Recursive parameter estimation for discrete-time model of an electro-hydraulic servo system with varying forgetting factor

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In general, an electro-hydraulic servo (EHS) system inherently suffers from parameter uncertainties and variation which makes the modeling and controller design complicated. To encounter those difficulties, recursive least square (RLS) is often used with advanced control strategy for position, force and pressure control of the EHS system. In this paper, a new RLS estimator with varying forgetting factor was proposed for the recursive parameter estimation process. The experimental work began with an organized procedure in developing a linear discrete-time model by emphasizing the offline identification process. Best fitting criterion, final prediction errors, minimum of loss function and correlation analysis were utilized to investigate the validity of the developed model. In recursive estimation, the proposed technique gave better estimations in terms of convergence speed and accuracy as compared to the conventional approach. Furthermore, it also showed that algorithm is more sensitive to parameter changes and improves the estimation results for each estimated parameter instead of using fixed forgetting factor.

Key words: Electro-hydraulic servo system, system identification, parameter estimation, recursive least square.

INTRODUCTION

Electro-hydraulic servo (EHS) system is crucial as an actuator especially in heavy engineering applications, including transportation, manufacturing and construction field. The potential in providing large forces and high control accuracies due to the high power to weight ratio and compact design in EHS system makes this actuator to be more prominent nowadays (Merrit, 1967). The greatest technology in this actuator system caused many significant changes in the industrial community where the development in control and design of EHS system purposely make the machinery to be more economical, attractive, harmless and efficient for a wide range of applications.

Hydraulically-operated machinery, such cranes, bulldozers, backhoes, shovels and forklifts are recognizable in construction fields as earthmoving and lifting equipment. In manufacturing industry, plastic making process, primary metal extraction, furnace equipment, rubber process and textile machinery are fully utilizing the advantages of the fluid power control by incorporating the EHS system. The applications of EHS system currently have been extended to equipment and systems which are used in airplanes, rockets and spaceships for transmission and rudder control. Besides the industrial tractors, material handling equipment and construction...
machinery in heavy transportation, the application is comprehensively implemented in commercial vehicles. Research in EHS system has also attracted a great attention to researchers and academia in different applications over the past years, such as aircrafts (Karpenko and Sepehri, 2009), manufacturing machines (Renn and Tsai, 2005; Chiang et al., 2005; Jha and Jain, 2004), fatigue testing (Ruan et al., 2006; Shafigh et al., 2011), hydraulic excavator (Chiang and Huang, 2004), sheet metal forming process (Lo and Yang, 2004), hydraulic cranes (Safarzadeh et al., 2011) and all kinds of automation especially in automotive applications (Wu and Shih, 2003; Samek et al., 2004; Plummer, 2006). In general, it is difficult to identify and develop an accurate dynamics model of EHS system since the system inherently has many uncertainties, highly nonlinear and time varying which makes the modelling and controller design to be more complicated. Actuator frictions, nonlinear flow-pressure characteristics, fluid compressibility, internal-external leakages and valve hysteresis are identified as the major sources of nonlinearity existing in the EHS system (Jelali and Kroll, 2003).

Linearization of EHS system has been studied and employed over the past decades to encounter the nonlinearities subsists in the modeling process. There are numerous researchers who used that linear model in either continuous-time (Lee and Srinivasan, 1989; Knohl and Unbehauen, 2000; Mihajlov et al., 2002; Yanada and Furuta, 2007) or discrete-time (Finney et al., 1985; Plummer and Vaughan, 1998; Ziaei and Zepehri, 2001; Zulfatman and Rahmat, 2009) in their proposed control strategy. It is well-known that a linear model is sufficient for position tracking control with advanced control strategy, such as neural network (NN) (Knohl and Unbehauen, 2000), self-tuning fuzzy PID (Zulfatman and Rahmat, 2009; Cetin and Akkaya, 2010), variable structure control (VSC) with sliding mode (Mihajlov and Nicolic, 2002; Chuang and Shiu, 2004; Chin et al., 2005; Ghazali et al., 2010), adaptive feed-forward compensator (PFO) (Yanada and Furuta, 2007), model reference adaptive control (MRAC) (Ziaei and Zepehri, 2001; Kiraocchi et al., 2003) and generalized predictive control (GPC) (Sepehri and Wu, 1988). Most of the modelling approaches for discrete-time model that have been implemented in previous researches are developed from first principle or physical laws.

Although, the importance of physical modelling is always considered in the controller design, but in real implementation, validation of the physical plant model is necessary in optimizing the use of controller that is commonly designed via computer simulation. Furthermore, any changes in the EHS system's parameter may reduce the controller performance, and the desired specification possibly not achieved. Therefore, system identification with online estimation technique is always performed as an adaptive mechanism integrating with other control design. Many industrial control applications, such as adaptive control, adaptive prediction and adaptive filtering need information of the system to be available recursively while the system is in operation. The techniques for determining an online model are called recursive identification methods and the algorithms are usually named as recursive parameter estimation, adaptive parameter estimation, sequential estimation and online algorithms. Recursive least square is one of the online estimation algorithms that has gained much interest. The combination of RLS with various control strategy has also been widely implemented for many years (Lee and Srinivasan, 1989; Plummer and Vaughan, 1996; Ziaei and Zepehri, 2001; Bobrow and Lum, 1996; Richardson et al., 2001). When combined with contrc scheme as an adaptive mechanism, the convergence speed of plant parameters is very important to make the controller responds with any changes in the system; where in real application, the control and plant parameters are achieved simultaneously.

A number of researchers have reported the implementation of RLS algorithm in different industrial applications. Ziaei and Sepehri (2000) discussed some practical issues in EHS system and estimated periodically the unknown parameters using standard RLS algorithm. Based on one-step-ahead prediction errors, it is observed that third order model is adequate to represent the system. The experimental works verified that the multiple-step-ahead prediction performance was inadequate because of nonlinear behavior in the system. Similar algorithm and configuration have been realized by Eker (2004a, b) for electromechanical system. The selection of the initial value of covariance matrix is achieved using a Bierman's upper-diagonal (U-D) factorization algorithm to guarantee the robustness in the estimation process. Subsequently, the work has been extended in (Kara and liker, 2004a, b) by implementing the RLS algorithm in their experimental study for linear and nonlinear cases. The Hammerstein structure is used in representing the direct-current (DC) motor with load for low speed two-directional operation. Followed by Tutunji et al. (2007), two mechatronic systems which are servo DC motor and Gyroscope system have been tested to identify the input-output patterns with excellent accuracy. The method also has been adopted in hydrostatic transmission system (Rabbo and Tutunji, 2008). Recently, Saleem et al. (2009) utilized the RLS estimator for pneumatic system where the best performance results obtained using one-step prediction for the discrete-time model and effects of the step predictions are explained in the study. Then, the identified model has been adopted in the controller design.
It can be summarized from the presented research studies, that the RLS estimator is extensively used as the capability to reduce bias, simple numerical solution and fast parameter convergence (Ghazali et al., 2011). Since the RLS algorithm is greatly influenced by the forgetting factor, researchers attempt to choose the most suitable forgetting factor depending on the plant response. It is common to choose the parameter via trial and error or based on heuristic method where the value depends on the experience of previous research. It is also noted that there is much less attention in validating the model structure selection. Some researchers only focused on the online estimation while ignoring the importance of offline identification. The proper modelling procedure in system identification should start with offline identification either parametric or nonparametric as an important step to determine the system properties, such as model order and pure time delay. The estimated parameters in offline identification can be used as initial value of parameters in the online identification to obtain better convergence and less computation cost (Plummer and Vaughan, 1995). Besides, the estimated value from offline identification can serve as a reference while estimating the plant parameters recursively.

In this paper, the RLS estimator with varying forgetting factor is introduced for discrete-time model of an EHS system. A third order discrete-time model is validated and clearly discussed using off-line identification by analyzing its residual function. Correlation of residual, final prediction error, minimum of loss function and best fitting criterion are taken into account to ensure the validity of the EHS system. Then, the on-line estimation is implemented to determine the estimated parameters with fast convergence and high accuracy.

**MATHEMATICAL MODEL OF THE EHS SYSTEM**

Electro-hydraulic system combines the advantages of hydraulic actuation with the flexibility of electronic instrumentation. The schematic of the position control of the EHS system is illustrated in Figure 1 and its nomenclature in Table 1 described the physical parameter consisting of hydraulic actuator and servo valve with continuous supply pressure which can be simply connected with computer through data acquisition card with digital-to-analog (DAC) and analog-to-digital (ADC) converter. A complete mathematical model with internal leakage, actuator leakage and friction model have been discussed lately (Kalyoncu and Haydim, 2008; Rahmat et al., 2011) with advanced control strategy.

However, the physical models derived in that simulation study are highly complex and difficult to utilize in some of the control designs, especially in industrial field. Most of the parameters involved in that mathematical model are usually not available in the manufacturer’s datasheet and vastly vary with time due to its nonlinearity. Besides, the parameters are affected by the hydraulic oil temperature, supply pressure changes and aging. A mathematical model of an EHS system can be developed by neglecting these nonlinearities, such as internal or external leakage and dynamics of the valve as explained (Knoll and Unbehauen, 2000). In servo valve design, the dynamics of the valve can be approximated as a single gain, where

### Table 1. Nomenclature.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_i, A_o$</td>
<td>Piston area</td>
<td>m²</td>
</tr>
<tr>
<td>$P_s$</td>
<td>Pump pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$P_b$</td>
<td>Return pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$P_i, P_o$</td>
<td>Piston pressure in and out</td>
<td>Pa</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Pressure drop across the load</td>
<td>Pa</td>
</tr>
<tr>
<td>$V_i, V_o$</td>
<td>Volume for each chamber</td>
<td>m³</td>
</tr>
<tr>
<td>$q_{in}, q_{out}$</td>
<td>Flow in and out in servo valve</td>
<td>m³/s</td>
</tr>
<tr>
<td>$Q_i$</td>
<td>Total oil flow</td>
<td>m³/s</td>
</tr>
<tr>
<td>$u$</td>
<td>Control signal</td>
<td>V</td>
</tr>
<tr>
<td>$x_v$</td>
<td>Spool valve position</td>
<td>m</td>
</tr>
<tr>
<td>$K_v$</td>
<td>Servo valve amplifier gain</td>
<td>m/V</td>
</tr>
<tr>
<td>$K_g$</td>
<td>Flow-gain coefficient</td>
<td>m³/s V</td>
</tr>
<tr>
<td>$K_x$</td>
<td>Flow-pressure coefficient</td>
<td>m²/Na</td>
</tr>
<tr>
<td>$F_a$</td>
<td>Actuator force</td>
<td>N</td>
</tr>
<tr>
<td>$M$</td>
<td>Load mass</td>
<td>kg</td>
</tr>
</tbody>
</table>
the spool valve structure is assumed as critical-centre and symmetrical. The equation relates the input signal (normally voltage) and the spool valve position as shown in Equation 1.

\[ x_v = K_x u \]  \hspace{1cm} (1)

With Equation 1, the dynamics of the electro-hydraulic system are derived from a Taylor series linearization by the following equation:

\[ Q_L = K_q u - K_C P_L \]  \hspace{1cm} (2)

Defining the load pressure, \( P_L \), is the pressure across the actuator piston; its derivative is given by the total load flow through the actuator divided by the fluid capacitance as shown in Equation 3.

\[ \dot{P}_L = \frac{4\beta_c}{V_i} (Q_L - C_p - A_P \dot{y}) \]  \hspace{1cm} (3)

And

the force of the actuator can be determined:

\[ F_a = A_p P_L = M_i \dot{y} \]  \hspace{1cm} (4)

Substituting Equations 2 and 3 into the derivative of Equation 4 and taking a Laplace transform that yields:

\[ Y(s) = \frac{K \omega_n^2}{s(s^2 + 2\xi \omega_n s + \omega_n^2)} \]  \hspace{1cm} (5)

where

\[ K = \frac{K_x K_v}{A_p}, \quad \omega_n = A_p \sqrt{\frac{4\beta_c}{V_i M_i}} \]  \hspace{1cm} and

\[ \xi = \frac{4\beta_c M_i (K_e + C_p)}{2A_p V_i} \]

Open loop transfer function of the EHS system relates between the control signal from the computer/controller and position of the hydraulic actuator. The block diagram as shown in Figure 2 represents the modelling process. The corresponding discrete-time model follows by transforming the continuous-time model in Equation 5 with zero-order-hold as in Equation 6.
\[ G(z^{-1}) = \frac{y(k)}{u(k)} = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}} \]

(6)

In practical, it is difficult to accurately derive and implement the mathematical model of the nonlinear system derived in Kalayconciliation and Haylim (2009). Therefore, to design a controller that is applicable and easy to realize in practice, simplified system model must be obtained around operating point or model order reduction. The parameters of the linear model of the EHS system given by Equation 6 that depend on the coefficients derived by linearization are all affected by the nonlinearity and disturbance which are varied for different environments. The linearized model from the EHS system can be determined by using system identification technique. Subsequently, the system identification technique employed in this research was discussed.

**SYSTEM IDENTIFICATION**

System identification is utilized to develop mathematical model of dynamic systems from experimental data. The identification process consists of estimating the unknown parameters of the systems dynamics. The identification method based on least square algorithm has been recommended for the identification process for ease of implementation in real systems. For linear identification process, a discrete time model for the EHS system can be utilized. The discrete-time model for the linear system is given as shown in Equation 7 (Ljung, 1999).

\[ A(q^{-1})y(k) = q^{-n_b}B(q^{-1})u(k) + e(k) \]

(7)

where

\[ A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + ... + a_{n_b} q^{-n_b} \]

\[ B(q^{-1}) = b_1 + b_2 q^{-1} + b_3 q^{-2} + ... + b_{n_b} q^{-n_b+1} \]

and the symbol \( q^T \) denotes the backward shift operator, \( u(k) \) and \( y(k) \) are the system input and output, respectively and \( e(k) \) is the white noise of the system with zero mean. The parameter \( a_i \) and \( b_j \) are real coefficient while \( n_k \) is the number of poles, \( n_b \) is the number of \( b \) parameters (or equal to the number of zeros plus 1). \( n_b \) is the number of samples before the input affects output of the system which is also called the delay or dead time of the model.

Equation 7 can be put into linear regression form as shown in Equation 8.

\[ y(k) = \phi^T(k)\theta(k) + e(k) \]

(8)

where

\[ \phi(k) = (-y(k-1), -y(k-n_k), u(k-n_k), ..., u(k-n_k-n_b+1)) \]

\[ \theta(k) = (a_1, a_2, ..., a_{n_b}, b_1, b_2, ..., b_{n_b}) \]

System identification for off-line identification or batch processing required two data sets for estimation and validation purpose. From the available input-output data, it can be separated into two sets of data instead of acquiring data from two different experiments. Some preprocessing might also be needed to ensure that acquired data samples are free from external noise, scaling problems, outliers and other corruptions. It is common to follow the procedure depicted in Figure 3 when attempting to identify a model of dynamic system (Soderstrom and Stoica, 1989; Ljung, 1999).
The least square method is used to estimate the parameters of the system, denoted by \( \theta(k) \), such that it minimizes the sum of squares of the residual, \( \varepsilon(k) \). The system parameters, \( \theta(k) \), is obtained by minimizing the quadratic loss function as in Equations 9 and 10.

\[
J(\theta) = \frac{1}{2} \sum [\varepsilon(k)]^2
\]  

(9)

\[
\varepsilon(k) = y(k) - \phi^T(k)\hat{\theta}(k)
\]  

(10)

By setting \( \frac{dJ}{d\theta} = 0 \), the loss function can be minimized and giving Equation 11 as:

\[
\hat{\theta}(k) = [\phi^T(k)\phi(k)]^{-1}\phi^T(k)y(k)
\]  

(11)

A number of analyses on different data sets have to be completed to ensure the reliability and accuracy of the identified model. This process also requires a sufficient set of experimental data of the system which might be time consuming. In addition, data recollection for the estimation process must be carried out if there are some changes or replacement of any components in the system. For that reason, online identification saves time in data collection and any changes in the system components and structure affecting the system model can be resolved without the need of data recollection which is more practical to overcome the disadvantage of offline identification. Recursive least square (RLS) algorithm has been recommended for on-line identification due to its advantages as stated in (Soderstrom and Stoica, 1989; Ljung, 1994). For the RLS algorithm to be able to update the parameters at each sample time, it is necessary to define an error from Equation 8:

\[
\varepsilon(k) = y(k) - \phi^T(k)\hat{\theta}(k-1)
\]

(12)

\[
P(k) = \frac{1}{\lambda}P(k-1) - \frac{\phi(k)\phi^T(k)P(k-1)}{\lambda + \phi^T(k)P(k-1)\phi(k)}
\]

(13)

\[
\hat{\theta}(k) = \hat{\theta}(k-1) + P(k)\phi(k)\varepsilon(k)
\]

(14)

where \( \hat{\theta}(k) \) is the estimated parameter, \( P(k) \) is the covariance matrix, the subscript 'p' is the dimension of the identity matrix, \( P = n_p + n_h \), and \( \lambda \) is the forgetting factor, \( 0 < \lambda \leq 1 \).

The selection of \( \lambda \) in the algorithm is very important and highly affects the estimation results. Theoretically, one must have \( \lambda \to 1 \) to get convergence and no data will be forgotten. Therefore, if the parameters to be identified are time-varying, \( \lambda \) should take a value less than 1. The smaller its value is, the faster old data is discarded. On the other hand, the estimated parameter changes quickly because the algorithm becomes very sensitive. Therefore, by substituting \( \lambda \) in Equation 13 by \( \lambda(k) \), where a typical selection is to let \( \lambda(k) \) tends exponentially to 1, it is an advantage to fully utilize the important behaviour of the forgetting factor. This can be written as in Equation 15, where \( \lambda_0 \) is the initial value of the forgetting factor.

\[
\lambda(k) = \lambda_0\lambda(k-1) + (1 - \lambda_0)
\]

(15)

The process to update the forgetting factor, covariance matrix and estimated parameters can be seen in Figure 4. Root-mean-square-error (RMSE) criterion is used to determine if there are any significant changes in model parameters. It is required to reset or repose all the variables to its initial value for this algorithm with the aim of convergence and accuracy.
EXPERIMENTAL DESIGN

The electro-hydraulic system in this study consists of single-ended cylinder type of actuator and the pressurized fluid flow is controlled by a Rexroth servo valve. The double acting cylinder has 300 mm stroke length, 63 mm bore size and 36 mm rod size. The displacement sensor is mounted at the top of the cylinder rod and both input-output signals are acquired using data acquisition system. Figure 5 shows the experimental workbench where the input-output measurement was acquired for identification process. This experimental work utilized the mixed-reality environment (MRE) or hardware-in-the-loop (HIL) scheme for the recursive estimation process. The concept of MRE has been discussed clearly by Saleem et al. (2009) for pneumatic servo drives.

The choice of input or perturbation signals is elementary in system identification procedure. The input signals for the identification experiment should be chosen in such a way that all modes are sufficiently excited, because the signals may influence
the estimated parameter. The obtained parametric model is more accurate in frequency region where the input signal contains much energy and has to be rich enough to excite all interesting modes of the system (Soderstrom and Stolca, 1989).

For software implementation, Simulink and Real Time Windows Target in MATLAB® have been used for analysis and communication with the hardware. The stimulus input signal is also generated using a command window in MATLAB® software. The selection of sampling time is important as to ensure the accuracy of the system response. In practice, the sampling time must be a bit higher than the Nyquist sampling theorem to ensure that the resulting data will be useful in further analysis. It is established that, 50 ms sampling time for the EHS system is sufficient to avoid folding and aliasing problem occurring during the sampling process (Zulfatman and Rahmat, 2008).

Offline identification was performed, first in order to determine the sufficient model order and validity in the linear discrete-time model that has been derived earlier on after linearization of the EHS system. The measured signals must be analyzed and pre-processed before the estimation and validation process. The data was divided into two sets where one set was used for estimation process and the other is to validate the obtained model. A model validation process using an independent data set is also called a cross-validation.

**RESULTS AND DISCUSSION**

A set of data that consists of the input voltage and actuator displacement as shown in Figure 6 was observed for a 100 s experiment under the off-line model identification. Various authors have used mean-square-error (MSE) (Eker, 2004a), root-mean-square error (RMSE) (Plummer and Vaughan, 1996; Eker, 2004b) and prediction error (Saleem et al., 2009) for model validation. The performance indices in the validation results can be more conclusive and comparable by using different approaches. Several model orders have been tested in the experiment. The main purpose of using system identification approach in modelling is to obtain a parsimonious model where the smallest number of parameters or model orders should be preferred in representing a dynamic system. Figure 7 shows the estimated output for the EHS system while the analysis of the validation process is tabulated in Table 2.

From Table 2, it can be observed that the higher order gives better performance in the observations of simulated outputs. Among the analysis of the different model order, a third order model appears to be suitable for the system, since there is no significant improvement in the model order increment in terms of percentage of fit, final prediction error and loss function criteria. Then, correlation analysis of the residuals is performed as a final procedure in validation process. Residuals are differences between the one-step-predicted output from the estimated model and the measured output from the
Simulated Output

![Graph showing simulated output](image)

Figure 7. Measured and simulated model output.

Table 2. Offline system identification analysis.

<table>
<thead>
<tr>
<th>Model order</th>
<th>Percentage of fit (%)</th>
<th>Final prediction error (FPE)</th>
<th>Loss function</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd order</td>
<td>91.27</td>
<td>0.0325882</td>
<td>0.0323295</td>
</tr>
<tr>
<td>3rd order</td>
<td>95.48</td>
<td>0.0230747</td>
<td>0.0228011</td>
</tr>
<tr>
<td>4th order</td>
<td>95.53</td>
<td>0.0217047</td>
<td>0.0213622</td>
</tr>
<tr>
<td>5th order</td>
<td>95.51</td>
<td>0.0215606</td>
<td>0.0211378</td>
</tr>
</tbody>
</table>

validation data set. Therefore, residuals can be considered as the portion of the validation data not presented by the model.

Residual analysis in the validation process consists of the whiteness test and the independence test. These tests will indicate the validity of the estimated model. In whiteness test analysis, a good model has the residual autocorrelation function inside the confidence interval and indicating that the residuals are uncorrelated. For independence test criteria, a good model has residuals uncorrelated with past inputs. The residual analysis of third order model is as shown in Figure 8. Thus, the following discrete-time model was identified for third order system as shown in Equation 16.

\[
G(z^{-1}) = \frac{y(k)}{u(k)} = \frac{0.3850z^{-1} - 0.5810z^{-2} + 0.2954z^{-3}}{1 - 2.2390z^{-1} + 1.7820z^{-2} - 0.5427z^{-3}} \quad (16)
\]

In online identification, the RLS with different forgetting factor were conducted simultaneously in the experiment for proper comparison. The parameters obtained from the offline identification were used as a reference value. Figures 9 and 10 show the estimated parameters \(A_q^j\) and \(B_q^j\) and their steady state values were tabulated as shown in Table 2. The results show that the proposed method improved significantly in the estimation process for every estimated parameter. Due to the behavior of forgetting factor, the small value gives fast convergence but very sensitive with noise which sometimes appears
during the experiment. The effect of the small value of forgetting factor is very obvious in estimation of parameter $B(q^{-1})$. Large oscillations can be observed from the graph and the forgetting factor is worst in terms of accuracy.

For large value of forgetting factor where no data will be discarded during the estimation process, the result shows that the convergence speed is too slow but intend to converge to its reference value and also robust with noise. It may take longer time to converge to the reference value. Even when the experimental time has reached 100 s, it requires longer time to converge to the reference value. Therefore, the trade-off between the convergence speed and accuracy is always taken into account in RLS estimation (Jelali and Kroll, 2003). The technique of choosing this value is mostly implemented by setting the forgetting factor between the possible values. With the proposed dynamic forgetting factor, the smaller forgetting factor is set at the early stage of estimation which improved slightly the estimation convergence speed and the large forgetting factor is implemented when the graph reaches the reference value to guarantee its accuracy and robustness with noise. It is obvious that the prediction error in Figure 11 and RMSE for different forgetting factor in Table 3 have been observed where the proposed method improved in estimation of the parameters $A(q^{-1})$ and $B(q^{-1})$.

**Conclusion**

In this paper, a complete study has been performed in the modelling of EHS system by introducing varying forgetting factor in RLS algorithm to improve the estimation process. A discrete-time model has been used to represent the EHS system with linearization based on the first principles law. Then, offline identification was conducted to validate the third order model with residual analysis. The results show that the derived model can be used as a linear model of the EHS system. Finally, RLS was implemented with the proposed technique of forgetting factor to estimate the parameters of discrete-time model and the results show significant improvement in terms of convergence speed and accuracy. The estimation process also gives improvement in its insensitivity towards noise. In conclusion, the proposed identification methods are capable of representing the EHS system dynamics for particular range without the knowledge of the actual physical system, thus reducing the engineering effort required to develop the system’s model for control design purpose.

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Figure 9. Estimated parameters of $A(q)$: (a) Parameter of a1, (b) parameter of a2 and (c) parameter of a3.
Figure 10. Estimated parameter of B(q): (a) parameter of $b_1$, (b) parameter of $b_2$ and (c) parameter of $b_3$.

Table 3. Estimated parameters using on-line identification.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Proposed forgetting factor</th>
<th>$\lambda = 0.99$</th>
<th>$\lambda = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>-2.2553</td>
<td>-2.2502</td>
<td>-2.2020</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1.8098</td>
<td>1.8148</td>
<td>1.6833</td>
</tr>
<tr>
<td>$a_3$</td>
<td>-0.5545</td>
<td>-0.5542</td>
<td>-0.4813</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.3753</td>
<td>0.3760</td>
<td>0.0344</td>
</tr>
<tr>
<td>$b_2$</td>
<td>-0.5660</td>
<td>-0.5694</td>
<td>0.1058</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.2878</td>
<td>0.2924</td>
<td>-0.0078</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.1817</td>
<td>0.3876</td>
<td>0.0987</td>
</tr>
</tbody>
</table>

Figure 11. Prediction error of third order model.
the present work.

REFERENCES


