of the largest circle in the element (refer Figure 2.8). Mathematically, this can be illustrated as:

$$D = \frac{H}{R}$$

where

D = Distortion factor

H = Size of the element

R = Diameter of the largest circle in the element

There may be some highly distorted elements present, due to the geometric constraints. Distortion tends to be worse in the area of the curved in the region. If the element is highly distorted, it leads to following errors:

- High inaccuracy as the element has high influence in that domain.
- Low convergence as the solution oscillates over the exact solution.
- Erroneous elemental behavior with very high and very low stiffness with respect to the nodes.

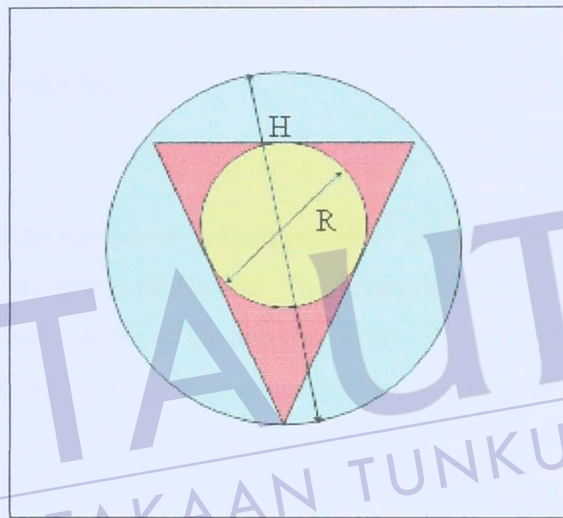


Figure 2.8 Mesh distortion of an element.

2.5.4 Node Numbering

The elements nodes can be labeled with separate sets of integers for identification. Since each element is related to several nodes, a node can be assigned a local label in the associated element in addition to its global label relative to the entire system. To relate the global node number, local node number and the element number, an integer array is introduced. This array is called the connectivity array $n(i,e)$. The $n(i,e)$ array contains the global node number indexed by the local node

number i and the element number e . this integer array includes all information concerning the numbering of the elements and nodes.

2.5.5 Element Interpolation

The edges of these elements are defined in terms of linear polynomial equations or linear shape functions. A one dimension linear polynomial has the form

$$\Phi = a + bx$$

The equation has two unknowns (a and b) and the straight line it represents is fully defined by two points on the line. Therefore, the straight edge of the element can be fully defined in terms of the two end nodes. When a linear element deforms, its sides must deform as straight lines in accordance with the linear polynomial form.

In reality, a real element of material may deform to a more complex shape and a number of elements must be used to capture the true deformation (a convergence problem). Elements with nodes along the edges are based on higher order polynomial shape functions. Elements with one (mid-side) node between the corners are based on quadratic shape functions. A one dimension quadratic polynomial has the form

$$\Phi = a + bx + cx^2$$

The equation has three unknowns (a , b and c) and the curved line it represents is fully defined by three points on the line. Therefore, the curved edge of the element

can be fully defined in terms of the two end nodes plus another node on the curve that usually located at the mid-point.

Higher order elements can be used to define elements with straight or curved boundaries. When a quadratic element deforms, its sides must deform as quadratic curves in accordance with the quadratic polynomial form. Thus, a higher order element can capture more complex deformation behavior than a linear element and are consequently more accurate. This, however, is at the cost of more degrees of freedom and a bigger model to solve. As with linear elements, a number of elements must be used to capture more complex deformations.

For two dimension linear polynomial, the most common form within an element is as given below.

$$\Phi(x,y) = a + bx + cy$$

for triangular element

$$\Phi(x,y) = a + bx + cy + dxy$$

for quadrilateral element

The function Φ is nonzero within the element but outside the element, it is equal to zero.

2.6 Element Governing Equation

For a linear triangular element as shown in Figure 2.8, there are three nodes located at the vertices of the triangle. The function Φ_1 , Φ_2 and Φ_3 respectively are:

$$\Phi_1(x,y) = a + bx_1 + cy_1$$

$$\Phi_2(x,y) = a + bx_2 + cy$$

$$\Phi_3(x,y) = a + bx_3 + cy_3$$

Where, x_j and y_j ($j = 1, 2, 3$) denotes the coordinate value of the j^{th} node in the element. Solving for the constant coefficient a , b and c in terms of Φ_j yields

$$\phi(x,y) = \sum_{j=1}^3 N_j(x,y)\phi_j$$

Where $N_j(x,y)$ is the interpolation or expansion function given by the equation below:

$$N_j(x,y) = \frac{1}{2A}(a_j + b_jx + c_jy) \quad j = 1, 2, 3.$$

with

$$\begin{array}{lll} a_1 = x_2y_3 - x_3y_2 & b_1 = y_2 - y_3 & c_1 = x_3 - x_2 \\ a_2 = x_3y_1 - x_1y_3 & b_2 = y_3 - y_1 & c_2 = x_1 - x_3 \\ a_3 = x_1y_2 - x_2y_1 & b_3 = y_1 - y_2 & c_3 = x_2 - x_1 \end{array}$$

and A is the area of the element.

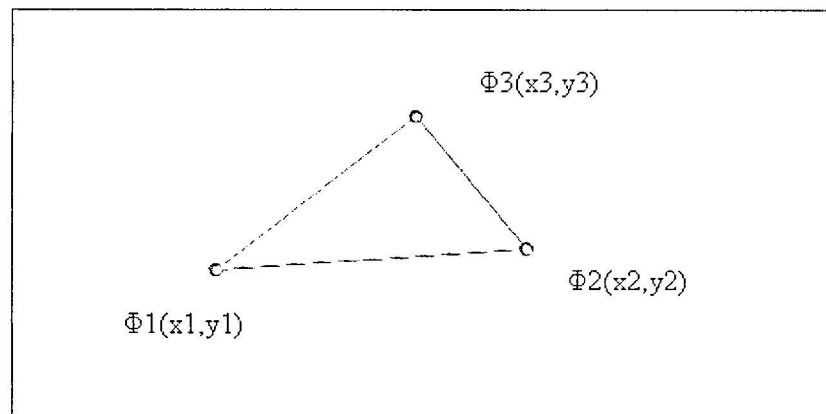


Figure 2.9 Linear triangular element.

2.6.1 Element Coefficient Matrix

The element coefficient matrix C is composed from the matrix elements C_{ij} . These matrix elements can be regarded as the coupling between nodes i and j . Its value is obtained from:

$$C_{ij} = \int \nabla N_j \cdot \nabla N_i dS$$

By combining the entire matrix element, the corresponding element coefficient matrix for element e , is represented by

$$C^{(e)} = \begin{bmatrix} C_{11}^{(e)} & C_{12}^{(e)} & C_{13}^{(e)} \\ C_{21}^{(e)} & C_{22}^{(e)} & C_{23}^{(e)} \\ C_{31}^{(e)} & C_{32}^{(e)} & C_{33}^{(e)} \end{bmatrix}$$

2.7 Assembling of All Elements

Having considered a typical element (i.e triangular element), the next step is to assemble all such elements in the solution region. The assembly procedure combines each element approximation of the field variable (as defined in the previous steps) to form a piecewise approximation of the behavior over the entire solution domain.

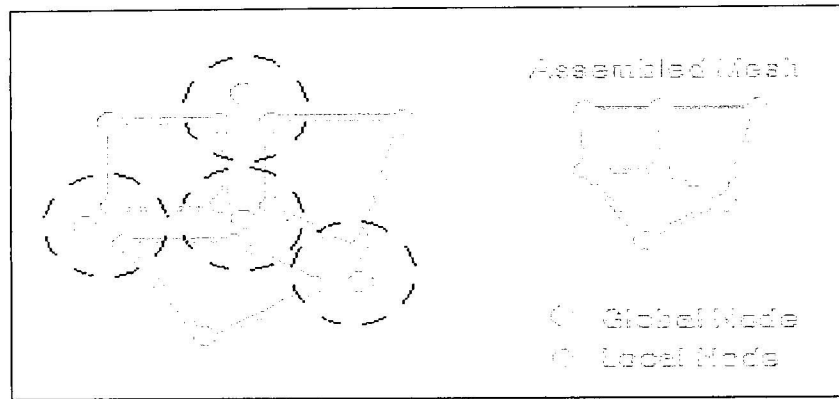


Figure 2.10 Assembling of all elements.

Assembly is accomplished using the following basic rule of compatibility, which stated that, the value of the field variable at a node must be the same for each element that shares that node. This step is handled automatically by the finite element package.

Theoretically, the process would involve the formation of a global coefficient matrix. This matrix is constructed by assembling individual element coefficient matrices. The global coefficient matrix is expected to have a form:

$$C = \begin{bmatrix} C_{11} & \dots & C_{1n} \\ \dots & & \dots \\ C_{n1} & \dots & C_{nn} \end{bmatrix} \quad i, j = 1, 2, \dots, n.$$

Where n = no of nodes. Again, C_{ij} is the coupling between nodes i and j . the global coefficient matrix have the following properties:

- It is symmetric ($C_{ij} = C_{ji}$).
- Since $C_{ij} = 0$ if no coupling exist between nodes i and j , for a large number of elements the global coefficient matrix will be sparse and banded.
- It is singular.

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