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Review

The performance evaluation of unsteady MHD non-Darcy nanofluid flow over a porous wedge due to renewable (solar) energy

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ABSTRACT

Solar energy has been used since the beginning of time and is vital to all living things. In addition to solar energy being a constant resource, heat and electricity are other forms of energy that can be made from solar energy. Technology allows solar energy to be converted into electricity through solar thermal heat. The main advantages of solar energy are that it is clean, able to operate independently or in conjunction with traditional energy sources, and is remarkably renewable. Nanofluid-based direct solar receivers, where nanoparticles in a liquid medium can scatter and absorb solar radiation, have recently received interest to efficiently distribute and store the thermal energy. The objective of the present work is to investigate theoretically the effect of copper nanoparticles in the presence of magnetic field on unsteady non-Darcy flow and heat transfer of incompressible copper nanofluid along a porous wedge due to solar energy. It is of special interest in this work to consider that unsteady flow. Copper nanofluid flow over a porous wedge incident solar radiation and transits it to the working fluid by convection.

1. Introduction

Energy plays an important role in the development of human society. However, over the past century, the fast development of human society leads to the shortage of global energy and the serious environmental pollution. All countries of the world have to explore new energy sources and develop new energy technologies to find the road to sustainable development. The sun is probably the most important source of renewable energy available today. Renewable energy sources which include solar energy (which comes from the sun and can be turned into electricity and heat), wind energy, geothermal energy (from inside the earth), biomass from plants, and hydropower from water are also renewable energy sources. Solar energy as the renewable and environmental friendly energy, it has produced energy for billions of years. Solar energy that reaches the earth is around $4 \times 10^{15}$ MW, it is 2000 times as large as the global energy consumption. Thus the utilization of solar energy and the technologies of solar energy materials attract much more attention. Nano-material is a new energy material, since its particle size is the same as or smaller than the wavelength of de Broglie wave and coherent wave. Therefore, nanoparticle becomes to strongly absorb and selectively absorb incident radiation. Based on the radiative motion properties of nanoparticle, the utilization of nanofluids in solar thermal system becomes the new study hotspot. Scientists and engineers today seek to utilize solar radiation directly by converting it into useful heat or electricity.

The varieties of solar energy applications and advantages are enormous, scientists and engineers today seek to utilize solar radiation directly by converting it into useful heat or electricity. The inadequacy and inability and the inherent danger in the use of fossil fuels energy and other conventional sources of energy to meet the worlds demands for energy both now and in the nearest future is highlighted and emphasized. The world eventually turning to the renewable energy sources, solar energy in particular, is inevitable, expected and wise. The inevitable impediment such as the earth’s atmosphere and its effect on the passage of solar radiation, to the realization of full utilization of solar energy are identified. Nano-material is a new energy material, since its particle size is the same as or smaller than the wavelength of de Broglie wave and coherent wave. Therefore, nanoparticle becomes to strongly absorb and selectively absorb incident radiation. Based on the radiation properties of nanoparticle, the utilization of nanofluids in solar thermal system becomes the new study hotspot. Radiative transport in porous media has important engineering applications in solar collectors and the porous medium acts as a means to absorb or emit radiant energy that is transferred to or from a fluid. Generally, the fluid itself can be assumed to be transparent to radiation, because the dimensions for radiative transfer among the solid structure
elements of the porous medium are usually much less than the radiative mean free path for scattering or absorption in the fluid. Solar energy is one of the best sources of renewable energy with minimal environmental impact. Angstrom [1] and Mostafa [2] have pointed out that solar tower solar collectors could benefit from the potential efficiency improvements that arise from using a nanofluid as a working fluid. The basic concept of using particles to collect solar energy was studied in the 1970s by Hunt [3]. It has been shown that mixing nanoparticles in a liquid (nanofluid) has a dramatic effect on the liquid thermophysical properties such as thermal conductivity. Nanoparticles also offer the potential of improving the radiative properties of liquids, leading to an increase in the efficiency of direct absorption solar collectors. The study of heat transfer in the presence of nanofluids due to solar energy radiation is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Numerous models and group theory methods have been proposed by different authors to study convective flows of nanofluids, e.g., Birkoff [4], Yurusoy and Pakdemirli [5] and Yurusoy et al. [6].

Convective flow in porous media has been widely studied in the recent years due to its wide applications in engineering as post-accidental heat removal in nuclear reactors, solar collectors, drying processes, heat exchangers, geothermal and oil recovery, building construction, etc. The effects of heat and mass transfer laminar boundary layer flow over a wedge have been studied by many authors (for example, Kafoussias and Nanousis [7], Kundasamy et al. [8] and Cheng and Lin [9]) in different situations. Nanofluids are suspensions of nanoparticles in fluids that show significant enhancement of their properties at modest nanoparticle concentrations, which was investigated by Abdul-Kahar et al. [10], Kundasamy et al. [11], Vajravelu et al. [12] and Rana and Bhargava [13]. Nanofluids may be used in various applications which include electronic cooling, vehicle cooling transformer and coolant for nuclear reactors.

In this paper, we apply the so-called symmetry methods for a particular problem of fluid mechanics. The main advantage of such methods is that they can successfully be applied to nonlinear differential equations. The method of Lie group transformations is used to derive all group-invariant similarity solutions of the unsteady two-dimensional laminar boundary-layer equations. On the other hand, it is now well known that the classical Lie symmetry method can be used to find similarity solutions, invariants, integrals motion, etc. systematically. Ovsyannikov [14] and Avramenko et al. [15] analyzed the application of Lie group theory to the boundary layers. Solar energy is currently one kind of important resource for clean and renewable energy, and is widely investigated in many fields. For this reason, it is of special interest in this work to consider natural convection due to solar radiation non-Darcy flow from a wedge embedded in a porous medium with variable porosity distribution. The inertia effect is expected to be important at a higher flow rate and it can be accounted for through the addition of a velocity squared term in the momentum equation, which is known as the Forchheimer's extension. Several researchers have studied natural convection heat transfer in porous medium by considering Forchheimer's extension.

The motivation of the present study is to investigate the development of the unsteady non-Darcy boundary layer flow and heat transfer over a porous wedge sheet in a nanofluid due to solar radiation. Lie symmetry group transformation is utilized to convert the governing partial differential equations into ordinary differential equations and then the numerical solution of the problem is accomplished by using Runge Kutta Gill method [16] with shooting technique. This method has the following advantages over other available methods: (i) it utilizes less storage register (ii) it controls the growth of rounding errors and is usually stable and (iii) it is computationally economical. The analysis of the results obtained shows that the flow field is influenced appreciably by the presence of convective radiation and nanoparticles deposition in the presence of nanofluid past a porous wedge sheet.

2. Mathematical analysis

Let us consider an unsteady laminar two-dimensional non-Darcy flow of an incompressible viscous nanofluid past a porous wedge in the presence of solar energy radiation (see, Fig. 1). We consider influence of a constant magnetic field of strength $B_0$ which is applied normally to the surface. The temperature at the wedge surface takes the constant value $T_\infty$ while the ambient value, attained as $y$ tends to infinity, takes the constant value $T_\infty$. Far away from the wedge plate, both the surroundings and the Newtonian, absorbing fluid are maintained at a constant temperature $T_\infty$. It is further assumed that the induced magnetic field is negligible in comparison to the applied magnetic field (as the magnetic Reynolds number is small). The porous medium is assumed to be transparent and in thermal equilibrium with the fluid. The thermal dispersion effect is minimal when the thermal diffusivity of the porous matrix is of the same order of magnitude as that of the absorbing fluid. This viewpoint of assuming that the effective thermal diffusivity remains constant when the porosity of the porous medium varies with the normal distance is shared by many other investigators such as Vafai et al. [17] and Tien and Hong [18]. The non reflecting absorbing ideally transparent wedge plate receives an incident radiation flux of intensity $q_{rad}$. This radiation flux penetrates the plate and is absorbed in an adjacent fluid of absorption coefficient $\alpha$. Due to heating of the absorbing nanofluid and the wedge plate by solar radiation, heat is transferred from the plate to the surroundings and the solar radiation is a collimated beam that is normal to the plate. The fluid is a water based nanofluid containing copper nanoparticles. As mentioned before, the working fluid is assumed to have heat absorption properties. For the present application, the porous medium absorbs the incident solar radiation and transits it to the working fluid by convection. The thermophysical properties of the nanofluid are given in Table 1 (see Ref. [20]). Under the above assumptions, the boundary layer equations governing the flow and thermal field can be written in dimensional form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

![Fig. 1. Physical flow model over a porous wedge sheet.](image-url)
where \( \rho \) is the density of the base fluid and nanoparticle, respectively, \( \rho_f \) and \( \rho_l \) are the density of the base fluid and nanoparticle, respectively, \( k \) and \( \rho_l \) are the thermal conductivity of the base fluid and nanoparticle, respectively, \( \rho_f \) and \( \rho_f \) are the density of the base fluid and nanoparticle, respectively, \( k_f \) is the mean absorption coefficient. The Rosseland approximation is used to describe the radiative heat transfer in the limit of the optically thick fluid (nanofluid).

By introducing the following non-dimensional variables

\[
X = \frac{x}{X_0}, \quad Y = \frac{y}{Y_0}, \quad U = \frac{u}{U_0}, \quad V = \frac{v}{V_0}, \quad T = \frac{T - T_w}{T_e - T_w}
\]

Equations (1)–(4) take the non-dimensional form

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \frac{1}{1 - \xi} \left( \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} - \frac{F}{\sqrt{\kappa}} \left( u^2 - U^2 \right) \right)
\]

with the boundary conditions

\[
\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \frac{1}{1 - \xi} \left( \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial y^2} \right) + \left( \frac{k_f}{\rho_f c_p} \right) \left( \frac{\partial u}{\partial y} \right)^2
\]
\( U = 0, \ \psi = -V_0, \ T = T_w \) at \( y = 0 \)

\( U = \frac{\nu x^m}{\delta^{m+1}}, \ T = T_w \) as \( y \to \infty \) 

\[ (10) \]

where \( \Pr_f = \nu/\alpha_f \) is the Prandtl number, \( \delta = \delta^{m+1}/(kK) \) is the porous media parameter, \( \gamma = (\gamma (\rho / \delta)_{T_0}) \left( \rho U^2 k_{X} \right) \) is the buoyancy or natural convection parameter, \( N = (4 \gamma (\rho _{T_0}) / (k k_{+})) \) is the conductive radiation parameter, \( F_c = (\mu (\rho _{T_0}) \left( U^2 / (T_w - T_0) \right) ) \) is the Eckert number, \( M = (\mu (\rho _{T_0}) / \rho \delta) \) is the magnetic parameter and \( \epsilon = T_w / (T_0 - T_0) \) is the temperature ratio where \( \epsilon \) assumes very small values by its definition as \( T_w - T_0 \) is very large compared to \( T_0 \). In the present study, it is assigned the value 0.1. It is worth mentioning that \( \gamma > 0 \) aids the flow and \( \gamma < 0 \) opposes the flow, while \( \gamma = 0 \) i.e., \( (T_w - T_0) \) represents the case of forced convection flow. On the other hand, if \( \gamma \) is of a significantly greater order of magnitude than one, then the buoyancy forces will be predominant. Hence, combined convective flow exists when \( \gamma = O(1) \).

Following the lines of Kafoussias and Nanoussis [7], the changes of variables are

\[ \eta = \sqrt{\frac{1 + m}{2}} \sqrt{\Delta_{m+1}}, \ \psi = \sqrt{\frac{2}{1 + m}} \frac{\nu x^m}{\delta^{m+1}} f(\eta) \] and

\[ \theta = \frac{T - T_w}{T_w - T_0} \] 

By introducing the stream function \( \psi \), which defined as \( u = \partial \psi / \partial y \) and \( v = -\partial \psi / \partial x \), then the system of equations (7)-(9) become

\[ \frac{\partial^2 \psi}{\partial x^2} + \frac{\psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} - \frac{1}{1 - \zeta} \left[ \left( 1 - \zeta - C_T \theta \right) \frac{\partial f}{\partial \eta} \right] \]

\[ \times \left( 1 + \frac{\partial \psi}{\partial \eta} \right) + \frac{1}{1 - \zeta} \frac{\partial^3 \psi}{\partial x^3} \left( \frac{\partial U}{\partial x} + \frac{\partial U}{\partial x} \right) - \left( \frac{\partial \psi}{\partial x} \right)^2 \]

\[ \left( 1 + \frac{\partial \psi}{\partial \eta} \right) + \frac{1}{1 - \zeta} \frac{\partial \psi}{\partial x} \left( \frac{\partial \psi}{\partial x} \right) \]

(12)

with the boundary conditions

\[ \frac{\partial \psi}{\partial y} = 0, \ \frac{\partial \psi}{\partial x} = -V_0, \ T = T_w \] at \( y = 0 \)

\[ \frac{\partial \psi}{\partial y} \frac{\nu x^m}{\delta^{m+1}}, \ T = T_w \) as \( y \to \infty \) 

\[ (14) \]

The symmetry groups of Equs. (12) and (13) are calculated using classical Lie group approach (see Ref. [11]). With the help of these relations, the (12) and (13) become

\[ \frac{\partial f}{\partial \eta} = \frac{n^2}{2 \left( 1 - \zeta \right)^{2/3}} \left( \frac{M + \lambda}{1 - \zeta} \right)^{1/3} - \frac{n^2}{2 \left( 1 - \zeta \right)^{2/3}} \left( \frac{M + \lambda}{1 - \zeta} \right)^{1/3} \]

where \( S \) is the suction parameter if \( S > 0 \) and injection if \( S < 0 \) and \( \xi = k_{X} (1-m)/2 \) is the dimensionless distance along the wedge \( (\xi > 0) \). In this system of equations, it is obvious that the non-similarity aspects of the problem are embodied in the terms containing partial derivatives with respect to \( \xi \). This problem does not admit similarity solutions. Thus, with \( \xi \)-derivative terms retained in the system of equations, it is necessary to employ a numerical scheme suitable for partial differential equations for the solution. Formulation of the system of equations for the local non-similarity model with reference to the present problem will now be discussed.

At the first level of truncation, the terms accompanied by \( \xi \)-derivative are small. This is particularly true when \( \xi \ll 1 \). Thus the terms with \( \xi \)-derivative on the right-hand sides of Equations (20) and (21) are deleted to get the following system of equations:

\[ \theta'' + \frac{4 k_l}{3 k_p} N \left( (C_T + \theta)^3 \right) \theta'' + \frac{Pr_f}{(1 - \zeta)^{2/3}} \left( \frac{M + \lambda}{1 - \zeta} \right)^{1/3} \left( \frac{M + \lambda}{1 - \zeta} \right)^{1/3} = 0 \]

(15)

The boundary conditions take the following form

\[ \frac{\partial f}{\partial \eta} = 0, \ \frac{m + 1}{2} f - \frac{1 - m}{2} \frac{\partial f}{\partial \eta} = -S, \ \theta = 1 \] at \( \eta = 0 \) and

\[ \frac{\partial f}{\partial \eta} = 1, \ \theta = 0 \] as \( \eta \to \infty \) 

(17)

where \( S \) is the suction parameter if \( S > 0 \) and injection if \( S < 0 \) and \( \xi = k_{X} (1-m)/2 \) is the dimensionless distance along the wedge \( (\xi > 0) \). In this system of equations, it is obvious that the non-similarity aspects of the problem are embodied in the terms containing partial derivatives with respect to \( \xi \). This problem does not admit similarity solutions. Thus, with \( \xi \)-derivative terms retained in the system of equations, it is necessary to employ a numerical scheme suitable for partial differential equations for the solution. Formulation of the system of equations for the local non-similarity model with reference to the present problem will now be discussed.

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The boundary conditions take the following form

\[ f' = 0, \quad f = \frac{-25}{m+1}, \quad \theta = 1 \text{ at } \eta = 0 \text{ and } \]

\[ f' = 1, \quad \theta \to 0 \text{ as } \eta \to \infty \quad (20) \]

Further, we suppose that \( \lambda_0 = c/\lambda^{m-1} \) where \( c \) is a constant so that \( c = (d^\theta/d\eta)(d\delta/d\eta) \) and integrating, it is obtained that \( \delta = c[(\eta + 1)\eta_1]^{1/(m-1)} \). When \( c = 2 \) and \( m = 1 \) in \( \delta \) and we get \( \delta = 2 \sqrt{\eta} \) which shows that the parameter \( \delta \) can be compared with the well established scaling parameter for the unsteady boundary layer problems (see Ref. [25]).

For practical purposes, the functions \( f(\eta) \) and \( \theta(\eta) \) allow us to determine the skin friction coefficient

\[ C_f = \frac{\mu_f}{\rho_f u^2} \left( \frac{\partial u}{\partial y} \right)_{y=0} = \frac{1}{(1-\eta^2)3} \left( \frac{Re_f}{\eta} \right)^{-1} f''(0) \quad (21) \]

and the Nusselt number

\[ Nu_x = \frac{xk_x}{k_f(T_w - T_u)} \left( \frac{\partial T}{\partial y} \right)_{y=0} \right|_{y=0} = -\left( \frac{Re_f}{2} \right) \frac{\partial^2 f}{\partial \eta^2} \left( 1 + \frac{4}{3} N(\gamma + \theta(\eta))^3 \right) \quad (22) \]

respectively. Here, \( Re_f = Ux/\nu_f \) is the local Reynolds number.

The set of Equs. (18) and (19) are highly nonlinear and coupled, and therefore, it cannot be solved analytically. Therefore, the nonlinear systems consisting of Equs. (18) and (19) along with the boundary conditions (20) have been solved numerically by applying Runge–Kutta–Gill [18] integration scheme together with shooting iteration technique (see also Ref. [8]) with \( Pr_f, \zeta, \lambda, m, S, \Omega, M, F_0 \), and \( N \) as prescribed parameters. A step size of \( \Delta \eta = 0.01 \) was selected to be satisfactory for a convergence criterion of \( 10^{-6} \) in all cases. The case \( \gamma \gg 1.0 \) corresponds to pure free convection, \( \gamma = 1.0 \) corresponds to mixed convection and \( \gamma \ll 1.0 \) corresponds to pure forced convection. Throughout this calculation we have considered \( \gamma = 2.0 \) unless otherwise specified. In order to validate our method, we have compared the results of \( f(\eta), f'(\eta) \) and \( f''(\eta) \) for various values of \( \eta \) (Table 2) with those of White [26] whereas \( f''(0) \) and \( \theta'(0) \) for various values of \( \zeta \) (Table 3) with those of Vajravelu et al. [12] and found them in excellent agreement.

In order to ascertain the accuracy of our numerical results, the present study is compared with the available exact solution in the literature. The velocity profiles for different values of \( m \) are compared with the available exact solution of Ref. [25], is shown in Fig. 2. It is observed that the agreement with the theoretical solution of velocity profile is excellent.

Table 2

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>Present works</th>
<th>White [26]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta )</td>
<td>( f(\eta) )</td>
<td>( f'(\eta) )</td>
</tr>
<tr>
<td>0.0</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
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</tr>
<tr>
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<tr>
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<td>0.96055</td>
</tr>
<tr>
<td>4.0</td>
<td>2.78388</td>
<td>0.99777</td>
</tr>
</tbody>
</table>

Fig. 2. Effects of \( m \) on the velocity distribution in the laminar flow past a wedge.

Table 3

Comparison of the current results with previous published work for \( f''(0) \) and \( \theta'(0) \).

| \( \gamma \) | \( \zeta \) | Present works | Vajravelu et al. [12]
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( f''(0) )</td>
<td>( \theta'(0) )</td>
<td>( f''(0) )</td>
<td>( \theta'(0) )</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>-1.001411</td>
<td>-2.972286</td>
</tr>
<tr>
<td>0.0</td>
<td>0.1</td>
<td>-1.175203</td>
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</tr>
<tr>
<td>0.0</td>
<td>0.2</td>
<td>-1.318301</td>
<td>-2.084192</td>
</tr>
</tbody>
</table>

Figs. 3 and 4 present typical profiles for temperature for different values of magnetic parameter in the case of pure water and Cu-water (nanofluid). Due to the uniform convective radiation, it is clearly shown that the above mentioned two cases, the temperature of the fluid accelerates with increase of the strength of magnetic field, which implies that the applied magnetic field tends to heat the fluid and enhances the heat transfer from the wall. As it
moves away from the plate, the effect of $M$ becomes less pronounced. The effects of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive-type force called the Lorentz force. This force clearly indicates that the transverse magnetic field opposes the transport phenomena and it has the tendency to slow down the motion of the fluid and to accelerate its temperature profiles. In all cases, the temperature vanishes at some large distance from the surface of the wedge. This result qualitatively agrees with the expectations, since magnetic field exerts retarding force on the natural convection flow. Physically, it is interesting to note that the temperature of the nanofluid (Cu Water) increases significantly as compared to that of the base fluid, because the copper Cu has high thermal conductivity. Magnetic nanofluid is a unique material that has both the liquid and magnetic properties. Many of the physical properties of these fluids can be tuned by varying magnetic field. These results clearly demonstrate that the magnetic field can be used as a means of controlling the flow and heat transfer characteristics.

Fig. 5 illustrates the effect of nanoparticle volume fraction $\zeta$ on the nanofluid temperature profile. It is clear that as the nanoparticle volume fraction increases, the nanofluid temperature increases and tends asymptotically to zero as the distance increases from the boundary. Increasing the volume fraction of nanoparticles increases the thermal conductivity of the nanofluid and we predict a thickening of the thermal boundary layer. We also observe that the temperature distribution in Silver--water and Alumina--water nanofluids are higher than that of Cu--water nanofluid. It is observed that with increasing $\zeta$, the thermal boundary layer thickness increases. The sensitivity of thermal boundary layer
thickness with $\zeta$ is related to the increased thermal conductivity of the nanofluid. In fact, higher values of thermal conductivity are accompanied by higher values of thermal diffusivity. The high value of thermal diffusivity causes a drop in the temperature gradients and accordingly increases the boundary thickness as demonstrated in Fig. 5. This agrees with the physical behavior, when the volume of copper nanoparticles increases the thermal conductivity increases, and then the thermal boundary layer thickness increases. Changes in the size, shape, material and volume fraction of the nanoparticles allow for tuning to maximize spectral absorption of solar energy throughout the fluid volume because the nanoparticle volume fraction parameter depends on the size of the particles. Enhancement in thermal conductivity can lead to efficiency improvements, although small, via more effective fluid heat transfer. In convective heat transfer in nanofluids, the heat transfer depends not only on the thermal conductivity but also on other properties, such as the specific heat, density, and dynamic viscosity of a nanofluid. Based on the experimental data [27], utilization of nanofluid instead of conventional base fluids results in remarkable heat transfer enhancement. In straight tubes heat transfer rate goes up as the nanoparticle mass concentration increases. Besides, nanofluid flows showed much higher Nusselt numbers compared to the base fluid flow. Finally, it was observed that combination of the two enhancing methods has a noticeably high capability to the heat transfer rate.

Fig. 8. Temperature profiles for various values of unsteadiness parameter ($\xi = 0.05$, $N = 0.5, M = 1.0$ and $m = 0.0909$ ($Q = 30^\circ$)).

Fig. 9. Forchheimer number over the velocity profiles ($\xi = 0.05, \lambda = 0.1, N = 0.5$ and $m = 0.0909$ ($Q = 30^\circ$)).
nanofluid increases with the increase of unsteadiness parameter $\lambda_0$ and also it is found to decrease with the increase of $\eta$. The variation of the Prandtl number within the boundary layer for different values of the unsteadiness parameter $\lambda_0$ plays a dominant role on nanofluid flow field. Significant change in the rate of decrease of $\theta$ for increasing values of $\lambda_0$ is noticed. Temperature at a point on the sheet decreases significantly with the increase in $\lambda_0$, i.e., rate of heat transfer increases with increasing unsteadiness parameter $\lambda_0$. The reason for this behavior is that the inertia of the porous medium provides an additional resistance to the fluid flow mechanism, which causes the fluid to move at a retarded rate with reduced velocity.

Figs. 9 and 10 illustrate typical profiles for velocity and temperature for different values of inertial parameter $Fn$ in the case of Cu-water. In the presence of uniform magnetic field, it is clearly shown that the velocity increases and temperature decreases as the inertial parameter (Forchheimer number) increases. In particular, the velocity of the Cu-nanofluid increases whereas the temperature of the fluid gradually changes from higher value to the lower value only when the strength of inertial parameter $Fn$ is higher than the nanoparticle volume fraction parameter. For heat transfer characteristics mechanism, interesting result is the large distortion of the temperature profile caused for $0.05 \leq Fn \leq 0.5$. The nanofluid becomes unstable and has no physical application at solid volume fractions greater than 0.05. In the case of Cu-water, negative value of the temperature profile is seen in the outer boundary region for $Fn = 0.1$ and 0.2. Non-Darcy behavior is important for describing fluid flow in porous media in situations where high velocity occurs. This is consistent with the fact that non-Darcy behavior is more severe in low permeability porous media. All these physical behavior are due to the combined effects of the strength of volume fraction of the nanoparticles in the presence of Non-Darcy flow.

4. Conclusions

In this work, the effect of copper nanoparticles in the presence of magnetic field on unsteady non-Darcy flow and heat transfer of incompressible copper nanofluid along a porous wedge due to solar energy have been analyzed. It is of special interest in this work to consider the similarity transformation is used for unsteady flow.

1. Thermal boundary layer thickness of copper nanofluid is stronger than that of the base fluid as the strength of the magnetic field increases because the driving force to the nanofluid decreases as a result of temperature profiles increase.

2. It is noticed that the temperature of a nanofluid is decelerated significantly as compared to that of the base fluid with increase of convective radiation.

3. It has been shown that mixing nanoparticles in a liquid (nanofluid) has a dramatic effect on the liquid thermophysical properties such as thermal conductivity. It implies that the thermal conductivity of nanofluid is strongly dependent on the nanoparticle volume fraction.

4. Increase of thermal boundary layer field due to increase in nanoparticle volume fraction and magnetic parameters show that the temperature decreases gradually as we replace Copper nanofluid, $\zeta = 0.05$ by Silver, $\zeta = 0.10$ and Alumina nanofluid, $\zeta = 0.15$ in the said sequence.

5. It is seen that the temperature of the nanofluid increases with the increase of unsteady parameter $\lambda_0$.

6. In the presence of uniform magnetic field, it is clearly shown that the velocity increases and temperature decreases as the inertial parameter (Forchheimer number) increases.

7. Copper nanofluid flow over a porous wedge plays a significant role and absorbs the incident solar radiation and transits it to the working fluid by convection.

The impact of nanoparticles on the absorption of radiative energy has been of interest for many years for a variety of applications. More recently researchers have become interested in the radiative properties of nanoparticles in liquid suspensions especially for medical and other engineering applications. Besides the benefits to the optical and radiative properties, nanofluids provide other benefits such as increased thermal conductivity and particle stability over micron-sized suspensions, which provide potential improvements to the operating efficiency of a direct absorption solar collector. Nanofluids have been shown to possess improved heat transport properties and higher energy efficiency in a variety of thermal exchange systems for different industrial applications, such as transportation, electronic cooling, military, nuclear energy, aerospace etc. Nanofluids due to solar energy are important because they can be used in numerous applications involving heat transfer and other applications such as in degreasing, solar collectors, drying processes, heat exchangers, geothermal and oil recovery, building construction, etc.

References

[7] Kazandzhi IS, Muhaimin I, Hashim Iskait, Ruhaila. Thermophoresis and chemical reaction effects on Non-Darcy mixed convective heat and mass transfer past a porous wedge with variable viscosity in the presence of suction or injection. Nucl Eng Design 2008:238:2699-

Nomenclature

$B_0$: magnetic field strength
$C$: nanoparticles volume fraction
$C_{wp}$: nanoparticle volume fraction at the wall
$C_p$: specific heat at constant pressure
$C_T$: temperature ratio
$C_{f,0}$: Eckert number
$f$: dimensionless stream function
$F$: empirical constant in the second-order resistance
$Fs$: Forchheimer number
$g$: acceleration due to gravity
$E$: thermal conductive

$k^*$: mean absorption coefficient
$k$: permeability of the porous medium
$M$: magnetic parameter
$N$: conductive radiation parameter
$P_f$: Prandtl number
$q_{inc}$: incident radiation flux of intensity
$s$: suction/injection parameter
$T$: temperature of the fluid
$T_a$: temperature at the wall
$u$, $v$: velocity components along x- and y-axes
$U(x)$: uniform velocity of the free stream flow
$w_p$: velocity of suction/injection

Greek symbols

$\alpha$: thermal diffusivity of the nanofluid
$\beta$: coefficient of thermal expansion
$\theta$: dimensionless temperature
$\phi$: dimensionless nanoparticle volume fraction
$\eta$: similarity variable
$\mu$: dynamic viscosity
$\mu_e$: effective dynamic viscosity of the nanofluid
$\sigma$: electric conductivity of the fluid
$\sigma_f$: Stefan–Boltzman constant
$\rho$: density of the base fluid
$\rho_e$: effective density of the nanofluid
$\rho_f$: heat capacity of the base fluid
$\rho_{np}$: effective heat capacity of the nanoparticle material
$\nu$: kinematic viscosity
$\psi$: stream function
$\sigma_f$: Stefan–Boltzman constant
$\theta$: angle of the wedge