THE USE OF HORIZONTAL AND INCLINED BARS AS SHEAR REINFORCEMENT

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THE USE OF HORIZONTAL AND INCLINED BARS AS SHEAR REINFORCEMENT

NOOR AZLINA BT ABDUL HAMID

A project report submitted in partial fulfillment
of the requirements for the award of the degree of
Master of Engineering (Civil – Structure)

FACULTY OF CIVIL ENGINEERING
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NOV, 2005

I declare that this thesis entitled " *The use of horizontal and inclined bars as shear reinforcement* " is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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To mama and papa,

Thanks for your support

My dream has come true just because of you

To my beloved husband,

Thanks for your understanding and support

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ABSTRACT

Shear failure in beams are caused by diagonal cracks near the support. Any form of effectively anchored reinforcement that intersects these cracks will be able to resist the shear stress to a certain extent. This project presents the results of an experimental investigation on six reinforced concrete beams in which their structural behaviour in shear were studied. All the beams were cast with the same grade of concrete, and provided with identical amount of main reinforcement. In order to investigate the contribution of the additional horizontal and independent bent-up bars to the shear carrying capacity of the beam, two specimens each were provided with horizontal longitudinal bars and bent-up bars in the high shear region. Two different quantities of additional bars in each of these cases were adopted. The fifth specimen was provided with sufficient amount of shear reinforcement in terms of vertical links, while the other one was cast without any shear reinforcement to serve as control specimens. The performances of the beams in resisting shear in the form of deflection, cracking, strain in the shear reinforcement and ultimate load were investigated. The results show that the shear capacities of the beams with additional horizontal and independent bent-up bars larger than 1.2% of their cross-sectional area are higher than that of the conventionally designed beam with vertical links. It may therefore be suggested that these types of shear reinforcement be used to ease the congestion of links near the supports.

ABSTRAK

Kegagalan ricih dalam rasuk adalah disebabkan oleh keretakan condong yang berlaku berdekatan dengan penyokong. Sebarang bentuk tetulang tambatan yang melintasi keretakan ini berkeupayaan untuk menghalang ricih pada suatu takat yang tertentu. Kajian ini memaparkan keputusan dari ujikaji makmal yang telah dijalankan ke atas enam rasuk konkrit bertetulang dimana kelakunannya terhadap ricih telah dikaji. Semua sampel rasuk dibina dengan kekuatan gred konkrit yang sama, dan menggunakan bilangan dan jenis tetulang utama yang sama. Bagi mengkaji sumbangan atau kesan bar ufuk tambahan dan bar yang dibengkok terhadap keupayaan menanggung ricih, dua sampel rasuk dimana setiap satunya disediakan bar ufuk tambahan dan bar yang dibengkok pada satah kegagalan ricih maksimum. Dua perbezaan kuantiti untuk setiap jenis tetulang tambahan disediakan. Spesimen yang kelima disediakan dengan bilangan tetulang ricih yang mencukupi dalam bentuk perangkai pugak, manakala satu lagi rasuk dibina tanpa menggunakan sebarang tetulang ricih dan bertindak sebagai rasuk kawalan. Kelakunan rasuk dalam menghalang ricih dikaji berdasarkan kepada nilai pesongan, keretakan, keterikan dan beban muktamad. Keputusan ujikaji menunjukkan bahawa rasuk yang menggunakan bar ufuk tambahan dan bar yang dibengkokkan sebagai tetulang ricih lebih daripada 1.2% daripada keratan rentas rasuk boleh menanggung keupayaan ricih lebih daripada rasuk yang menggunakan perangkai pugak. Oleh yang demikian, tetulang ricih jenis ini dicadangkan bagi memudahkan kerja-kerja pemasangan perangkai ricih yang disusun rapat berhampiran dengan penyokong rasuk.

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A	-	Area of a cross-section
A_s	-	Area of tension reinforcement
A_{sb}	-	Area of steel in bent-up bars
$A_{s,prov}$	-	Area of tension reinforcement provided
$A_{s,req}$	-	Area of tension reinforcement required
A_{sv}	-	Total cross-sectional area of links at the neutral axis
a_v	-	Shear span
b	-	Width of a section
b_v	-	Breadth of member for shear resistance
c	-	Cover to reinforcement
d	-	Effective depth of tension reinforcement
f_{cu}	-	Characteristic concrete cube strength at 28 days
f_s	-	Service stress in reinforcement
f_{tt}	-	Design tensile stress in concrete at transfer
f_y	-	Characteristic strength of reinforcement
f_{yb}	-	Characteristic strength of inclined bars
f_{yv}	-	Characteristic strength of link reinforcement
L	-	Effective span of a beam
M_{max}	-	Maximum bending moment
s_b	-	Spacing of bent-up bars
s_v	-	Spacing of links
V	-	Shear force at ultimate design load
V_b	-	Design ultimate shear resistance of bent-up bars
V_c	-	Design ultimate shear resistance of a concrete section

v	-	Shear stress
v_b	-	Design shear stress resistance of bent-up bars
v_c	-	Design ultimate shear stress resistance of a singly reinforced concrete beam
α	-	Angle between a bent-up bar and the axis of a beam
β	-	Bond coefficient
θ	-	Angle
ϕ	-	Bar diameter

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CHAPTER 1

INTRODUCTION

Reinforced concrete (RC) beams are the important structural elements that transmit the loads from slabs, walls, imposed loads etc. to columns. A beam must have an adequate safety margin against bending and shear stresses, so that it will perform effectively during its service life.

At the ultimate limit state, the combined effects of bending and shear may exceed the resistance capacity of the beam and causes tensile crack. Since the strength of concrete in tension is considerably lower than its strength in compression, design for shear is of major importance in concrete structures. However, shear failure is difficult to predict accurately. In spite of many decades of experimental research, it is not fully understood.

The behaviour of reinforced concrete beams at failure in shear is distinctly different from their behaviour in flexure, which may be more dangerous than flexure failure. They fail abruptly without sufficient advanced warning[1] and the diagonal cracks that develop are considerably wider than the flexural cracks.

Shear failures in beams are caused by the diagonal cracks near the support and it had been tested before at Cornell University under third point loading. With no shear reinforcement provided, the member failed immediately upon formation of the critical crack in the high-shear region near the support.

Whenever the value of actual shear stress exceeds the permissible shear stress of the concrete used, the shear reinforcement must be provided. The purpose of shear reinforcement is to prevent failure in shear, such increases the ductility of the beam and considerably reduces the likelihood of a sudden failure.

Normally, the inclined shear cracks start at the bottom near the support at approximately 45° and extend towards the compression zone. Any form of effectively anchored reinforcement that intersects these diagonal cracks will be able to resist the stress to a certain extent. In practice, shear reinforcement is provided whether in the form of vertical links, inclined links or combination system of links and bent-up bars.

In building construction, vertical links are most commonly used as shear reinforcement, because of their simplicity in fabricating and installing. Normally, links are arranged closely or sometimes double or more shear links are used to resist high shear stress. Congestion near the support of RC beam due to the presence of the closely spaced links can increase the cost and time required in fixing the reinforcement.

The use of bent-up bars along with vertical links had been practical before. In situations where all the tensile reinforcement is not required to resist bending moment, some of the bar was bent-up in the region of high shear to form the inclined legs of shear reinforcement. For example, beams which provide 4 bars of main tensile reinforcement, 2 bars may be bent-up diagonally in shear region and used as shear reinforcement, while the other 2 bars would be left to continue to the support.

However, its application has been less preferred nowadays. The difficulties to form as bent-up bars and required adequate amount of main reinforcement make it rarely used in construction. In beams with small number of bars provided, the bent-up system is not suitable because insufficient amount of reinforcement would be left to continue to the support as required by the code of practice.

Due to the problems of conventional shear reinforcement, the use of independent inclined and horizontal bars provided in the high shear region are recommended in this project and expected would be able to serve the same purposes. The main advantages of these types of shear reinforcement system are structural effectiveness, flexibility, simplicity and speed of construction.

In this project, the experimental investigation of the system was carried out in which their structural behaviours in shear were studied. Six reinforced concrete beams, which contained different types of shear reinforcement were designed and prepared for laboratory testing. In this investigation, all the beams are allowed would be fail only in shear, so adequate amount of tension reinforcement were provided to give a sufficient of bending moment resistance.

In order to investigate the contribution of the additional horizontal and inclined bars to the shear carrying capacity of the beam, two specimens each were provided with horizontal longitudinal bars and inclined bars in the high shear region. The other two specimens each were cast without shear reinforcement as control specimen and the other one was provided with sufficient amount of shear reinforcement in terms of vertical links. External forces were loaded within a sufficient distance near the support. The performances of the beams in general and in resisting shear in particular were compared in terms of deflection, cracking and ultimate load.

The results from the laboratory testing are very useful to determine the effectiveness of independent inclined and horizontal bars as shear reinforcement. It is anticipated that both types of additional bars increase the shear capacity of the beam and therefore be suggested to use as alternative shear reinforcement.

1.1 Objectives

In general, the aim of this project is to investigate the behaviour of rectangular beams in shear. In a more specific terms, the objective of this study are as follow :

a) To study the effectiveness of additional longitudinal bars in resisting shear forces in rectangular beams.

b) To study the effectiveness of independent inclined bars as shear reinforcement.

c) To determine the optimum amount of both types of shear reinforcement to achieve a shear capacity similar to that of a normal links system.

1.2 Scope of the study

This study is based fully on the experimental investigation to be carried out with the scope given below :

a) The study was based on experimental investigation on six rectangular reinforced concrete beams.

b) All specimens were of the same size and reinforced with identical amount of longitudinal steel.

c) The beams were tested to failure with two point loads near the support to give a shear span to effective depth ratio of 2.5

d) The concrete compressive strength of the specimens on the testing day was in the range 30 to 35 N/mm^2.

e) The variables in these specimens are the shear reinforcement systems, which are vertical links, independent inclined bars and additional horizontal bars.

CHAPTER 2

LITERATURE REVIEW

2.1 Shear

Shear is a term used when the two sections on either side of a plane in a member have a tendency to slide along that plane. The shear forces that act in the opposite directions as shown in Figure 2.1 cause this tendency.

This shear is caused by the variation of bending moment along the span. The relationships between shear V and bending moment M is given below. Once the shear V is known, then shear stress v can be obtained by dividing the shear force with the area of cross section on which the shear is acting.. :

$$V = \frac{dM}{dx} \qquad \text{Equation 2.1}$$

Figure 2.2 represents the distribution of principal stresses across the span of a concrete beam. The direction of the principal compressive stresses takes the form of an arch whilst the tensile stresses have the curve of a suspended chain. Towards mid-span where the shear is low and the bending stresses are dominant the direction of the stresses tends to be parallel to the beam axis.

Near the support where the shearing forces are greater the principal stresses are inclined at a steeper angle, so that the tensile stresses are liable to cause diagonal cracking. If the diagonal tension exceeds the limited tensile strength of the concrete then shear reinforcement must be provided. This shear reinforcement is either in the form of vertical links or inclined bars.

The shear in a reinforced concrete beam without shear reinforcement is carried by a combination of concrete in the compression zone, dowel action and aggregate interlock across flexural cracks. The actual behaviour of shear is complex, and difficult to analyze theoretically, but by applying the results from many experimental investigations, reasonable simplified procedures for analysis and design have been developed.

2.2 Shear stress variation in rectangular beams

In rectangular beams, the shear stresses are caused by the variation of the bending moments as bending moment is zero at supports and maximum at the center in case of simply supported beams loaded with uniform load. This variation from supports to any point on the beam will depend upon the type of loading and end supports of the beam. The vertical shear forces causes complimentary shear stress and diagonal tensile and compressive stresses of the same magnitude, as shown in Figure 2.3 (a) if a small square element located at the neutral axis is isolated. If the only stresses present, the element would be not in equilibrium. Therefore, at any point on the beam, the horizontal shear stresses are equal in magnitude to the vertical shear stresses.

The action of two pairs of shear stresses on the vertical and horizontal faces is the same as that of two pairs of normal stresses, one tension and one compression, acting on the 45°. As can be seen from Figure 2.3 (b), the distribution of shear stresses in rectangular beams is parabolic and giving the maximum shear stress at the neutral axis of the section equal to $1.5V/bd$.

The shear stress, v at any section is given by :

$$v = \frac{V.A.\overline{Y}}{I.b} \qquad \text{Equation 2.2}$$

where,

v	=	shear stress
V	=	shear force at the section
A	=	area above the point under consideration
$\overline{Y}$	=	distance of center of gravity of area A from neutral axis
b	=	width of section
I	=	moment of inertia

2.3 Shear stress variation in reinforced rectangular beams

The shear stress variation in reinforced rectangular beams is in the forms of parabolic and uniform as shown in Figure 2.4. The section above neutral axis is in compression and below neutral axis is in tension. As $T+\Delta T$ is greater than T, the lower portion has a tendency to slide towards right, while the upper portion has a tendency to slide towards left due to $C+\Delta T$ greater than C.

The dimension of the cross-section of a reinforced concrete beam and the area of longitudinal steel are usually determined from calculation which consider resistance to bending moment at the ultimate limit state. As the load is increased in a plain concrete beam without reinforcement, a tension crack will form where the tension stresses are largest and will immediately cause the beam to fail. However, the situation is different when tension reinforcement is provided. Even though tension cracks form in the concrete, the flexural tension strength will resist by the steel and much higher loads can be carried.

Shear stresses increase proportionally to the loads. In consequence, the diagonal tension stresses of significant intensity are created in regions of high shear forces, which is close to the supports. The longitudinal tension reinforcement does not reinforce the tensional weak concrete against the diagonal tension stresses that caused by shear or combined effect of shear and flexure. Eventually, these strains attain magnitudes sufficient to form tension cracks in a direction perpendicular to the local tension stress. These are known as diagonal cracks, in distinction to the vertical flexural cracks.

2.4 Shear failure in beams without shear reinforcement

The purpose of shear reinforcement is to prevent failure in shear and this makes it necessary to understand the behaviour of beams in which reinforcement are not provided. In a beams which does not contain shear reinforcement, the behaviour after inclined cracking depends very much on the conditions of load and support. The inclined cracks may develop and extend gradually with increasing load to produce the patterns illustrated in Figure 2.5.

The causes of the cracks are, when the diagonal tension stresses reach the tensile strength of the concrete, and a diagonal crack will develop. The shear mechanism is complex and depends on the shear span ratio a_v/d. When this ratio is large the failure is as shown in Figure 2.6.

The are three actions form the mechanism resisting shear in the beam :

1. Shear stresses in the compression zone with a parabolic distribution.

2. Aggregate interlock along the cracks

3. Dowel action in the bars where the concrete between the cracks transmits shear forces to the bars.

The behaviour of reinforced concrete beams is much influenced by the shear stresses and depending on configuration, support conditions and load distribution. At a location of large shear force *V* and small bending moment *M*, there will be little flexural cracking. The average shear stress prior to crack formation is given in BS 8110: Part 1, clause 3.4.5.2 :

$$v = \frac{V}{b_v d} \qquad \text{Equation 2.3}$$

where, V = the ultimate shear force

b_v = the beam width ($b_v = b$ for rectangular beam)

d = the effective depth

This value is then use as guide to determine whether the section is of the correct size and whether shear reinforcement is required. The design concrete shear stress v_c is used to determine the shear capacity of the concrete alone. Values of v_c depend on the percentage of steel in the member, the depth and the concrete grade. An increase in the amount of tension steel as well as increase in the dowel action component increases the aggregate interlock value by resisting the width of the shear crack.

The design concrete stress is given by the following formula :

$$v_c = \frac{0.79(100A_s/(b_v d))^{1/3}(400/d)^{1/4}}{\gamma_m} \qquad \text{Equation 2.4}$$

Where $100A_s/(b_v d)$ should not be greater than 3.0 and 400/d should not be less than unity. The code notes (clause 3.4.5.4 in BS 8110) that for tension steel to be counted in calculating A_s it must continue for a distance at least equal to *d* beyond the section being considered. This formula gives values of v_c for concrete grade 25. For higher grades of concrete values should be multiplied by $(f_{cu}/25)^{1/3}$. The value of f_{cu} should not be greater than 40 and the value of γ_m is 1.25.

The code states that shear failure in beam sections without shear reinforcement normally occurs at about 30° to the horizontal. If the angle is steeper due to the load causing or because the section is close to the support, the shear capacity is increased. The increase is because the concrete in diagonal compression resist shear. For a maximum shear stress requirement, the nominal shear stress $v=V/b_vd$ must in no case exceed $0.8f_{cu}^{1/2}$ or $5\ N/mm^2$ even if the beam is reinforced to resist shear. This upper limit prevents failure of the concrete in diagonal compression.

2.5 Types of shear failure

The vertical cracks are produced by the bending moment and these are linked to diagonal cracks produced by the shear forces. Many test have been attempted and establish that rectangular beam fails eventually as the shear force V is increased and the failure mode is strongly dependent on the shear-span/depth ratio a_v/d. There are five types of shear failure, which four of it are related to the shear span ratio a_v/d. As this ratio increase, the shear resistance decreases. The following cases have been observed in tests :

(1) **Case 1** : ($a_v/d > 6$)

The bending moment is large in comparison to the shear force and the mode of failure is similar to that in pure bending. The initial vertical bending cracks become inclined due to the action of the shear stresses and failure occurs in the compression zone as shown in Figure 2.7 (a). The stresses in the tensile steel are approximately at yield and only minimum reinforcement is required in design situations.

(2) **Case 2 :** ($6 > a_v/d > 2$)

The initial bending cracks become inclined early in the loading sequence and at collapse horizontal cracks form running along the line of the tensile reinforcement (see Figure 2.7 (b). The horizontal cracks reduce the shear resistance of the section by destroying the dowel force and reducing the bond stresses between the steel and the concrete. The steel in the tension zone does not reach yield.

(3) **Case 3 :** ($a_v/d < 2$)

The influence of shear tends to be critical and the diagonal crack form independently. The beam continues to carry additional shear force and failure takes place by crushing of concrete in compression near the loads as the cracks penetrates into this zone. Therefore, in this case bending cracks do not develop but shear cracks (at approximately 45^{o}) suddenly appear and often run through to the compression zone and produce collapse as shown in Figure 2.7 (c). The steel in the tension zone does not reach yield.

(4) **Case 4 :** ($a_v/d = 0$)

Punching shear failure occurs when the plane of failure is forced to run parallel to the shear forces as shown in Figure 2.7 (d)). This can occur when the opposing shear forces are close together or, if shear links have been added, when the failure plane forms which does not intercept the shear links. When this type of failure occurs the shear resistance of the section is at a maximum. The addition of shear reinforcement in the form of vertical links does not increase the shear resistance above the punching shear value and if a section is unable to resist this force it must be increased in size.

(5) **Case 5 :** shear reinforcement

The addition of shear reinforcement increases the shear resistance for cases of failure I, II and III. Numerous diagonal cracks develop as shown in Figure 2.7 (e) and at failure the shear reinforcement and the longitudinal steel yield, provided that the steel is anchored and the member is not over-reinforced.

2.6 Shear reinforcement in beams

The function of the shear reinforcement is to provide a satisfactory margin of safety against shear failure and to ensure ductile behaviour at overload. Shear reinforcement have to be provided against the diagonal tensile stress caused by shear forces. For this reason, shear reinforcement is designed on an ultimate strength basis. The longitudinal reinforcement does not prevent this tension failure.

Although the truss analogy has formed a simple explanation for vertical links behaviour in beams for many years, it does not include several significant components of shear force transmission. Certainly shear reinforcement increase the shear strength[2] of a member, but such reinforcement contributes little to the shear resistance prior to the formation of inclined cracks.

To provide shear strength by allowing a redistribution of internal forces, across any inclined crack that may form, the shear reinforcement has three primary functions :

(1) To carry part of the shear, V

(2) To restrict the growth of the inclined crack and thus help maintain aggregate interlock

(3) To tie the longitudinal bars in place and thereby increase their dowel capacity

If the amount of shear reinforcement is too little, it will yield immediately at the formation of the inclined crack, and the beam then fails. If the amount of shear reinforcement is too high, there will be a shear compression failure before yielding of the web steel. The optimum amount of shear reinforcement should be such that both the shear reinforcement should be such that both the shear reinforcement and the compression zone continue to carry increasing shear after the formation of the inclined crack until yielding of the shear reinforcement, thus ensuring a ductile failure.

Whenever the value of actual shear stress exceeds the permissible shear of the concrete used, the shear reinforcement must be provided with adequate spacing and length. Taking a simple view, concrete is weak in tension so shear failure is caused by failure in diagonal tension with cracks running at 45° to the beam axis. Any form of shear reinforcement, which intercepts the diagonal crack, would suddenly carry a portion of the shear force, V.

In order to determine the inclination of crack, for the principal stresses developed, we have :

$$\tan 2\theta = -\frac{2\tau}{\sigma_t}$$

(a) At a section where bending moment = 0, tensile stress = $\sigma_t = 0$

Maximum principal stress = τ

$$\tan 2\theta = -\frac{2.\tau}{0} = -\infty$$

$$\therefore \quad \theta = 45^{\circ} \text{ or } 135^{\circ}$$

Therefore, at or near the supports of a simply supported beam where bending moment is zero, the inclination cracks shall be 45°.

(b) At a section where bending moment is maximum,

Shear stress $\tau = 0$

Maximum principal stress = σ_τ

$\tan 2\theta = 0$

$\therefore \quad \theta = 0$

Therefore, at or near the mid-span of a simply supported beam where the bending moment is maximum, the inclination cracks with the vertical is zero. Providing shear reinforcement for diagonal tension can prevent the cracks.

2.6.1 Types of shear reinforcement

In practice, vertical links are most commonly used as shear reinforcement because of their simplicity in fabricating and installing. Inclined links at an angle about 40 to 60 degrees are also used. For construction purposes, a longitudinal bar is placed in each corner the compressive face for attachment of the vertical or inclined links.

While in bent-up bars system, when a portion of the main tensile reinforcement is not needed at section to resist bending moment, it can be bent up diagonally and used as shear reinforcement. The angles of inclination bars are usually about 45° in order to ensure the reinforcement intercepts the crack effectively. The combination system of vertical links and bent-up bars also had been practical before. For shear reinforcement to be effective, it must be properly anchored at each end and intersects the potential cracks at a large angle at least 45°. See Figure 2.8.

2.6.2 Shear resistance of a beam with vertical links

At the ultimate limit state in situations where the average shear stress v exceeds the design shear stress $v_c + 0.4\,N/mm^2$ shear reinforcement is added. The most common type of shear reinforcement is in the form of vertical links spaced at intervals along the length of a member. Vertical links strengthen the web of a beam and prevent dowel failure. Vertical links in a beam intercept the cracks in the concrete, which are at approximately 45° as shown in Figure 2.9.

In order to derive simplified equations the action of a reinforced concrete beam in shear is represented by an analogous truss in which the longitudinal reinforcement forms the bottom chord, the links are the vertical members and the concrete acts as the diagonal and top chord compression members as indicated in Figure 2.10.

In the analogous truss, let

A_{sv} be the cross-sectional area of the two legs of the vertical links

f_{yv} be the characteristic strength of the links reinforcement

V be the shear force due to the ultimate loads

Using the method of section it can be seen at section XX in the figure that at the ultimate limit state the force in the vertical links member must equal the shear force V, that is,

$$0.87 f_{yv} A_{sv} = V$$

or $$0.87 f_{yv} A_{sv} = v b d \qquad \text{Equation 2.5}$$

where v = V/bd is the average shear stress on the section.

If Sv = the links spacing, equation 2.5 becomes,

$$0.87 f_{yv} A_{sv} = v b d \left(\frac{S_v}{d} \right)$$

or $$\frac{A_{sv}}{S_v} = \frac{vb}{0.87 f_{yv}}$$

Since the concrete is also capable of resisting a limited amount of shear this equation is rewritten as,

$$\frac{A_{sv}}{S_v} = \frac{b\left(v - v_c\right)}{0.87 f_{yv}} \qquad \text{Equation 2.6}$$

where v_c is the ultimate shear stress that can be resisted by the concrete with its longitudinal reinforcement.

Equation 2.6 permits the areas and spacing of the vertical links to be calculated. Rearrangement of the equation gives the shearing resistance for a given links size and spacing, thus

$$V = V(\text{singly reinforced beam}) + V(\text{links})$$

$$\text{Shear resistance} = v\text{bd} = \left(\frac{A_{sv}}{S_v} 0.87 f_{yv} + b v_c \right) d \qquad \text{Equation 2.7}$$

To ensure that bond failure does not occur in a link the strength of the links f_{yv} should not exceed 460 N/mm^2. In most building construction, smallest diameter link used is 6mm and the largest diameter is 10mm. Larger diameter links are not often used because of the difficulties involved in bending and because of the displacement of the longitudinal bars at the corners of links when large corner radii are used.

If too little shear reinforcement is provided it is not effective. Where shear stress exceed $0.5v_c$ and are less than $v_c + 0.4\, N/mm^2$, then minimum shear reinforcement should be added. Therefore, the links should have strength at least equivalent to the shear stress of 0.4 N/mm^2 and BS 8110 recommends that :

$$A_{sv} / S_v = b_v (v - v_c) / 0.87 f_{yv}$$

$$\therefore \ Sv = A_{sv} 0.87 f_{yv} / (0.4 b_v) \qquad \text{Equation 2.8}$$

The spacing of links should be such that every potential crack is crossed by at least one link, therefore the maximum longitudinal spacing allowed in BS 8110 is 0.75d. If the spacing is greater than the lever arm, approximately 0.75d, the links may not intercept the crack and therefore will not resist the applied shear force.

The code states that the minimum area of link is to be :

$$A_{sv} = 0.4b_v S_v / 0.87f_{yv}$$ Equation 2.9

This recommendation is to ensure that a link is adequately anchored. However, the minimum longitudinal spacing is controlled by the need for the concrete to flow around the reinforcement during casting, which means that a space of at least the diameter of the maximum size of aggregate plus 5mm or bar diameter if greater.

For deformed bars the longitudinal spacing should be twice the cover to the outside of the beam in order to ensure the necessary bond strength between steel and concrete. For anchorage of links, it is necessary to provide longitudinal bars in the bottom and top of a beam, which are enclosed by the link and used as an anchorage.

2.6.3 Shear resistance of a beam with bent-up bars

Shear reinforcement can also be provided as inclined bars. In situation where all the tensile reinforcement is not required to resist the bending moment at the ultimate limit state, some of the bars may be bent up to form shear reinforcement as shown in Figure 2.11. If the tensile reinforcement is used in this way it must be adequately anchored at the top of the beam to prevent bond failure.

The longitudinal bars can be bent-up at a distance from support which varies from L/5 to L/7 where L is the effective span of beam. The reinforcement may be bent-up at any angle to the longitudinal axis of the beam, but in practice they are bent-up at 45^{o} to 60^{o}.

These bars taking stresses in two ways :

1. They resist diagonal tension

2. They act as support to hanger bars which are provided to give sufficient support to vertical stirrups

Generally, there are insufficient bent-up bars available to resist the total shear force and a maximum of 50% of the total shear is allowed, the other 50% is to be provided by vertical links. When bent-up bars and vertical links are used together their effect is added. The vertical shear reinforcement is to be provided in a distance $0.5L-X$ from the supports where X is the distance from the point of zero shears to the point of maximum shear strength.

The shear resistance of bent-up bars may be calculated using the truss analogy, which assumes that the bent-up bars act as tension members and the concrete forms the compression members as shown in Figure 2.12.

From the figure, to determine the shear resistance of bent-up bars is :

$$V_b = A_{sb}0.87f_{yb}\left[\cos\alpha + \sin\alpha\cot\beta\right]\left(d - d'\right)/S_b$$

rearranging

$$= A_{sb}0.87f_{yb}\sin\alpha\left(\cot\alpha + \cot\beta\right)\left(d - d'\right)/S_b$$

and simplifying

$$= A_{sb}0.87f_{yb}\sin\alpha$$

divide by $b_v d$, the shear resistance:

$$V_b = A_{sb}0.87f_{yb}\sin\alpha/\left(b_v d\right) \qquad \text{Equation 2.10}$$

This form of the equation shows that the shear resistance is calculated from the vertical resistance of bent-up bars at a section, and is independent of the spacing S_b. However, α and β should be greater than 45°, giving a minimum value of $S_b = 1.5d$. If $\alpha = 45^\circ$, then $\beta = 90 - \frac{\alpha}{2} = 67\frac{1}{2}^\circ$.

With this, the first bar is to be bent-up at a distance, $y = \frac{d}{\sin 45^\circ} = d.\sqrt{2}$ or $1.414d$ from the supports. However, if the bars are bent-up at half the above distance which is $0.705d$, then two bars shall provide resistance to diagonal tension and thus known as double system. This restriction is introduced to ensure that the diagonal cracks in the concrete are intercept by the bent-up bars.

2.7 Reinforced concrete beams

2.7.1 Stress-strain relations

The loads on a structure cause distortion of its member with resulting stresses and strain in the concrete and the steel reinforcement. To carry out the analysis and design of a member it is necessary to have knowledge of the relationship between these stresses and strains.

2.7.1.1 Concrete

Concrete is a very variable material, having a wide range of strengths and stress-strain curves. A curve representative of the actual conditions in a structure at the ultimate limit state has the shape shown in Figure 2.13. As the load is applied, the ratio between the stresses and strain is approximately linear at first ant the concrete behaves almost as an elastic material.

Eventually, the curve is no longer linear and the concrete behaves more and more as a plastic material. The maximum stress is reached at a strain of about 0.002 and tends to be a constant value of 0.0035. The maximum stress is to be only 80% of the characteristic strength f_{cu}. However, it reduced to 0.67fcu/γ_m where γ_m = 1.5, the partial safety factor for concrete.

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