Scaling group transformation for MHD boundary-layer flow of a nanofluid past a vertical stretching surface in the presence of suction/injection

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1. Introduction

A nanofluid is a new class of heat transfer fluids that contain a base fluid and nanoparticles. The use of additives is a technique applied to enhance the heat transfer performance of base fluids. The thermal conductivity of the ordinary heat transfer fluids is not adequate to meet today's cooling rate requirements. Nanofluids have been shown to increase the thermal conductivity and convective heat transfer performance of the base liquids. One of the possible mechanisms for anomalous increase in the thermal conductivity of nanofluids is the brownian motions of the nanoparticles inside the base fluids. A variety of nuclear reactor designs featured thermal properties at modest nanoparticle concentrations. Many of the publications on nanofluids are about understanding their behavior so that they can be utilized where straight heat transfer enhancement is paramount as well as in many industrial applications, nuclear reactors, transportation, electronics as well as biomedicine and food. In nuclear reactors, the heat is removed from fuel elements via forced convection, making this a much more important heat transfer process. Although nanofluids exhibit better heat transfer properties than pure substances, they also have a higher viscosity, which corresponds to an increase in pumping power. Nanofluids are engineered colloidal suspensions of nanoparticles in water and exhibit a very significant enhancement of the boiling critical heat flux at modest nanoparticle concentrations. They are also used in other electronic applications which use microfluidic applications.

Magnetic nanofluid is a unique material that has both the liquid and magnetic properties. Many of the physical properties of these fluids can be tuned by varying magnetic field. In addition, they have been wonderful model system for fundamental studies. As the magnetic nanofluids are easy to manipulate with an external magnetic field, they have been used for a variety of studies. Particle transport and deposition in flowing suspensions onto collector surfaces is of importance in a broad field of applications. Brownian diffusion and thermophoresis are important slip mechanisms in nanofluids. Nanofluids are suspensions of submicronic solid particles (nanoparticles) in common fluids. The term "nanofluid" refers to a liquid containing a suspension of submicronic solid particles (nanoparticles). The term was coined by Choi (1995). The characteristic feature of nanofluids is thermal conductivity enhancement, a phenomenon observed by Masuda et al. (1993). This phenomenon suggests the possibility of using nanofluids in advanced nuclear systems (Buongiorno and Hu (2005)). A comprehensive survey of convective transport in nanofluids was made by Buongiorno (2006), who says that a satisfactory explanation for the abnormal increase of the thermal conductivity and viscosity is yet to be found. He focused on the further heat transfer enhancement observed in
Nomenclature

\( R_0 \)  magnetic field strength
\( C \)  nanoparticles volume fraction
\( C_f \)  skin-friction parameter
\( C_{\infty} \)  nanoparticle volume fraction at the wall
\( C_{\infty} \)  ambient nanoparticle volume fraction
\( c_p \)  specific heat at constant pressure
\( D_B \)  Brownian diffusion coefficient
\( D_T \)  thermophoretic diffusion coefficient
\( f \)  dimensionless stream function
\( g \)  acceleration due to gravity
\( k \)  thermal conductivity
\( Le \)  Lewis number
\( M \)  magnetic parameter
\( N_B \)  Brownian motion parameter
\( N_T \)  thermophoresis parameter
\( N_r \)  buoyancy ratio
\( Pr \)  Prandtl number
\( P \)  pressure
\( R_e \)  local Rayleigh number
\( S \)  suction/injection parameter
\( T \)  temperature of the fluid
\( T_w \)  temperature at the wall
\( T_o \)  ambient temperature
\( \vec{v} \)  velocity vector
\( u, v \)  velocity components along \( x \) and \( y \) axes
\( U(x) \)  uniform velocity of the free stream flow
\( V_0 \)  velocity of suction/injection

Greek symbols

\( \alpha \)  thermal conductivity
\( \beta \)  coefficient of thermal expansion
\( \theta \)  dimensionless temperature
\( \phi \)  dimensionless nanoparticle volume fraction
\( \eta \)  similarity variable
\( \mu \)  dynamic viscosity
\( \sigma \)  electric conductivity of the fluid
\( \rho_f \)  density of the base fluid
\( \rho_f \)  density of the base fluid
\( \rho_{nf} \)  nanoparticle mass density
\( \rho_{nf} \)  heat capacity of the base fluid
\( \rho_{nf} \)  effective heat capacity of the nanoparticle material
\( \tau \)  heat capacity ratio
\( \nu \)  kinematic viscosity
\( \psi \)  stream function

Different reductions to ordinary differential equations. They studied the effects of a moving surface with vertical suction or injection through the porous surface and also analyzed the exact solution of boundary layer equations of a special non-Newtonian fluid over a stretching sheet by the method of Lie group analysis. Yurusoy et al. (2001) investigated the Lie group analysis of creeping flow of a second grade fluid. They constructed an exponential type of exact solution using the translation symmetry and a series type of approximate solution using the scaling symmetry and also discussed some boundary value problems. The objective of the present study is to analyze the development of the steady boundary layer flow, heat transfer and nanoparticle volume fraction over a stretching surface in a nanofluid for various parameters using scaling group of transformations viz., Lie group transformations (Fig. 1).

2. Mathematical analysis

We consider a two-dimensional problem. We select a coordinate frame in which the \( x \)-axis is aligned vertically upwards. We consider a vertical plate at \( y = 0 \). A uniform transverse magnetic field of strength \( B_0 \) is applied parallel to the \( y \)-axis. It is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible. At this boundary the temperature \( T \) and the nanoparticle fraction \( \phi \) take constant values \( T_0 \) and \( \phi_0 \), respectively. The ambient values, attained as \( y \) tends to infinity, of \( T \) and \( \phi \) are denoted by \( T_\infty \) and \( \phi_\infty \), respectively.

The Oberbeck–Boussinesq approximation is employed. The following four field equations embody the conservation of total mass, momentum, thermal energy, and nanoparticles, respectively.

The field variables are the velocity \( \vec{v} \), the temperature \( T \) and the nanoparticle volume fraction \( \phi \).

\[ \nabla \cdot \vec{v} = 0 \]  
\[ \rho_f \left( \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p - \sigma B_0^2 \vec{B}_0 + \mu \nabla^2 \vec{v} \]

\[ + \left[ C_{\phi} (1 - C) \left\{ \rho_f (1 - \beta (T - T_\infty)) \right\} \right] \hat{z} \]

(2)
where $\rho_f$ is the density of the base fluid, $\beta_0$ is a constant magnetic field, $\alpha$ is the electric conductivity, $\mu$ is the density, $\kappa$ is the thermal conductivity, $v$ is the volume expansion coefficient of the nanofluid, while $\rho_p$ is the density of the particles. The gravitational acceleration is denoted by $g$. The coefficients that appear in Eqs. (3) and (4) are the Brownian diffusion coefficient $D_\psi$ and the thermophoretic diffusion coefficient $D_T$. Details of the derivation of Eqs. (3) and (4) are given in the paper by Buongiorno (2008) and Nield and Kuznetsov (2009). The boundary conditions are taken to be

$$u = U(x), \quad v = V(x), \quad C = C_\infty, \quad T = T_\infty \text{ at } y = 0;$$
$$u = 0, \quad C = C_\infty, \quad T = T_\infty \text{ as } y \to \infty$$

We consider a steady state flow.

In keeping with the Oberbeck–Boussinesq approximation and an assumption that the nanoparticle concentration is dilute, and with a suitable choice for the reference pressure, we can linearize the momentum equation and write Eq. (2) as

$$\rho_f \left( \frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} \right) = -\nabla p - \sigma B_0^2 \psi + \mu \nabla^2 u$$

$$+ \left[ (\rho_f - \rho_{\infty})(C - C_\infty) + (1 - C_\infty)\rho_{\infty}(C - C_\infty) \right] \phi$$

We now make the standard boundary-layer approximation, based on a scale analysis, and write the governing equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial y}{\partial x} = -\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \left( \frac{\partial y}{\partial x} \right)$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial y}{\partial y} = 0$$

$$\frac{\partial T}{\partial x} + \frac{\partial y}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_\phi \frac{\partial C}{\partial T} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right]$$

$$\frac{\partial C}{\partial x} + \frac{\partial y}{\partial y} = D_\psi \frac{\partial C}{\partial T} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y}$$

where $u$ and $v$ are the velocity components along the $x$ and $y$ axes, respectively, $\alpha = \beta(\rho C_p) \rho_f$ is the thermal diffusivity of the fluid, $\nu$ is the kinematic viscosity coefficient and $\tau = (\rho C_p) (\rho C_p)$. The streamwise velocity and the suction/injection velocity are taken as

$$U(x) = C(x), \quad V(x) = V_0 x^{-m-1/2}$$

Here $c>0$ is constant, $T_\infty$ is the wall temperature, the power-law exponent $m$ is also constant. In this study we take $c = 1$.

One can eliminate $p$ from Eqs. (8) and (9) by cross-differentiation. At the same time one can introduce a stream function $\psi$ defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \theta = \frac{T - T_\infty}{T_\infty - T_\infty}, \quad \text{and} \quad \phi = \frac{C - C_\infty}{C_\infty - C_\infty}$$

So that Eq. (7) is satisfied identically. We are then left with the following three equations.

$$\frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} = -\alpha \frac{\partial^2 \psi}{\partial y^2}$$

$$+ (1 - \phi_\infty) \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial y} \Delta \theta - (\rho_f - \rho_{\infty}) \frac{\partial \psi}{\partial y} \Delta \phi$$

$$+ \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} = -\alpha \frac{\partial^2 \psi}{\partial y^2} + \frac{D_T}{T_\infty} \left( \frac{\partial \psi}{\partial y} \right)^2$$

$$\frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} = \frac{D_T}{T_\infty} \frac{\partial \psi}{\partial y}$$

$$\frac{\partial \psi}{\partial y} = \frac{D_T}{T_\infty} \frac{\partial \psi}{\partial y}$$

$$\theta \to 0, \quad \phi \to 0, \quad \psi \to 0 \text{ as } y \to \infty$$

where $\Delta \theta = T_\infty - T_\infty$ and $\Delta \phi = C_\infty - C_\infty$.

We now introduce the simplified form of Lie-group transformations namely, the scaling group of transformations,

$$T' = x', \quad y' = y' \phi' \psi, \quad \psi' = \psi' \phi' \psi$$

$$u' = u \phi \psi', \quad \theta' = \theta \phi \psi' \phi, \quad \phi' = \phi' \phi' \phi$$

Eq. (18) may be considered as a point-transformation which transforms co-ordinates ($x, y, \psi, u, \theta, \phi$) to the coordinates ($x', y', \psi', u', \theta', \phi'$). Substituting (18) in (14)-(16) we get,

$$\psi(x', y', x_0, y_0) \frac{\partial \psi(x', y', x_0, y_0)}{\partial x'} = \alpha \phi(x, y, x_0, y_0) \frac{\partial \psi(x, y, x_0, y_0)}{\partial x}$$

$$+ e^{-\phi(x, y, x_0, y_0)} \frac{\partial \psi(x, y, x_0, y_0)}{\partial y} \Delta \theta - \phi(x_0, y_0) \frac{\partial \psi(x, y, x_0, y_0)}{\partial y} \Delta \phi$$

$$e^{-\phi(x, y, x_0, y_0)} \frac{\partial \psi(x, y, x_0, y_0)}{\partial y} \Delta \theta - \phi(x_0, y_0) \frac{\partial \psi(x, y, x_0, y_0)}{\partial y} \Delta \phi$$

$$\phi(x_0, y_0) \frac{\partial \psi(x, y, x_0, y_0)}{\partial y} \Delta \theta - \phi(x_0, y_0) \frac{\partial \psi(x, y, x_0, y_0)}{\partial y} \Delta \phi$$

$$\phi(x_0, y_0) \frac{\partial \psi(x, y, x_0, y_0)}{\partial y} \Delta \theta - \phi(x_0, y_0) \frac{\partial \psi(x, y, x_0, y_0)}{\partial y} \Delta \phi$$

$$\phi(x_0, y_0) \frac{\partial \psi(x, y, x_0, y_0)}{\partial y} \Delta \theta - \phi(x_0, y_0) \frac{\partial \psi(x, y, x_0, y_0)}{\partial y} \Delta \phi$$

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$$\phi(x_0, y_0) \frac{\partial \psi(x, y, x_0, y_0)}{\partial y} \Delta \theta - \phi(x_0, y_0) \frac{\partial \psi(x, y, x_0, y_0)}{\partial y} \Delta \phi$$

The system will remain invariant under the group of transformations $\Gamma$, we would have the following relations among the parameters, namely

$$\alpha_1 + 2\alpha_2 = 3 \alpha_2 - \alpha_3 = \alpha_2 - \alpha_3 = -\alpha_3$$

$$\alpha_1 + \alpha_2 = 3 \alpha_2 - \alpha_3 = \alpha_2 - \alpha_3 = -\alpha_3$$

$$\alpha_1 + \alpha_2 = 2 \alpha_2 - \alpha_3 = \alpha_2 - \alpha_3 = -\alpha_3$$

$$\alpha_1 + \alpha_2 = 2 \alpha_2 - \alpha_3 = \alpha_2 - \alpha_3 = -\alpha_3$$

Hence, $\alpha_1 + 2\alpha_2 = 3 \alpha_2 - \alpha_3$ gives the value $\alpha_2 = 0$

$$\alpha_1 + \alpha_2 = 2 \alpha_2 - \alpha_3 = \alpha_2 - \alpha_3 = -\alpha_3$$

$$\alpha_1 + \alpha_2 = \alpha_2 - \alpha_3 = -\alpha_3$$

The boundary conditions yield $\alpha_4 = \alpha_2 = 1/2 \alpha_1$

$$\alpha_4 = \alpha_2 = 1/2 \alpha_1$$

$$\alpha_4 = (m - 1)/2 \alpha_1 = -1/4 \alpha_1$$
In view of these, the boundary conditions become

\[ \frac{\partial \phi^*}{\partial y^*} = x^* \left( \frac{\partial \phi}{\partial x} \right) = x^* \left( \frac{\partial \phi}{\partial x} \right) = -V_0 x^* \left( \frac{\partial \phi}{\partial x} \right), \quad \theta^* = \phi^* = 1 \quad \text{at} \quad y^* = 0 \]

and \( \frac{\partial \phi^*}{\partial y^*} \rightarrow 0, \quad \theta^* \rightarrow 0, \quad \phi^* \rightarrow 0 \quad \text{as} \quad y^* \rightarrow \infty \) \hspace{1cm} (22)

The set of transformations \( x^* \) reduces to \( x = x e^\alpha_1, \quad y^* = ye^{\alpha_1/4}, \quad \psi^* = \psi^* e^{(3\alpha_1/4)}, \quad u^* = u e^{\alpha_1/2}, \quad v^* = v e^{\alpha_1/4}, \quad \theta^* = \theta, \quad \phi^* = \phi \)

Expanding by Taylor's method in powers of \( \varepsilon \) and keeping terms up to the order \( \varepsilon \) we get

\[ x^* = x = x e^\alpha_1, \quad y^* = y = y e^{\alpha_1/4}, \quad \psi^* = \psi = \psi e^{3\alpha_1/4}, \quad u^* = u = u e^{\alpha_1/2}, \quad v^* = v = v e^{\alpha_1/4}, \quad \theta^* - \theta = \phi^* - \phi = 0 \]

From the above equations we get,

\[ y^* e^{\alpha_1/4} = \eta, \quad \psi^* = x^{(3/2)}(\eta), \quad \theta^* = \theta(\eta), \quad \phi^* = \phi(\eta) \]

With the help of these relations, Eqs. (19)-(21) become

\[ f'' + \frac{1}{Pr} \left[ \frac{0.75\phi''}{\varepsilon} - \frac{0.5\phi'}{\varepsilon^2} \right] + Rax \left[ \theta - N \phi' \right] + Mf' = 0 \]

\[ \theta'' + 0.75\phi'' + Nb\phi' + N\theta'' = 0 \]

\[ \phi'' + 0.75Ls\phi' + \frac{Nt}{Nb} \frac{\partial^2 \phi}{\partial x^2} = 0 \] \hspace{1cm} (26)

The boundary conditions take the following form

\[ f' = 1, \quad f = \frac{-4V_0}{3}, \quad \theta = \phi = 1 \quad \text{at} \quad \eta = 0 \text{ and } f' \rightarrow 0, \quad \theta' \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \] \hspace{1cm} (27)

where \( Pr = \rho c_p a_c^2 \) is the Prandtl number, \( Rax = (1 - \phi_{m}) \left( \frac{\partial \theta}{\partial x} + \frac{\partial \psi}{\partial y} \right) \) is the local Rayleigh number, \( Nb = \left( \frac{\rho \phi - \rho_{m}}{\rho_{m}} \right) \left( \frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x} \right) \left( 1 - \phi_{m} \right) \) is the buoyancy ratio, \( Nb = - \left( \frac{\mu_s \phi_s D_D A}{\varepsilon} \right) \left( \varepsilon \right) \) is the Brownian motion parameter, \( rt = \left( \frac{\alpha R^2(\varepsilon)}{\mu_s} \right) \) is the thermophoresis parameter, \( M = \frac{\alpha R^2(\varepsilon)}{\mu_s} \) is the magnetic parameter and \( Le = v/D \) is the Lewis number.

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where \( S = -(4/3) V_0 \), \( S > 0 \) corresponds to suction and \( S < 0 \) corresponds to injection whereas \( V_0 \) is the velocity of suction if \( V_0 < 0 \) and injection if \( V_0 > 0 \).

3. Numerical solution

The set of non-linear ordinary differential equations (24)-(26) with boundary conditions (28) have been solved by using the Runge-Kutta-Gill algorithm (Gill, 1951) with a systematic guessing of \( f'(0), \theta(0), \phi(0) \) by the shooting technique with until the boundary conditions at infinity \( f'(0), \theta'(0), \phi'(0) \) decay exponentially to zero. The step size \( \Delta \eta = 0.001 \) is used while obtaining the numerical solution with \( n_{max} \), and accuracy to the fifth decimal place is sufficient for convergence. The computations were done by a program which uses a symbolic and computational computer language Matlab. A step size of \( \Delta \eta = 0.001 \) was selected to be satisfactory for a convergence criterion of \( 10^{-7} \) in nearly all cases. The value of \( n_{max} \) was found to each iteration loop by assignment state \( n_{max} = n_{min} + \Delta n \). The maximum value of \( n_{max} \) to each group of parameters \( Pr, Le, M, Nb, Nt \) determined when the values of unknown boundary conditions at \( \eta = 0 \) not change to successful loop with error less than \( 10^{-7} \) effects of development of the steady boundary layer flow, heat transfer and nanoparticle volume fraction over a stretching surface in a nanolfoil are studied for different values of Brownian motion parameter, thermophoresis parameter, magnetic parameter and Lewis number. In the following section, the results are discussed in detail.

4. Results and discussion

Numerical analysis are carried out for \( 0.5 < Nt < 2.5, \quad 0.5 < Nt < 2.0, \quad 0.5 < M < 2.0 \) and \( -2.0 < S < 1.0 \). Eqs. (24)-(26) subject to the boundary conditions (28) have been solved numerically for some values of the governing parameters \( Pr, Le, M, Nb, Nt \) and \( Nt \) using Runge-Kutta-Gill algorithm with shooting technique. Neglecting the effects of \( Nb \) and \( Nt \) numbers, the results For the reduced Nusselt number \( -f'(0) \) are compared with those obtained by Khan and Pop (2010), Wang (1989), and Coria and Sidawi (1994) for different values of \( Pr \) in Table 1. We notice that at the comparison shows good agreement for each value of \( Pr \). Therefore, we are confident that the present results are very accurate.

In the absence of Local Rayleigh number \( Rax \), in order to ascertain the accuracy of our numerical results, the present study is compared with the available exact solution in the literature. The nanoparticle fraction profiles for Lewis number \( Le \) are compared...
Table 1: Comparison of results for \(\theta(0)\) with previous published works.

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With the available exact solution of Khan and Pop (2010), are shown in Fig. 2a and b. It is observed that the agreements with the theoretical solution of nanoparticle fraction profiles are excellent. For a given \(Nb\) and \(Nt\), it is clear that there is a fall in nanoparticle fraction with increasing the Lewis number. This is due to the fact that there would be a decrease of nanoparticle volume fraction boundary layer thickness with the increase of Lewis number as one can see from Fig. 2a and b by comparing the curves with \(Le = 10\), \(Le = 20\) and \(Le = 30\).

Volume fraction of nanoparticles is a key parameter for studying the effect of nanoparticles on flow fields and temperature distributions. Thus Figs. 3 and 4 are prepared to present the effect of Brownian motion on temperature distribution and volume fraction of nanoparticle. Figs. 3 and 4 illustrate the typical temperature and nanoparticle volume fraction profiles for various values of Brownian motion parameter, \(Nb\). Temperature of the fluid increases and the nanoparticle volume fraction decreases with increase of \(Nb\). It is interesting to note that Brownian motion of nanoparticles at the molecular and nanoscale levels is a key nanoscale mechanism governing their thermal behavior. In nanofluid systems, due to the size of the nanoparticles Brownian motion takes place which can affect the heat transfer properties. As the particle size scale approaches to the nanometer scale, the particle Brownian motion and its effect on the surrounding liquids play an important role in heat transfer.

Figs. 5 and 6 display the effect of thermophoretic parameter \(Nt\) on temperature and nanoparticle volume fraction profiles. It is to note that the temperature of the fluid increases whereas the nanoparticle volume fraction decreases with increase of \(Nt\). We notice that, positive \(Nt\) indicates a cold surface, but is negative to a hot surface. For hot surfaces, thermophoresis tends to blow the nanoparticle volume fraction boundary layer away from the surface since a hot surface repels the sub-micron sized particles from it thereby forming a relatively particle-free layer near the surface. As a consequence, the nanoparticle distribution is formed just outside. In particular, the effect of increasing the thermophoretic parameter \(Nt\) is limited to increasing slightly the wall slope of the nanoparticle volume fraction profiles but decreasing the nanoparticle volume fraction. This is true only for small values of Lewis number for which the Brownian diffusion effect is large compared to the convection effect. However, for large values of Lewis number, the diffusion effect is minimal compared to the convection effect.

![Fig. 2](image1.png)  
**Fig. 2.** Temperature profiles for various values of \(Nb\) when \(Pr = 2.0, Le = 3.0, Nt = 1.0, S = 2.0\) and \(Nt = 0.5, M = 1.0\).  

![Fig. 3](image2.png)  
**Fig. 3.** Temperature profiles for various values of \(Nb\) when \(Pr = 2.0, Le = 3.0, Nt = 1.0, S = 2.0\) and \(Nt = 0.5, M = 1.0\).  

![Fig. 4](image3.png)  
**Fig. 4.** Effect of \(Nb\) over the nanoparticle volume fraction profiles when \(Pr = 2.0, Le = 3.0, Nt = 1.0, S = 2.0\) and \(Nt = 0.5, M = 1.0\).  

![Fig. 5](image4.png)  
**Fig. 5.** Temperature profiles for various values of \(Nt\) when \(Pr = 2.0, Le = 3.0, Nb = 1.0, S = 2.0\) and \(Nt = 0.5, M = 1.0\).  

![Fig. 6](image5.png)  
**Fig. 6.** Temperature profiles for various values of \(Nt\) when \(Pr = 2.0, Le = 3.0, Nb = 1.0, S = 2.0\) and \(Nt = 0.5, M = 1.0\).
and, therefore, the thermophoretic parameter \( N_t \) is expected to alter the nanoparticle volume fraction boundary layer significantly. Although thermophoresis effect is important in natural convection of nanofluids, there are other parameters that may have effect and should be considered. These effects include increase in effective viscosity of nanofluids due to the presence of nanoparticles and density variation due to variable volume fraction. More volume fraction of nanoparticles makes nanofluid much visous and the mixture's convection takes place weaker, thus natural convective Nusselt number decreases due to high viscose fluid. On the other hand it is shown that the separation factor for common nanofluids is positive and density variation due to variable volume fraction of nanoparticles, called particulate buoyancy force, helps nanofluid to have strong convection heat transfer.

Figs. 7–9 depict the influence of the suction/injection parameter \( S \) on the velocity, temperature and nanoparticle volume fraction profiles in the boundary layer when the thermophoretic particle deposition is uniform, i.e. \( N_t = 1.0 \). With the increasing value of the suction \( (S > 0) \), the velocity is found to decrease (Fig. 7), i.e. suction causes to decrease the velocity of the fluid in the boundary layer region. In case of suction, the heated fluid is pushed towards the wall where the buoyancy forces can act to retard the fluid due to high influence of the Brownian motion effects. This effect acts to decrease the wall shear stress. Figs. 8 and 9 exhibit that the temperature \( \theta(\eta) \) and nanoparticle volume fraction \( \phi(\eta) \) in boundary layer also decrease with the increasing suction parameter \( (S > 0) \) (the fluid is of uniform thermophoretic particle deposition, i.e. \( N_t = 1.0 \)) \( (Pr = 2.0 \text{ and } \alpha = 1.0) \). The explanation for such behavior is that the fluid is brought closer to the surface and reduces the thermal and nanoparticle volume boundary layer thickness in case of suction. As such then the presence of wall suction decreases velocity boundary layer thicknesses but decreases the thermal and nanoparticle volume fraction boundary layers thickness, i.e. thins out the thermal and nanoparticle volume fraction boundary layers. However, the exact opposite behavior is produced by imposition of wall fluid blowing or injection. These behaviors are also clear from Figs. 7–9. The samples of velocity, temperature and the nanoparticle volume fraction profiles are given in Figs. 7, 8 and 9, respectively. These profiles satisfy the far field boundary conditions (28) asymptotically, which support the numerical results obtained.

Figs. 10 and 11 present typical profiles for velocity and temperature for different values of magnetic parameter. Due to the
uniform thermophoresis particle deposition, it is clearly shown that the velocity of the fluid decreases and the temperature of the fluid increases whereas the nanoparticle volume fraction of the fluid is not significant with increase of the strength of magnetic field. The effects of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive-type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid and to increase its temperature profiles. This result qualitatively agrees with the expectations, since magnetic field exerts retarding force on the natural convection flow. Application of a magnetic field moving with the free stream has the tendency to induce a motive force which decreases the motion of the fluid and increases its boundary layer.

5. Conclusions

In the present paper, we have studied theoretically the problem of steady MHD boundary-layer flow of a nanofluid past a vertical stretching surface in the presence of suction/injection. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. It is found that the volume fraction of nanoparticles is a key parameter for studying the effect of nanoparticles on flow fields and temperature distributions. It is interesting to note that the impact of thermophoresis particle deposition in the presence of magnetic field with Brownian motion have a substantial effect on the flow field and, thus, on the heat transfer and nanoparticle volume fraction rate from the sheet to the fluid. Particularly, the temperature of the fluid increases whereas the nanoparticle volume fraction decreases with increase of Brownian motion and thermophoretic parameter, respectively. Brownian motion of nanoparticles at the molecular and nanoscale levels is a key nanoscale mechanism governing their thermal behavior. In nanofluid systems, due to the size of the nanoparticles Brownian motion and thermophoresis takes place which can affect the heat transfer properties.

The analysis has helped engineers understand the mechanisms that are most important in the deposition process. Free convective flow through porous media is an area of research undergoing rapid growth in the fluid mechanics and heat transfer and nanoparticle volume fraction field due to its broad range of scientific and engineering applications. One of the technological applications of nanoparticles that hold enormous promise is the use of heat transfer fluids containing suspensions of nanoparticles to confront cooling problems in thermal systems. Hence, the subject of nanofluids is of great interest worldwide for basic and applied research. Nanofluids are important because they can be used in numerous applications involving heat transfer and other applications such as heat exchanger. Colloids which are also nanofluids have been used in the biomedical field for a long time, and their use will continue to grow. Nanofluids have also been demonstrated for use as smart fluids.