Probabilistic Analysis of Surface Crack in Round Bars under Bending Moments
A.E Ismail
Department of Engineering Mechanics, Faculty of Mechanical & Manufacturing Engineering,
Universiti Tun Hussein Onn Malaysia, Parit Raja, Batu Pahat, 86400 Johor, Malaysia
emran@uthm.edu.my

Abstract—This paper presents the probabilistic analysis of surface crack in round bars subjected to bending moment using Monte Carlo simulation (MCS). A probabilistic model based on an elastic finite element analysis (FEA) was developed to evaluate the failure probability obtained using a K-estimation method. This method is based on the stress intensity factor (SIF) where the failure occurred when the SIF was assumed to exceed the critical SIF. It was found that, the K-estimation method was adequate to evaluate the failure probability of the bars when compared with the results obtained using the probabilistic FEA approach. The advantage of K-estimation methods was the reduction of the computational time and cost significantly.

Keywords-component; Probabilistic; Monte Carlo Simulation; Surface Cracks; Bending Moment

I. INTRODUCTION

Probabilistic fracture mechanics (PFM) is used to evaluate the fracture response and reliability of cracked structures. The PFM has incorporated the statistical uncertainties of mechanical, geometrical and loading properties to determine the probability of failure of the components. Currently, there are many methods and applications for probabilistic fracture mechanics in oil and gas, nuclear, automotive, naval, aerospace, and other industries, nearly all of which have been developed based on linear elastic fracture mechanics models [1-3]. Probabilistic fracture mechanics also provides a more rational means to describe the actual behavior and reliability of structures than traditional deterministic methods [4]. Monte Carlo simulation (MCS) is normally used to estimate the probability of failure. However, the use of MCS requires longer computational time when incorporated with finite element analysis (FEA). Therefore, an alternative method such as K-estimation is proposed to reduce the computational time and then, they are compared with the results obtained using FEA. In this work, K-estimation method is based on the linear elastic fracture mechanics. Then, the K-estimation method is coded into ANSYS finite element analysis. If K is a relevant fracture parameter that can be calculated from elastic finite element analysis. Suppose that the structure fails when $K > K_c$. This requirement cannot satisfied with certainty, because $K$ depends on input vector $X$ which is random and $K_c$ itself is a random variable. Hence, the performance of the cracked structure should be evaluated by the reliability, $P_r$ or its complement, the probability of failure, $P_f (P_r = 1 - P_f)$ defined as [8]

$$P_f = \Pr\{g(X) < 0\} = \int_{g(X) < 0} f_X(X) dX$$

(3)

<table>
<thead>
<tr>
<th>Random variables</th>
<th>Mean, $\mu$</th>
<th>COV</th>
<th>Distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus, $E$</td>
<td>222 MPa</td>
<td>0.08</td>
<td>Normal</td>
</tr>
<tr>
<td>Crack depth, $a$</td>
<td>Arbitrary</td>
<td>0.10</td>
<td>Normal</td>
</tr>
<tr>
<td>Diameter, $D$</td>
<td>50 mm</td>
<td>0.01</td>
<td>Normal</td>
</tr>
<tr>
<td>Toughness, $KIC$</td>
<td>24.8 MPa/$\sqrt{m}$</td>
<td>0.05</td>
<td>Lognormal</td>
</tr>
<tr>
<td>Poisson ratio, $\nu$</td>
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where \( f_{\theta}(X) \) is the joint probability density function of \( X \) and \( g(X) \) is the performance function given by

\[
g(X) = K_{IC} - K_{I,b} \tag{4}
\]

Substitute (1) and (2) into (4) then produces

\[
g(X) = K_{IC} - F_{I,b} \left( \frac{32M}{\pi D^3} \right) \sqrt{\frac{a}{\pi}} \tag{5}
\]

Codes of APDL are developed to incorporate the performance function as in (5) with the all random variables listed in Table 1. Then, MCS with Latin Hypercube Sampling is used to generate the random numbers and therefore calculating the probability of failure. The results of probability of failure from ANSYS are given in terms of cumulative density function (CDF).

III. FINITE ELEMENT MODELING

Finite element model (FEM) is constructed using ANSYS through the use of ANSYS Parametric Design language (APDL). Due to the symmetrical analysis, quarter FEM is used in order to reduce the computational efforts as shown in Fig. 1 and it is also used in the probabilistic analysis.

FINITE ELEMENT MODELING

MPC184 elements or rigid element beams are used. These elements are connected from an independent node to the bar. Then, bending moment is applied remotely to the independent node. Singular elements are used around the crack tip to ensure stress/strain singularities. Mid-side nodes of the singular elements are shifted to quarter position from the crack tip and stress intensity factors (SIF) are determined along the crack front. The results of SIFs can be obtained elsewhere [9-11]. Different crack aspect ratio, \( a/b \) range between 0.2 to 1.2 and relative crack depth, \( a/D \) between 0.1 and 0.3 are considered. The crack geometries considered in this work are based on the experimental observations [11]. The present model is then compared with the previous model from the literature for validation purposes. It is found that the present model is well in agreements with others as shown in Fig. 2.

IV. RESULTS AND DISCUSSION

Fig. 4 shows the probability of failure subjected to bending moment, \( P_{f,b} \) plotted against bending moment, \( M \). Two methods are used to estimate the \( P_{f,b} \) such as FEA and \( K \)-estimation. The results reveal that the \( P_{f,b} \) are in agreement to each others. The \( P_{f,b} \) are estimated for three points situated along the crack front which are A, D and G as depicted in Fig. 3. According to the Fig. 4, higher \( P_{f,b} \) is obtained at point A compared with other positions. However, when \( a/D \) is increased the curves of \( P_{f,b} \) become closer to each other. The band of \( P_{f,b} \) for \( a/D < 0.2 \) is larger than the \( P_{f,b} \) for \( a/D > 0.2 \). This is indicated that for deeper cracks result higher \( P_{f,b} \) and the different positions along the crack front are not affected the \( P_{f,b} \). Increasing the \( a/D \) also influenced the \( P_{f,b} \), for \( a/D = 0.1 \), the curves of \( P_{f,b} \) are almost vertical compared with the curves obtained using \( a/D > 0.1 \). The reliability of the cracked bars is affected even lower applied bending moment is used. Fig. 4 shows the comparison in CPU time between FEA and \( K \)-estimation. It is revealed that the analysis using \( K \)-estimation method solves the problem faster than FEA. Therefore, \( K \)-estimation is used for further probabilistic analysis. In the practical point of view, \( K \)-estimation method provided an excellent analysis tool to estimate the failure probability of the components. Fig. 5 provides the probabilistic analysis carried out at point G for different crack geometry, \( a/b \). Probabilistic based FEA is not performed. This is due to the fact that the \( K \)-estimation method capable to predict the probability of failure similar to FEA results. Crack aspect ratio, \( a/b \) played an important role in determining the \( P_{f,b} \). It can be observed that, there is not significant different between \( P_{f,b} \) for \( a/b = 0.2 \) and \( a/b = 0.4 \). However, the for \( a/b \leq 0.4 \) produced higher \( P_{f,b} \) relative to \( a/b = 0.6 \). It is shown in Fig. 6 that at point G, lower SIFs are obtained for \( a/b = 0.6 \) compared to \( a/b < 0.6 \). It is shown that typical probability curves are observed compared with Fig. 3.
V. CONCLUSION

Three-dimensional finite element model (FEM) was developed using ANSYS and used to determine the failure probability. Stress intensity factor (SIF) based estimation method called $K$-estimation method was developed to estimate the failure probability. Monte Carlo Simulation (MCS) was used to assist the FEM and $K$-estimation method to estimate the failure probability. Comparisons were conducted and found that the results were agreed to each others. $K$-estimation method capable to reduce the computational time compared to FEA results.

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REFERENCES
