MODELLING AND CONTROL OF A BALANCING ROBOT USING DIGITAL STATE SPACE APPROACH

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JUDUL: MODELLING AND CONTROL OF A BALANCING ROBOT USING DIGITAL STATE SPACE APPROACH

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HERDAWATIE BINTI ABDUL KADIR

A project report submitted in partial fulfilment of the requirements for a award of the degree of Master of Engineering (Electrical-Mechatronics and Automatic Control)

Faculty of Electrical Engineering
Universiti Teknologi Malaysia

NOVEMBER, 2005

I declare that this thesis "Modelling and Control a Balancing Robot using Digital State Space Approach" is the result of my own research except for works that have been cited in the reference. The thesis has not been accepted any degree and not concurrently submitted in candidature of any other degree.

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Date : 30 NOVEMBER 2005

To my dearest mother, father and family for their encouragement and blessing

To my beloved classmate for their support and caring … … …

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ABSTRACT

This thesis is concerned with the problems of modelling a complete mathematical model of a balancing robot and control the system using digital state space approach to pilots the motors so as to keep the system in equilibrium. The research work is undertaken in the following development stages. In order to analyze and design the control system the dynamic of model of the system was first established in discrete-time. Then the difference equation approach is used to obtain the dynamic equations of an actual experimental test-rig. The dynamic of the DC motors as well as chassis and wheels of balancing robot are incorporated in the overall dynamic model, which is in the form of continuous state-space. Two type of controllers, namely pole placement controller and LQR controller are considered in this work. The performance and reliability of both controller will be determined by performing extensive simulation using MATLAB/SIMULINK as the platform.

ABSTRAK

Tesis ini berkenaan dengan masalah untuk memformulasikan model lengkap dinamik robot dan juga kawalan sistem menggunakan ruang digital yang boleh mengawal motor bagi mengekalkan keseimbangan system. Kajian ini telah dibahagikan kepada beberapa peringkat. Bagi menganalisa dan mereka kawalan sistem model dinamik system diambil kira dalam bentuk digital. Selepas itu kaedah persamaan perbezaan digunakan untuk memperolehi dinamik bagi platfom ujian experimen yang sebenar. Dinamik bagi motor DC juga kasis dan tayar bagi robot seimbang itu telah diaplikasikan didalam model dinamik ,dimana ia telah diubah kepada keadaan berterusan dan diubah kepada ruang digital. Dua jenis pengawal yang digunakan adalah pengawal penentuan kutub dan LQR. Prestasi dan kepercayaan bagi kedua-dua pengawal akan ditentukan melalui simulasi secara extensive menggunakan MATLAB/SIMULINK sebagai platform.

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LIST OF SYMBOLS

x	-	Displacement
$\dot{x}$	-	Displacement velocity
ϕ	-	Angle
$\dot{\phi}$	-	Angular velocity
θ	-	Parameters position
ω	-	Velocity
V_a	-	Applied torque
H_{∞}	-	H-infinity
k_m	-	Torque constant
τ_m	-	Motor torque
I	-	Current that flow through the armature circuit
k_e	-	Back emf constant
R	-	Lumped armature winding resistance
L	-	Self inductance of the armature winding
J_a	-	Moment inertia of the armature
k_f	-	Frictional constant
R	-	Radius of the wheels
Θ	-	Rotation angle of the wheels which is the same as the rotation angle of the armature
P_R	-	Reaction force between the wheel and the chassis of y-component of the force
H_R	-	Reaction force between the wheel and the chassis of x-component
H_{fR}	-	Friction force between ground and the wheel
M_w	-	Mass of the wheel

J_w	-	Moment inertia of the wheels
H_{fRr}	-	Conversion of translational force into rotational force
G	-	Gravity 9.81 m/s^2
J_P	-	Moment inertia of the robot chassis
l	-	Distance between the centre of the wheel and the robot's centre gravity
M_P	-	Mass of the robot's chassis

CHAPTER 1

INTRODUCTION

1.1 Overview

Balancing robots are characterised by the ability to balance on its two wheels and spin on the spot similar to inverted pendulum. The inverted pendulum problem is common in the field of control engineering thus the uniqueness and wide application of technology derived from this unstable system has drawn interest from many researches and robotics enthusiasts around the world. In recent years, researchers have applied the idea of a mobile inverted pendulum model to various problems like designing walking gaits for humanoid robots, robotic wheelchairs and personal transport systems.

This nonlinear control problem is surprisingly difficult to solve in a methodological approach due to two degrees of freedom, i.e, the balancing robot position and chassis angle using only one control input force. A practical problem with regard to control the balancing robot is similar to the concept designing a controller to swing the inverted pendulum up from a pendant position, achieve inverted stabilization, and simultaneously position.

In this thesis the balancing robots are characterized by the ability to balance on its two wheels and spin on the spot. The robot is composed of a chassis based on a stack of 130mm x 130mm Perspex plates carrying a Faulhaber DC motor, the Mark 4 Eyebot controller running on Robios version 5.2, a HOTEC GY-130 digital rate gyroscope, a SEIKA N3 digital inclinometer as described in (Thomas, 2002). The wheels of balancing robot are directly coupled to the output of the dc motor.

The balancing robot chasis is constructed from a single sheet of aluminium, drilled with holes for the easy mounting of motors, controller, sensors and battery pack. A pair of Faulhaber DC motors drive the robot's wheels. Each motor has a gear reduction of 54.2:1 and a torque constant of 6.9203×10^{-4} kg^{-m}/A. These motors have encapsulated encoders, and can be used to measure displacement and velocity of the robot. The robot is controlled by an EyeBot. A Mark 4 Eyebot controller running on Robios version 5.2 is used as the 'brain' of the balancing robot system.

The controller consists of a powerful 32-Bit microcontroller running at 33MHz, there is 512k ROM and 2048k RAM on board. The gyroscope modifies a servo control signal by an amount proportional to its measure of angular velocity. Instead of using the gyro to control a servo, we read back the modified servo signal to obtain a measurement of angular velocity. An estimate of angular displacement is obtained by integrating the velocity signal over time. The Inclinometer outputs an analogue signal, proportional to the angular displacement of the sensor (Braunl, 2002).

The balancing robot with two degree of freedom (DOF), is able to move along x, y axes describe by displacement, x and displacement velocity, $\dot{x}$ and chassis angle corresponding the angle, ϕ and angular velocity, $\dot{\phi}$. These four state space variable fully describe the dynamics of the 2 DOF system.

The balancing robot balance the load with its wheels while dragging the weight around on a pivot in a regular differential drive robot. This thesis will delve into the suitability and performance analysis of Pole Placement and LQR controllers in balancing the balancing robot in discrete-time environment.

1.2 Objective

The objectives of this research are as follows:

1. To formulate the complete mathematical dynamic model of the Balancing Robot using differential equation method.

2. To establish the state space model of the Balancing Robot using Digital State Space Approach.

3. To show mathematically that the Balancing Robot system is controllable and observable in discrete-time.

4. To design digital state feedback regulators for the Balancing Robot using pole placement approach and Optimal Controller (LQR).

5. To simulate the Balancing Robot continuous system and hybrid system using MATLAB-SIMULINK.

6. To demonstrate that the digital state space approach is as accurate as the continuous state space approach.

1.3 Scope of Project

The work undertaken in this project are limited to the following aspects:

1. Balancing Robot as described by Thomas Braunl (2002).

2. Digital State Space Approach as described by Richard J. Vaccaro (1995)

3. State feedback with Pole Placement Approach and Linear Quadratic Regulator.

4. Simulation on MATLAB-SIMULINK.

1.4 Research Methodology

The research work is undertaken in the following nine developmental stages:

1. Formulate the complete mathematical dynamic model using differential equation method.

2. Establish continuous state space mathematical model.

3. Linearization: Nonlinear equations of motion are linearized around the operating point .

4. Choose Sampling Interval.

5. Discretize the linearized continuous state space model to digital state space model.

6. Check the controllability and observability of ZOH Equivalent Models.

7. Design continuous-time and discrete-time state feedback controller using the pole placement method and LQR.

8. Verify the controller design of the balancing robot simulated on MATLAB SIMULINK.

9. Evaluate results

1.5 Literature Review

The research on balancing robot has gained momentum over the last decade. This is due to the nonlinear and inherent unstable dynamics of the system. The balancing problem extensively studied by numerous researchers (Mori, 1976). An understanding of how to control such a system will allow us to easily solve the other related control problems, such as single-link flexible manipulators (Yeung et al., 1990) and stabilization of a rocket booster by its own thrust vector. Below is several research that has been done by researches.

Yangsheng Xu (2004), developed a dynamic model for the single wheel robot and verified it through simulations and experiments. Using the linearization method, a linear state feedback approach to stabilize the robot at any desired lean angle was developed. This feedback provides means for controlling the steering velocity of the robot. Line following controller is developed for tracking any desired straight line while keeping balance. The controller is composed of two parts: the velocity control law and the torque control law. In the velocity control law, the velocity input (steering velocity) is designed for ensuring the continuity of the path curvature. Then, the robot can be stabilized for tracking a lean angle trajectory in which the steering velocity is identical to the desired value.

Henrik Niemann (2003) has derived a linear model of a double inverted pendulum system together with a description of the model uncertainties. For the double inverted pendulum system the trade-off between robust stability and performance is quite limited. There is not much space for reduction of the robustness to increase the performance of the system. The reason is the nonlinearities in the system together with the limitations/saturations in the system. The limitations in the system are e.g. maximal power to the motor (maximal acceleration of the cart), maximal length of the track for mention the two most important limitations.

Rich Chi Oii (2003) discusses the processes developed and considerations involved in balancing a two-wheeled autonomous robot based on the inverted pendulum model. The experimental examines the suitability and evaluates the performance of a Linear Quadratic Regulator (LQR) and a Pole-placement controller in balancing the system. The LQR controller uses several weighting matrix to obtain the appropriate control force to be applied to the system while the Pole placement requires the poles of the system to be placed to guarantee stability.

Felix Grasser (2002) has built a prototype of a revolutionary two-wheeled vehicle (JOE). The goal was to build a vehicle that could balance its driver on two coaxial wheels – a mobile, inverted pendulum. In order to reduce cost as well as danger for the test pilots it was decided on building a scaled down prototype carrying a weight instead of a driver. The control system used to guarantee stability of the system is based on two state space controllers, interfaced via a decoupling unit to the two DC motors driving the wheels. The performance of the system is shown that its have ability to reject force and angular disturbances as well as its capability of tracking a pilot's driving inputs. A control system varying the pole placement in real time depending on the states and inputs of the system has the potential to further increase JOE's performance. The implementation of these controllers can be seen in papers published by Nakajima et al. (1997), Shiroma et al. (1996), Takahashi et al. (2001) and Grasser et al (2002).

Chinichian (1990) design and analyze a controller for balancing one pendulum with two degrees of freedom, "spatial inverted pendulum". The pendulum, with two degrees of freedom, has a three dimensional motion, and it will be more analogous to the design of a controller for attitude control during launching a rocket. A full state-variable feedback controller design for a state-space linear model of a three dimensional inverted cart/pendulum system is presented. This design was based on pole-placement technique. Alternative solutions to the simple pole-placement technique were also proposed to exploit non-uniqueness of the feed-back gains for a certain closed-loop pole locations and the closed-loop system response was simulated on a digital computer.

Shiroma et al. (1996) presented the 'Cooperative Behaviour of a Wheeled Inverted Pendulum for Object Transportation' by showing the interaction of forces between objects and the robot by taking into account the stability effects due to these forces. This research highlights the possibility of cooperative transportation between two similar robots and between a robot and a human.

The rapid increase of the aged population in countries like Japan has prompted researchers to develop robotic wheelchairs to assist the infirm to move around (Takahashi et al. 2000). The control system for an inverted pendulum is applied when the wheelchair manoeuvres a small step or road curbs.

On a higher level, Sugihara et al. (2002) modelled the walking motion of a human as an inverted pendulum in designing a real time motion generation method of a humanoid robot that controls the centre of gravity by indirect manipulation of the Zero Moment Point (ZMP). The real time response of the method provides humanoid robots with high mobility.

1.6 Layout of Thesis

This section outlines the structure of the thesis.

Chapter 2 deals with the mathematical modelling of the balancing robot. The formulation of the integrated dynamic model of this robot is presented in detail. First, the state space representations of the chassis and wheel dynamics comprising of DC motors are formulated. In addition, the assumptions and limitations that been added to the model will be described.

Chapter 3 discusses control algorithm design for controlling balancing robot. Analysis regarding on performance of designed controller will be conducted.

Chapter 4 explains the discretization method that can be used in discretizing an analog plant.

Chapter 5 discusses the simulation results. The performance of the continous-time and discrete-time of the state feedback and LQR controller is evaluated by simulation study using Matlab/Simulink.

Chapter 6 concludes the topics and suggests recommendation for future works.

CHAPTER 2

MODELING OF A BALANCING ROBOT

2.1 Introduction

Modeling is the process of identifying the principal physical dynamic effects to be considered in analyzing a system, writing the differential and algebraic equations from the conservation laws and property laws of the relevant discipline, and reducing the equations to a convenient differential equation model (Robert, 1999). In order to develop the control system, mathematical model is establish to predict the behavior before applied into real system. Actually, the dynamics refer to a situation which is varying with time (Ernest, 1972). The dynamic performance of a balancing robot depends on the efficiency of the control algorithms and the dynamic model of the system.

This chapter focuses on the formulation of integrated mathematical model of a balancing robot in state variable form. The different equation approach is used to obtain the dynamic equations of an actual experimental test-rig. The DC motor, chassis and wheels of balancing robot are incorporated in the dynamic model, which is organized to a continues state-space model and then transform into the digital state space model form.

The formulation of an integrated mathematical dynamic model of balancing robot is presented in this chapter. In this thesis, a 2 DOF system will be modeled using four state space variable ,namely:.

x =displacement (m)

$\dot{x}$ =displacement velocity (m/s)

ϕ =angle (rad)

$\dot{\phi}$ =angular velocity (rad/s)

Some assumptions and limitations have been added in order to make the modeling of the balancing robot a reality. There are:

a. Motor inductance is neglected and the current through the winding is not considered in the equation of motion of the motor.

b. The wheels of the robot will always stay in contact with the ground

c. There is no slip at the wheels.

d. Cornering forces are also negligible.

2.2 MATHEMATICAL MODELING FOR A BALANCING ROBOT

Figure 2.1 shows an autonomous balancing robot described in Ooi (2003). The mathematical model of this balancing robot is derived by decomposing it into 3 elements, i.e the linear model of direct current motor, chassis and wheels.

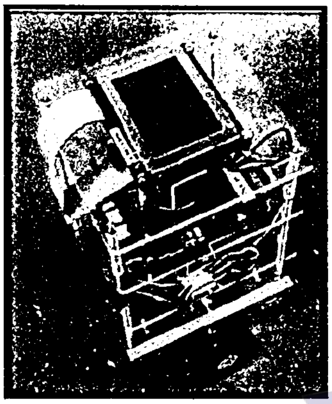

Figure 2.1 : Autonomous Balancing Robot (Ooi, 2003)

DC motor The robot is powered by two faulhaber DC motors. The dynamic model DC motor of balancing robot will provide a correlation between input voltage to the motors and the control torque needed to balance the robot. The block diagram of functional representing the information flow between the working component is as shown in Figure 2.2.

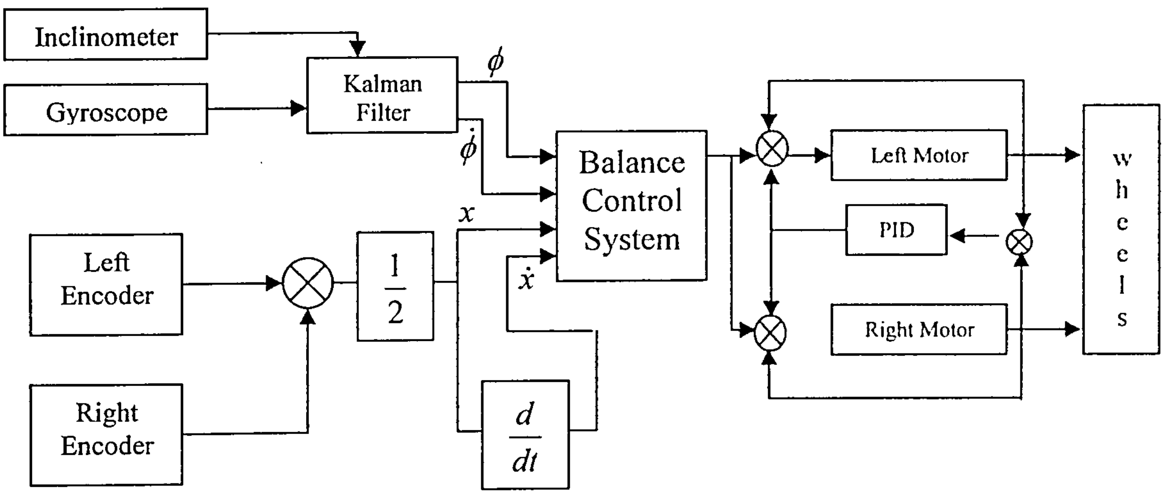

Figure 2.2 : Functional block diagram of balancing robot

In order to proceed with the formulation of the integrated dynamic model of the balancing robot, a complete set of dynamic equations of the DC motor must be formulated first. The block diagram for the DC motor as shown in Figure 2.3 is considered. Figure 2.4 shows DC motor circuit used in modeling.

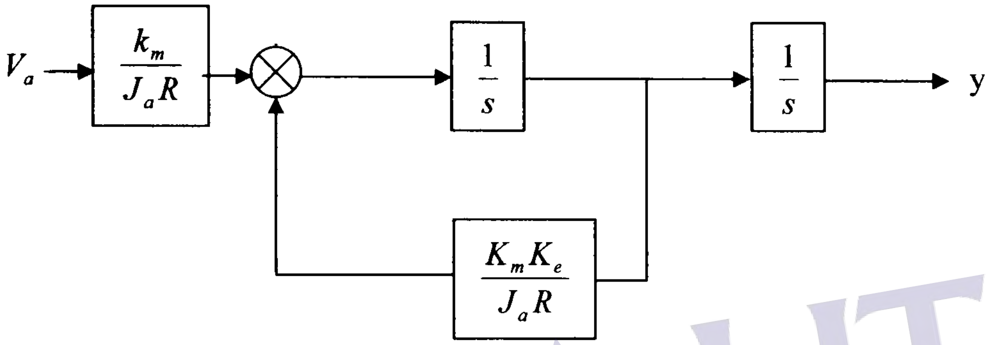

Figure 2.3 : Block Diagram of the DC motor

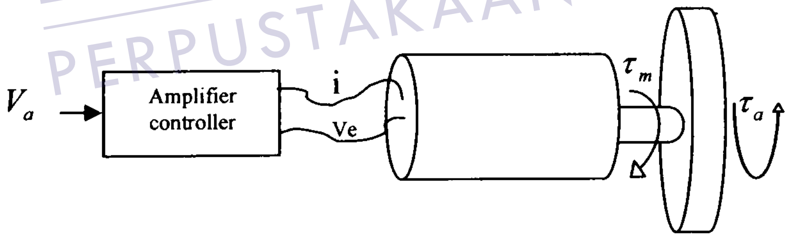

Figure 2.4 : DC motor circuit

A DC electric motor has natural relationship between its torque τ_m and its current i, as well as between its voltage V_e and its angular speed ω. For an ideal motor, the current required is directly proportional to the applied voltage (Robert, 2000). This relationship can be expressed as:

$$\tau_m = k_m i \tag{2.1}$$

The rotation of armature induced an emf across the armarure winding. Since permanent magnet DC motor is considered, the back emf can be written as

$$V_e = k_e \omega \tag{2.2}$$

The equation for DC motor electrical circuit can be stated as

$$V_a = V_e + Ri(t) + \frac{\mathrm{Ldi(t)}}{\mathrm{dt}} \tag{2.3}$$

Friction on the shaft of the motor is approximated as a linear function of the shaft velocity. Newton's Law of motion states, the sum of all torque produces on shaft is linearly related to the acceleration of the shaft by the inertial load of armature J_a.
The equation governing the motion become the following:

$$\sum M = J_a \omega \tag{2.4}$$

$$\tau_m - k_f \omega = J_a \dot{\omega} \tag{2.5}$$

Subtituting equation (2.1) and (2.2) into (2.3) and (2.5) gives ,

$$\frac{di}{dt} = \frac{V_a}{L} - \frac{Ri(t)}{L} - \frac{K_e \omega}{L} \tag{2.6}$$

$$\dot{\omega} = \frac{k_m i}{J_a} - \frac{k_e \omega}{J_a} \tag{2.7}$$

The motor inductance and motor friction is considered negligible and approximated as zero. Hence equation (2.7) can be approximated as

$$\dot{\omega} = \frac{k_m i}{J_a} - \frac{\tau_a}{J_a} \tag{2.8}$$

$$\mathrm{i} = -\frac{-k_e}{R}\omega + \frac{V_a}{R} \tag{2.9}$$

Substituting equation (2.8) into (2.9), an approximation for the DC motor only function of current motor speed, applied voltage and applied torque gives

$$\dot{\omega} = \frac{k_m K_e \omega}{J_a R} + \frac{K_m V_a}{J_a R} \tag{2.10}$$

$$\dot{\theta} = \omega \tag{2.11}$$

The motor 's dymanic can be represented with a state space model as follows where

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-k_m k_e}{J_a R} \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k_m}{J_a R} \end{bmatrix} [V_a] \tag{2.12}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} \tag{2.13}$$

where

θ = parameters position

ω =velocity

V_a=applied torque

Two wheeled The robot's behavior can be influenced by disturbances as well as the torque from the motor. As such the mathematical model will have to accommodate for such forces. Firstly the equations of motion associated with the left and right wheels are obtained. Figure 2.5 shows the free body diagram for both wheels. Since the equation for the left and right wheels are completely analogous, only the equation for the right wheel is considered.

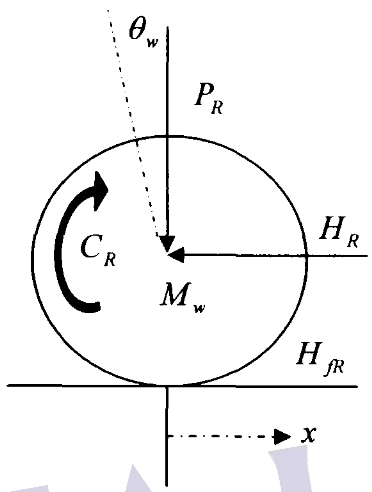

Figure 2.5: The free body diagram wheeled of balancing robot

A portion of a mechanical device may be idealized as a uniform, homogeneous wheel rolling without slipping on the horizontal surface. The wheel is not constrained to rotate about a fixed axis. Newton's law of motion state the sum of forces on the horizontal x direction gives

$$\sum F_x = Ma$$
$$M_w \ddot{x} = H_{fR} - H_R \tag{2.14}$$

The wheels rolls on a horizontal surface, there is no acceleration in the y direction. The sum of forces around the centre of the wheel gives

$$\sum M_0 = I\alpha$$
$$I_w \ddot{\theta}_w = C_R - H_{fR} r \tag{2.15}$$

From DC motor dynamics, the motor torque can be expressed as,

$$\tau_m = I_R \frac{d\omega}{dt} \tag{2.16}$$

By rearranging the equation and substituting the parameters from the DC motor derivation section, the output torque to the wheels is attained

$$C_R = I_R \frac{d\omega}{dt} = \frac{-k_m k_e}{R}\dot{\theta}_w + \frac{k_m}{R}V_a \tag{2.17}$$

Therefore, equation (2.15) becomes

$$I_w \ddot{\theta}_w = \frac{-k_m k_e}{R}\dot{\theta}_w + \frac{k_m}{R}V_a - H_{fR} r$$

Thus,

$$H_{fR} = \frac{-k_m k_e}{Rr}\dot{\theta}_w + \frac{k_m}{Rr}V_a - \frac{I_w}{r}\ddot{\theta}_w \tag{2.18}$$

Equation (2.17) is substituted into equation (2.14) to get the equation for the left and right wheels.

For left wheel

$$M_w \ddot{x} = \frac{-k_m k_e}{Rr}\dot{\theta}_w + \frac{k_m}{Rr}V_a - \frac{I_w}{r}\ddot{\theta}_w - H_L \tag{2.19}$$

For right wheel

$$M_w \ddot{x} = \frac{-k_m k_e}{Rr}\dot{\theta}_w + \frac{k_m}{Rr}V_a - \frac{I_w}{r}\ddot{\theta}_w - H_R \tag{2.20}$$

The linear motion is acting on the centre of the wheel, the angular rotation can be transformed into linear motion by simple transformation,

$$\ddot{\theta} r = \ddot{x}$$

$$\dot{\theta} r = \dot{x}$$

By the linear transformation, equation (2.19) and (2.20) give

For left wheel

$$M_w\ddot{x} = \frac{-k_m k_e}{Rr}\dot{x} + \frac{k_m}{Rr}V_a - \frac{I_w}{r^2}\ddot{x} - H_L \tag{2.21}$$

For right wheel

$$M_w\ddot{x} = \frac{-k_m k_e}{Rr}\dot{x} + \frac{k_m}{Rr}V_a - \frac{I_w}{r^2}\ddot{x} - H_R \tag{2.22}$$

Adding equation (2.21) and (2.22) together yields

$$2\left(M_w + \frac{I_w}{r^2}\right)\ddot{x} = \frac{-2k_m k_e}{Rr^2}\dot{x} + \frac{2k_m}{Rr}V_a - (H_L + H_R) \tag{2.23}$$

Robot's chassis Using Newton's law of motion robot chassis can modeled as an inverted pendulum. Figure 2.6 shows the free body diagram of the chassis.

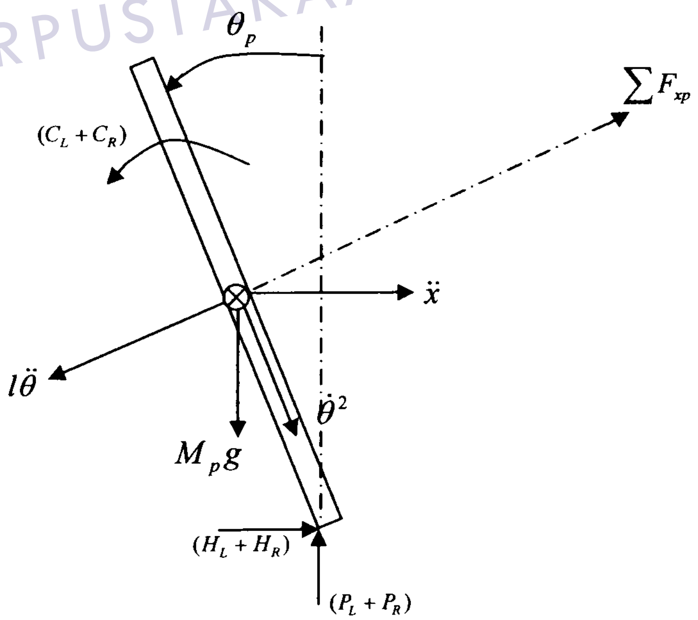

Figure 2.6: Free body diagram of the chassis

The sum of forces in the horizontal direction:

$$\Sigma F_x = M_p \ddot{x} = (H_L + H_R) - [Mpl\ddot{\theta}_p \cos\theta_p + M_p l\dot{\theta}_p^2 \sin\theta_p] \qquad (2.24)$$

The sum of forces perpendicular to the pendulum:

$$\sum F_{xp} = M_p \ddot{x}\cos\theta_p = (H_L + H_R)\cos\theta_p + (P_L + P_R)\sin\theta_p - M_p g\sin\theta_p - M_p l\ddot{\theta}_p \qquad (2.25)$$

The sum of moment around the centre of mass of pendulum

$$\sum M_a = I\alpha = -(H_L + H_R)l\cos\theta_p - (P_L + P_R)l\sin\theta_p - (C_L + C_R) = I_p l\ddot{\theta}_p \qquad (2.26)$$

The torque applied on the pendulum from the motor as defined in equation (2.17) and after linear transformation:

$$C_L + C_R = \frac{-2k_m k_e}{R}\frac{\dot{x}}{r} + \frac{2k_m}{R}V_a \qquad (2.27)$$

The final equation is shown as below

$$I_p\theta_p - \frac{2k_m k_e}{Rr}\dot{x} + \frac{2k_m}{R}V_a + M_p gl\sin\theta_p + M_p l^2\ddot{\theta}_p = -M_p l\ddot{x}\cos\theta_p \qquad (2.28)$$

$$2\left(M_w + \frac{I_W}{r^2}\right)\ddot{x} = -\frac{2k_m k_e}{Rr^2}\dot{x} + \frac{2k_m}{Rr}V_a - M_p\ddot{x} - M_p l\ddot{\theta}_p\cos\theta_p + M_p l\dot{\theta}^2{}_p\sin\theta_p \qquad (2.29)$$

Rearranging equation (2.28) and (2.29) gives the **non-linear equations** of motion of the system

$$(I_p + M_p l^2)\ddot{\theta}_p - \frac{2k_m k_e}{Rr}\dot{x} + \frac{2k_m}{R}V_a + M_p gl\sin\theta_p = -M_p l\ddot{x}\cos\theta_p \qquad (2.30)$$

$$\frac{2k_m}{Rr}V = \left(2M_w + \frac{2I_W}{r^2} + M_p\right)\ddot{x} + \frac{2k_m k_e}{Rr^2}\dot{x} +_a + M_p l\ddot{\theta}_p\cos\theta_p - M_p l\dot{\theta}^2{}_p\sin\theta_p \qquad (2.31)$$

Equation (2.30) and (2.31) can be linearized by assuming $\theta_p = \pi + \phi$, where ϕ represent s a small angle from the vertical upward direction.

Therefore,

$$\cos\theta_p = -1,\ \sin\theta_p = -\phi \qquad \text{and} \qquad \left(\frac{d\theta_p}{dt}\right)^2 = 0$$

The linearized equation of motion is

$$\ddot{\phi} = \frac{M_p l}{\left(I_p + M_p l^2\right)}\ddot{x} + \frac{2k_m k_e}{Rr\left(I_p + M_p l^2\right)}\dot{x} - \frac{2k_m}{R\left(I_p + M_p l^2\right)}V_a + \frac{M_p g l}{\left(I_p + M_p l^2\right)}\phi \tag{3.22}$$

$$\ddot{x} = \frac{2k_m}{Rr\left(2M_w + \frac{2I_W}{r^2} + M_p\right)}V_a + \frac{-2k_m k_e}{Rr^2\left(2M_w + \frac{2I_W}{r^2} + M_p\right)}\dot{x} + \frac{M_p l}{\left(2M_w + \frac{2I_W}{r^2} + M_p\right)}\ddot{\phi} \tag{3.23}$$

The continuous state space equation is obtained as:-

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\phi}\\ \ddot{\phi}\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0\\ 0 & \frac{2k_m k_e(M_p lr - I_p - M_p l^2)}{Rr^2 \mathrm{X}} & \frac{M_p^2 g l^2}{\mathrm{X}} & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{2k_m k_e(r\Upsilon - M_p l)}{Rr^2 \mathrm{X}} & \frac{M_p g l \Upsilon}{\mathrm{X}} & 0 \end{bmatrix} \begin{bmatrix} x\\ \dot{x}\\ \phi\\ \dot{\phi}\end{bmatrix} + \begin{bmatrix} 0\\ \frac{2k_m(I_p lr - M_p l^2 - M_p lr)}{Rr\mathrm{X}}\\ 0\\ \frac{2k_m(M_p l - r\Upsilon)}{Rr\mathrm{X}}\end{bmatrix} V_a \tag{3.33}$$

where

$$\mathrm{X} = \left[I_p\beta + 2M_p l^2\left(M_w + \frac{I_w}{r^2}\right)\right] \tag{3.34}$$

$$\Upsilon = \left(2M_w + \frac{2I_w}{r^2} + M_p\right) \tag{3.35}$$

CHAPTER 3

DISCRETIZATION OF CONTINOUS-TIME STATE SPACE MODELS

3.1 Introduction

Discrete-time dynamical systems are often expected to follow certain continuous-time models. This does not mean, however, that the discrete approximation of continuous systems is restricted to modelling of existing systems. Motivated by the importance and occurrence of analog systems and signals, as well as by associated engineering experience, the method of carrying out continuous design prior to digital mechanization is quite common (Jury, 1964).

3.2 Discretization Problem

Since there is no unique equivalence between the continuous-time and discrete-time systems, various techniques are used to convert an analog system into an approximate digital model.

Among the approximation procedures, two classes of digital system designs are widely recognized (Beauchamp, 1973). The first class represents the classical approach of digitizing a given (existing or theoretically derived) analog system as a whole by transforming it into the discrete-time domain. The method generally results in closed-form solutions for the digital systems. The second class constitutes the open-form approach by designing digital systems that are optimum with respect to some specified criteria.

3.2.1 Closed-Form Discretizing Transformations

The continuous-to-discrete transformations are the most popular in digital filter and controller design; they are also used in digital simulation. This method is sometimes considered to be digital simulation as it attempts to approach an imitation of the analog model, and as such, it may be useful in testing complex system designs or the behavior of existing systems. One has to stress, however, that since we are here interested in closed-form expressions, this is rather a limited simulation in the sense that multistep integration procedures have to be converted into single step discrete integration formulas or, at most, to multirate integration with limited access to the external measurements.

The essential criterion used here is the discrete-time error between the response of the digital model and the response of the analog prototype at sampling points (Glover, 1984). Since the error depends on the input signal applied, it is important to distinguish between two types of signals according to their spectral properties.

Working with the relatively unbounded input signals (undersampled processes) one has to cope with the frequency aliasing issue, apparently no matter which method was used in the design.

The best way to treat such cases is to design the system accurate for one particular input function, and then to analyze the correctness of this design. Whenever the input signal contains essentially only frequencies below ω_N, designs of arbitrary good accuracy can be sought. They are usually obtained by making the discrete system's frequency response approach the continuous system response in the Nyquist interval so that for any spectrally limited form of the input the discrete-time error will approach zero. These types of methods are referred to as the frequency transformations, in which left-half s-plane should be mapped inside the unit circle and the imaginary axis converted to the unit circle. The operators themselves can be derived in an optimization process.

3.2.2 Open-Form Discrete Approximation by Optimization

The second class of digital design uses different optimization techniques to obtain digital systems that are most desirable in view of certain measures.

Direct closed-form design of the transfer function coefficients can take place in the discrete time or frequency domain. Iterative methods have to be applied in search of a discrete system whose response satisfactorily approximates a prescribed impulse response. On the other hand, in the frequency domain, the magnitude- squared function fitting brings about problems with a suitable rational trigonometrical polynomial and with the factorization of the squared magnitude to determine the z-transfer function.

There is a general remark (which concerns all the discrete approximation designs) that there is no simple way of even approximately restricting the deviation in the time and the frequency responses at the same time.

The bulk of optimization-based digital system design methods represent an open approach with respect to approximation of a desired frequency characteristic (magnitude, phase, or group delay). They certainly must rely upon unaliased input signal samples. The minimum mean-squared error design (with a nonlinear optimization procedure of Fletcher-Powell, for instance), the minimum L_p , error design, and the method of Deczky are examples of the open approach (Temes et al., 1973).

These methods, however, have a common feature in that they cannot be solved explicitly to give the transfer function coefficients, and usually an iterative mathematical optimization procedure has to be used to determine the system coefficients that minimize a given error criterion. Primarily, the recently developed theory of the worst case frequency-domain design, where stability is the main concern, and which uses the minimax or Chebyshev or H^{∞} norm for the model matching also belongs to this class. Linear multivariable systems given by matrix transfer-function or state-space representations can be treated within this framework.

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