

INDUCTION MOTOR MODELLING USING FUZZY LOGIC

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ABSTRACT

Fuzzy logic has been widely used in many engineering applications since this can overcome the limitations of conventional method of data analysis, modelling and system identification, and control system. The capability of dealing with highly non-linear system modelling that is so complex that require absolute analytical design make these mathematical model architecture more popular in the engineering field. This project is addressed on the modelling of induction motor Auto-Regressive with exogenous input (ARX) model structure using fuzzy logic. In this case fuzzy logic is combined with neural network of said Neuro Fuzzy (ANFIS) is applied and has functioned as estimator of the ARX model parameters. The ARX model of induction motor is estimated from its input output data. Input variable is voltage and output variable is speed. The experimental results show that the best model responses have similarly trend with the motor actual responses, final prediction error is 0.00873, loss function is 0.00807, and fit to working data is 67.22%. It means the model produce from system identification able adopt the motor dynamic and can use for replacing real motor for analysis and control design.

ABSTRAK

Logik kabur telah digunakan secara meluas dalam aplikasi kejuruteraan, ini kerana ia boleh mengatasi keterbatasan dalam kaedah konvensional bagi membuat data analisis, model, sistem pengenalan dan sistem kawalan. Bagi sistem model bukan linear memerlukan analisis rekabentuk yang mutlak, dimana ia melibatkan matematik model yang popular dalam bidang kejuruteraan. Projek ini melibatkan model motor aruhan auto-regresif dengan model struktur input luarannya (ARX) menggunakan logik kabur. Dalam projek ini logik kabur digabungkan dengan rangkaian neural (ANFIS) yang berfungsi untuk menganggar parameter bagi ARX model. Di mana model ARX bagi motor aruhan ini dianggarkan daripada data bagi input dan output. Input bagi motor aruhan ini adalah voltan sementara output adalah kelajuan bagi motor aruhan. Hasil projek ini menunjukkan model yang terbaik untuk motor aruhan mempunyai model yang sama dengan motor aruhan sebenar, dimana ramalan ralat akhir adalah 0.00873, fungsi badan adalah 0.00807 dan sesuai untuk data yang berfungsi pada 67.22%. Ini bermakna model ini boleh diterima pakai bagi motor aruhan dinamik dan boleh digunakan untuk menggantikan motor sebenar bagi tujuan analisis dan rekabentuk kawalan.

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LIST OF SYMBOLS AND ABBREVIATIONS

ARX	Autoregressive Model with Exogenous input
AR	AutoRegressive
MA	Moving average
X	eXogeneous inputs
NARX	Nonlinear Autoregressive Model with Exogenous in
IM	Induction Motor
Ψ	Resultant flux vectors
emf	Electromotive force
V	Voltage
I_m	Magnetizing current
s	Synchronous speed
n	Speed of the rotor
ns	Speed of the magnetic field
mmf	Magnetomotive force
ϕ_r	Current lag by a angle
ω_r	Motor speed
L_s and L_r	Stator and rotor self-inductance
ω_s	Stator supply angular frequency
e	Noise
y	The matrix of observed outputs of a system
U	The input of the system
Θ	The system model parameters
\hat{y}	The estimate of y based on a system model and the inputs to the system

CHAPTER 1

INTRODUCTION

1.1 Project Background

The induction motor, which is the most widely used motor type in the industry, has been favoured because of its good self-starting capability, simple and rugged structure, low cost and reliability. Along with variable frequency AC inverters, induction motors are used in many adjustable speed applications which do not require fast dynamic response. The concept of vector control has opened up a new possibility that induction motors can be controlled to achieve dynamic performance as good as brushless DC motors.

In order to understand and analyze vector control, the dynamic model of the induction motor is necessary. It has been found that the dynamic model equations developed on a rotating reference frame is easier to describe the characteristics of induction motors. Any method for speed prediction is based on a model of the motor and the drive. The best accuracy of prediction for an induction motor is needed. Today, there are many choices of modelling techniques. One of them is system identification where it identifies the behaviour of a given system by estimating the model from input and output data. The estimated model is useful to simulate and predict the behaviour of the system. Not limited to that, the fitted model can be employed to regulate the output of plant.

An ARX model is one of the linear models that is usual used as a candidate model in system identification. The ARX stands for Auto-Regressive with

Exogenous Input where its autoregressive variable comprising of past output data and the exogenous input variable is represented as past input data (Bjorn Sohlberg, 2005). In other word, this model is used to estimate the parameters in the model structure from historical input-output data of a scrutiny system.

The ARX model is among the simple Models for linear process and it is easy to be implemented (Xiangdong Wang et all, 2008). This project is addressed the modelling of induction motor based on the ARX model structure using fuzzy logic. Fuzzy models become useful when a system cannot be defined in precise mathematical terms. The non-fuzzy or traditional representations require a well structured model and well defined model parameters. Even if the structure is known, numerical model representations usually become irrelevant and computationally inefficient as the complexity increases. Moreover, there may be a lot of uncertainties, unpredictable dynamics and other unknown phenomena that cannot be mathematically modelled at all. Therefore, when a system cannot be modelled with traditional methods for the reasons stated above, then fuzzy logic offers an efficient mathematical tool in handling many practical problems. The main contribution of fuzzy control theory is its ability to handle many practical problems that cannot be adequately handled by conventional control methods.

Fuzzy modelling of the systems has been observed in many scientific researches. Takagi and Sugeno have proposed a search algorithm for a fuzzy controller and generalized their research for fuzzy identification (T. Takagi & M. Sugeno, 1985). Sun has observed a modelling scheme for an adaptive-network-based fuzzy inference system (C.T. Sun, 1994). Chen, Pham and Weiss have shown that state space model of a linear system can be modelled fuzzily and extended their study on nonlinear systems (G. Chen et all, 1995). Eksin and Ayday have proposed an approach for fuzzy identification of nonlinear systems (I. Eksin & C.T. Ayday, 1993), Mouzouris and Mendel have implemented a search algorithm for dynamical non-singleton fuzzy control systems (G.C. Mouzouris & J.M. Mendel, 1997). In the studies listed above, except the one by Takagi and Sugeno, the models require some human knowledge and experience. The operator's experience in the system is involved in the mathematics of fuzzy control theory as a collection of "if...then" rules, known as "heuristic rules". The main goal of this paper is to present a

mathematical way of defining a fuzzy model of a nonlinear system without necessitating any human knowledge.

1.2 Problem Statements

System identification in control theory is about building a mathematical model, which identifies any dynamical system or signal, based on observed inputs and outputs data. The key problem in modelling induction motor is to find a suitable model structure. To avoid this problem, in this project nonlinear ARX modelling has been chosen as a candidate model of induction motor and fuzzy logic as an estimator algorithm.

1.3 Project Objectives

The objectives of this project consist of the following:

1. To design fuzzy logic for system modelling.
2. To derive ARX model structure for induction motor candidate model.
3. To collect input output data of induction motor through experimental test.
4. To identify mathematical model of induction motor using fuzzy logic. Algorithm based on induction motor input output data.
5. To validate the identified induction motor mathematical model.

1.4 Scopes of the project

The scopes of the project are:

1. Induction motor input output data are collected in experimental test.
2. Induction motor is assumed has ARX model structure

3. ARX model of induction motor is identified applying fuzzy logic
4. Identified model is validated with noise test and several prediction methods.
5. System identification is done off line.



PTTA UTHM
PERPUSTAKAAN TUNKU TUN AMINAH

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Mathematical modelling is of fundamental importance in science and engineering. It is very useful and presents a compact way of summarizing the knowledge about a process or system. This need for identifying and modelling the observation data appears in a wide range of applications. The process of model building consists of mapping the relationship between observed data from the system, onto a mathematical structure. Mathematical models can be constructed based on either known results from physics or data analysis.

2.2 Induction Motor Basics

The motor, used in this thesis work, is a two pole induction machine composed of a cage rotor and a stator containing windings connected to the three-phase power supply. In common for induction machines is that the stator is made up of a stack of steel laminations pressed into a aluminium or cast iron frame, and that the rotor consists of a stack of steel laminations with evenly spaced slots punched around the circumference where the rotor bars are placed, see Figure 2.1 both the rotor and stator lamination plates are insulated to prevent eddy currents¹ from flowing in the

iron. The rotor slots are also skewed to reduce non-linear effect such as harmonics and torque pulsation.

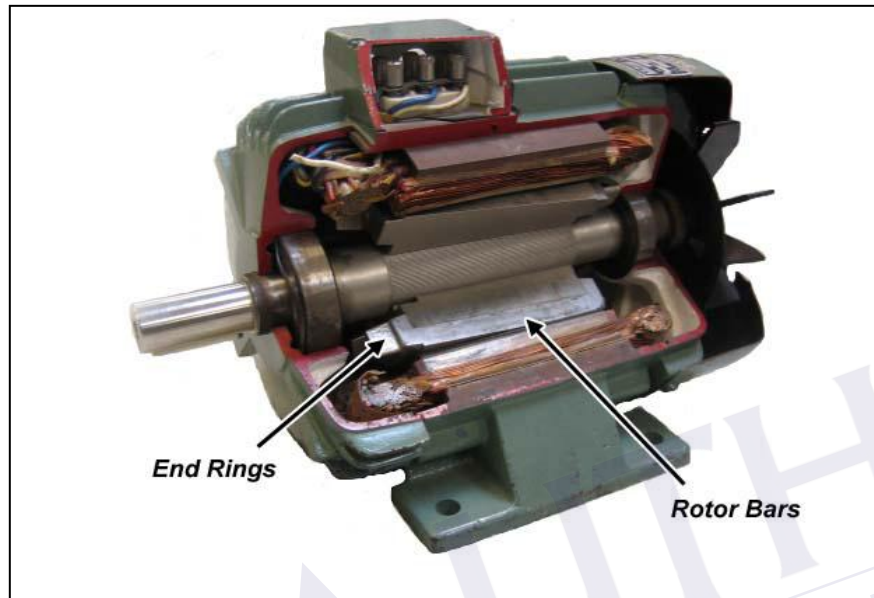


Figure 2.1: Inside view of an induction motor

2.2.1 The Stator

Each of the three alternating currents that flow in the three stator windings give rise to a magnetic field. Since the currents are alternating, the resultant magnetic field Figure 2.2 rotates with the stator current frequency and crosses the air-gap radially. It is the layout of the stator windings that determines the number of poles, see Figure 2.3 and hence the speed of the rotating magnetic field in relation to the supply frequency.

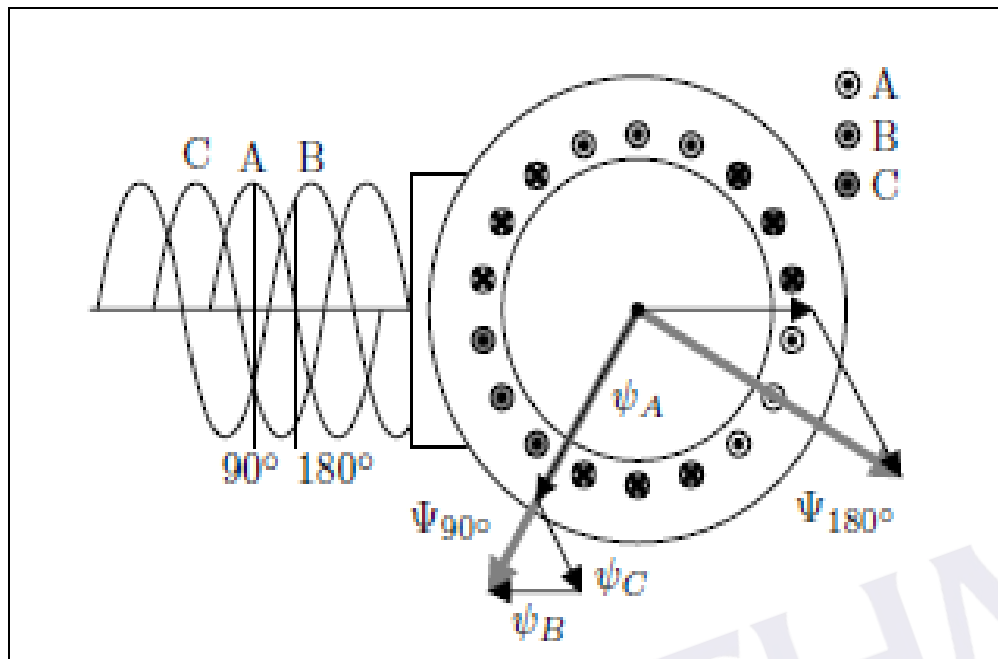


Figure 2.2 Resultant flux vectors as a sum of the three phase currents at 90° and 180°

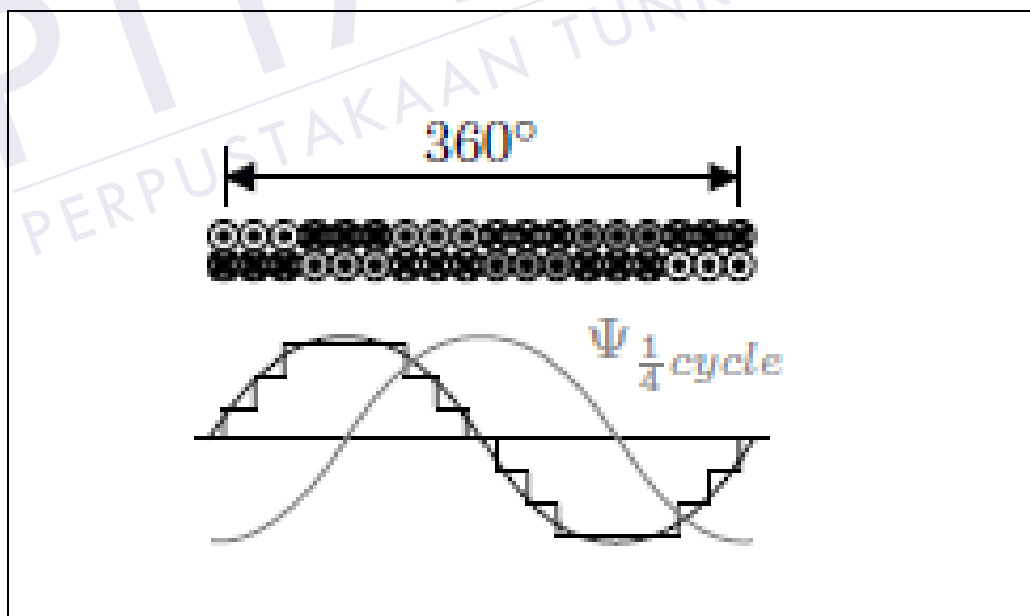


Figure 2.3 Air-gap flux density of a 3-phase, 2-pole, two-layer induction motor winding

Since every stator conductor is cut by the rotating magnetic field an alternating electromotive force (emf) E will be induced opposing the voltage V according to Lenz's law. The stator can thus be described by the equation $V = ImR + E$, which shows the relation between the applied voltage V , the magnetizing current Im (example the current that sets up the flux) and the induced (emf) E . If the flux decreases, so will the emf. This makes the magnetizing current increase which in turn makes the flux increase and hence the emf. The magnetizing current will adjust itself so that the emf always equals the applied voltage.

2.2.2 The Rotor

Torque producing currents are induced in the rotor bars by interaction with the air-gap flux wave, the rotor is dragged along by the rotating field. If the rotor is held stationary there will be a high current induced in the rotor bars since the wave will cut the bars at a high velocity. If, on the other hand, the rotor is rotating with the same velocity as the magnetic field, there will be no current induced in the rotor bars. The slip is defined as the relative velocity between the speed of the magnetic field (n_s), which is also known as the synchronous speed, and the speed of the rotor (n).

$$s = \frac{n_s - n}{n_s} \quad (2.1)$$

If a mechanical load is applied to the shaft, the rotor slows down, the slip increases and more current gets induced in the rotor bars. This results in a stronger magnetic field in the rotor bars and hence a higher torque is produced. The currents in the rotor bars also set up a magnetomotive force (mmf) wave in the stator that counteracts with the stator generated flux wave. Hence, a modest reduction of air-gap flux results in a reduction of e.m.f. Since the applied voltage is constant this increases the magnetizing current.

2.2.3 Torque Production

With a small slip, the frequency of the induced emf in the rotor is low which makes the reactance of the rotor low (in this case the rotor is predominantly resistive) and thus the rotor current in phase with the rotor emf which in turn is in phase with the air-gap flux. As a result the torque-speed relationship for small slip is approximately a straight line. As the slip increases, both the rotor emf and frequency increases. With increased frequency the rotor inductive reactance also increases which makes the current lag by a angle ϕ_r shown in the right part of Figure 2.4.

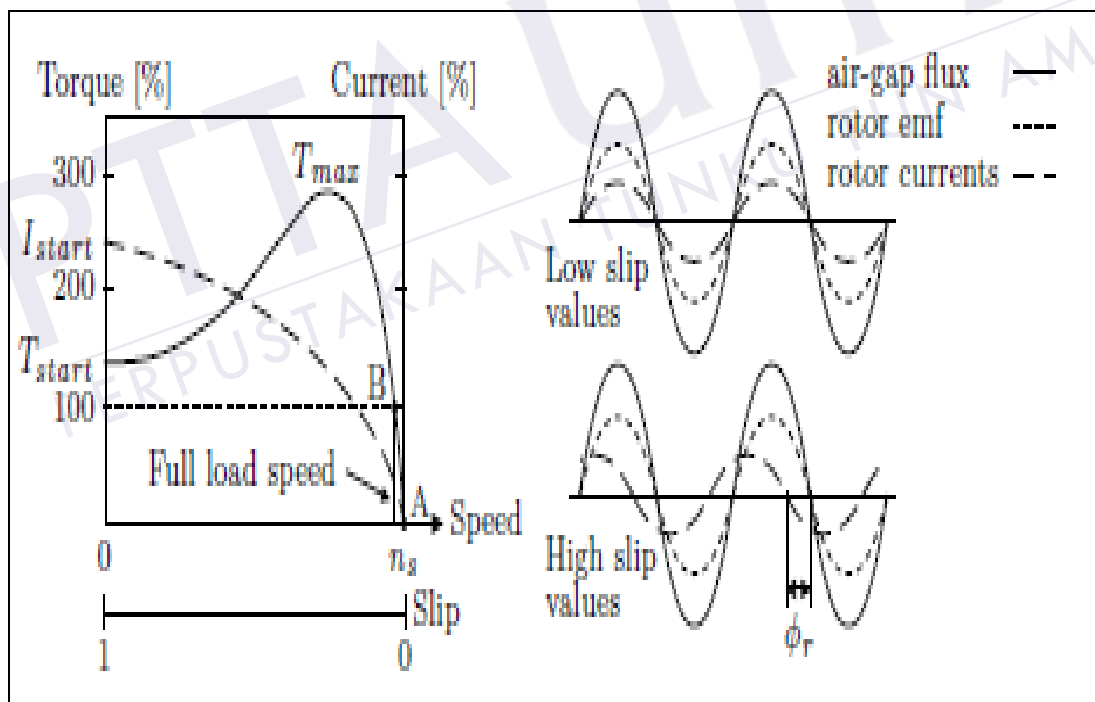


Figure 2.4 Torque speed relationship

2.3 Mathematical Dynamic Model of an Induction Motor

The induction motor can be represented in the stator stationary reference frame (α - β coordinate axes) by a second order differential equation relating the stator input voltages and currents as:

$$\frac{d^2 \mathbf{i}_s}{dt^2} + \mathbf{g}_1 \frac{d\mathbf{i}_s}{dt} + \mathbf{g}_0 \mathbf{i}_s = \mathbf{h}_1 \frac{d\mathbf{v}_s}{dt} + \mathbf{h}_0 \mathbf{v}_s \quad (2.2)$$

Where the coefficients of equation (2.2) are given in the following way

$$\begin{aligned} \mathbf{g}_1 &= \frac{R_s L_r + R_r L_s}{\sigma_s} - j\omega_r \\ \mathbf{g}_0 &= \frac{L_r R_s}{\sigma_s} \left(\frac{1}{T_r} - j\omega_r \right) \\ \mathbf{h}_1 &= \frac{L_r}{\sigma_s} \\ \mathbf{h}_0 &= \frac{L_r}{\sigma_s} \left(\frac{1}{T_r} - j\omega_r \right) \end{aligned} \quad (2.3)$$

These coefficients are functions of the machine parameters and motor speed (ω_r). Where L_s and L_r are stator and rotor self inductance respectively, and are defined by $L_s = L_m + L_{ls}$ and $L_r = L_m + L_{lr}$, $T_r = L_r/R_r$ is a rotor time constant and $\sigma_s = L_s L_r - L_m^2$ is known as leakage index. The stator phase voltages and currents can also be transformed to the stationary reference frame (α - β axes) (D. W. Novotny & T. A. Lipo, 1996). The stator voltage and current equations, which are complex, can be written as

$$\mathbf{v}_s = v_{s\alpha} + jv_{s\beta} \quad (2.4)$$

$$\mathbf{i}_s = i_{s\alpha} + ji_{s\beta} \quad (2.5)$$

Where $v_{s\alpha}, v_{s\beta}$ are the α -axis and β -axis stator voltage components in the stationary reference frame and $i_{s\alpha}, i_{s\beta}$ are the corresponding currents. Using equations (2.2)-(2.4), the second derivative of the stator current, components at constant speed can be derived as

$$\begin{bmatrix} \frac{d^2 i_{s\alpha}}{dt^2} + \omega_r \frac{di_{s\beta}}{dt} \\ \frac{d^2 i_{s\beta}}{dt^2} - \omega_r \frac{di_{s\alpha}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{di_{s\alpha}}{dt} \left(\frac{dv_{s\alpha}}{dt} + \omega_r v_{s\beta} \right) v_{s\alpha} - i_{s\alpha} - \omega_r i_{s\beta} \\ -\frac{di_{s\beta}}{dt} \left(\frac{dv_{s\beta}}{dt} + \omega_r v_{s\alpha} \right) v_{s\beta} - i_{s\beta} - \omega_r i_{s\alpha} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix} \quad (2.6)$$

Where $A_1 = \frac{R_s L_r + R_r L_s}{\sigma_s}, A_2 = \frac{L_r}{\sigma_s}, A_3 = \frac{R_r}{\sigma_s}, A_4 = \frac{R_r R_s}{\sigma_s}$ and $A_5 = \frac{L_r R_s}{\sigma_s}$

Applying the following constraints to equation (2.6),

Constraint 1: $v_{s\alpha} = |V_s| \cos \omega_s t$

Constraint 2: $v_{s\beta} = |V_s| \sin \omega_s t$

Constraint 3: $i_{s\alpha} = |I_s| \cos(\omega_s t - \theta)$

Constraint 4: $i_{s\beta} = |I_s| \sin(\omega_s t - \theta)$

Where ω_s is the stator supply angular frequency (rad./sec.) and θ is the phase angle between the stator voltage and current components. This results in the following equations:

$$i_{s\alpha} = \frac{1}{(\omega_s^2 + \omega_s \omega_r - A_4)} [A_2 \omega_s v_{s\beta} + A_3 v_{s\alpha} + A_1 \omega_s i_{s\beta} - A_2 \omega_r v_{s\beta} + A_5 \omega_r i_{s\beta}] \quad (2.7)$$

$$i_{s\beta} = \frac{1}{(\omega_s^2 + \omega_s \omega_r - A_4)} [A_3 v_{s\beta} + A_2 \omega_s v_{s\alpha} + A_1 \omega_s i_{s\alpha} + A_2 \omega_r v_{s\alpha} - A_5 \omega_r i_{s\alpha}] \quad (2.8)$$

Equations (2.7) and (2.8) represent the model of the induction motor. These equations may be put in the form

$$i_{s\alpha} = K_1 v_{s\beta} + K_2 v_{s\alpha} + K_3 i_{s\beta} + K_4 \omega_r v_{s\beta} + K_5 \omega_r i_{s\beta} \quad (2.9)$$

$$i_{s\beta} = K_2 v_{s\beta} + K_1 v_{s\alpha} + K_3 i_{s\alpha} - K_4 \omega_r v_{s\alpha} - K_5 \omega_r i_{s\alpha} \quad (2.10)$$

Where

$$K_1 = \frac{A_2 \omega_s}{(\omega_s^2 + \omega_s \omega_r - A_4)} \quad (2.11)$$

$$K_2 = \frac{A_3}{(\omega_s^2 + \omega_s \omega_r - A_4)} \quad (2.12)$$

$$K_3 = \frac{A_1 \omega_s}{(\omega_s^2 + \omega_s \omega_r - A_4)} \quad (2.13)$$

$$K_4 = \frac{-A_2}{(\omega_s^2 + \omega_s \omega_r - A_4)} \quad (2.14)$$

$$K_5 = \frac{A_5}{(\omega_s^2 + \omega_s \omega_r - A_4)} \quad (2.15)$$

This is for constant known motor speed and if the leakage inductance in both stator and rotor circuits are considered the same (M. Liwschitz-Garik and C. C. Whipple, 1961). ($L_{ls} \cong L_{lr}$), equations (2.11)-(2.15) may be solved together to obtain the electrical machine parameters in the form.

$$R_s = \frac{K_5 \omega_s}{K_1} \quad (2.16)$$

$$L_s = \frac{K_3 - R_s K_1 / \omega_s}{K_2} \quad (2.17)$$

$$R_r = \frac{K_2 \omega_r^2 L_s}{K_1} \quad (2.18)$$

$$\sigma_s = \frac{R_r + K_2 L_s R_s}{(\omega_r^2 + \omega_s \omega_r) K_2} \quad (2.19)$$

$$L_m = \sqrt{L_s^2 - \sigma_s} \quad (2.20)$$

2.4 System Identification

Higher quality standards, economic motives, and environmental constraints impose more stringent demands on productivity, accuracy, and flexibility of production processes and products. To meet these demands, control theory has become increasingly more important. Modern controllers are model based on a mathematical model of the process to be controlled. The achievable performance is limited, amongst other things, by the fidelity of the model. A model must capture the dynamic behaviour, and this is often accomplished using differential or polynomial equations. Black-box modelling from data, without trying to model internal physical mechanisms, is also referred to as system identification or time series analysis. Another way to come up with models is based on rigorous mathematical deduction and a prior knowledge of the process (D. C. Karnopp et al 1990). This route is

referred to simply as modelling (P. V. d. Bosch, 1994). System identification techniques are applied in, example the process industry to find reliable models for control design (Y. Zhu & T. Backx,1993). The input-output data is collected from experiments that are designed to make the data maximally informative on the system properties that are of interest. The model set specifies a set of candidate models in which the "best" model according to a well-defined criterion will be searched for. In prediction error methods, the sum of the square of prediction errors, example the mismatch between the real measured output and the model output is often used as a criterion (L. Ljung, 1987). Selecting the three entities, data, model set, and criterion are very important steps in an identification procedure. When the data is available, the model set is chosen, and a criterion is selected, the model in the model set that best fits the data according to the specified criterion has to be found. In general, a model set is parameterized and a parameter estimation algorithm is used to find the parameter values such that the model behaviour fits best to the data according to the criterion. Finally, model validation tests are performed.

A control-engineering inspired approach depicting the basic idea behind the identification process is presented in Figure 2.5. The process (the real-world system whose dynamics we want to identify) outputs are compared with the model outputs, when both are subject to the same input signals. After comparing the output difference an error criterion is established by which the model gets refined in the following iterations.



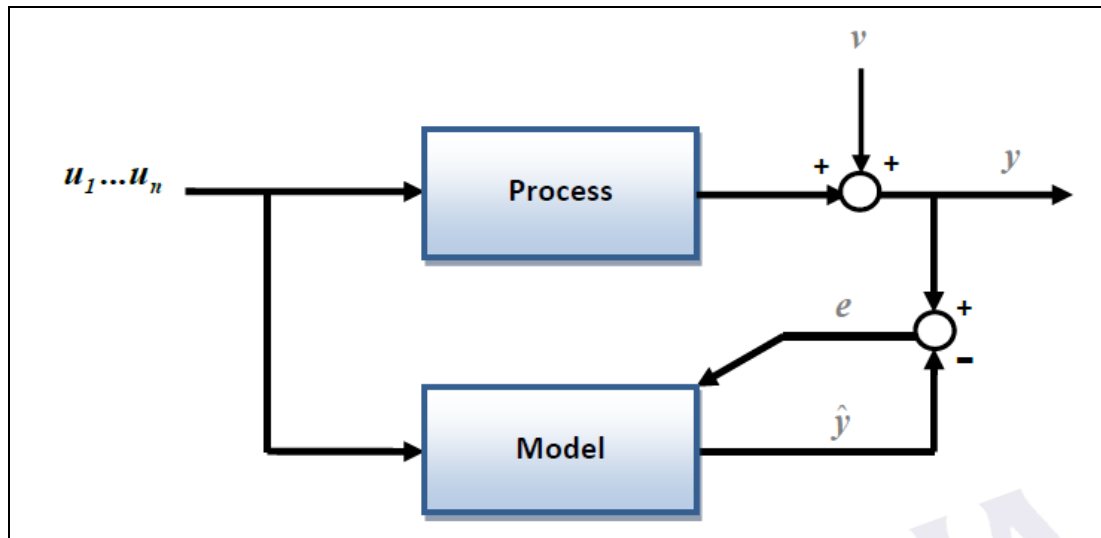


Figure 2.5: A set of inputs u is fed to the process and model. Output y is disturbed by noise v , denotes the model output and e the error difference $y - \hat{y}$

2.5 Linear model structures

The choice of model structure is a central concept in identification – in this section the most well-known structures are briefly introduced. This framework of model structures is often referred to as black box structure (A. Procházka, 1998), which covers a general set of systems without particular attention to physical constraints. A further distinction is often made between parametric and nonparametric models (O. Nelles, 2001). Parametric models represent the system through a finite set of parameters, example in a difference equation, which allows parametric methods to be used in identification. Hence, the discussion will deal almost exclusively with parametric models, with the parameters assumed to be time-invariant. Furthermore, the models presented are considered in discrete time only due to implementation reasons.

Identification of linear input-output data can be classified into two main categories: input-output model identification and state-space model identification. In

the first category, the most general representation (assuming a SISO system) can be obtained by filtering the input u by a linear filter G and filtering the stochastic white-noise term e by a corresponding filter H . The filters can generally be split into a nominator and denominator part (where A represents a possible common denominator of G and H), hence giving the model;

$$A(q^{-1})y(k) = G(q^{-1})u(k) + H(q^{-1})e(k) = \frac{B(q^{-1})}{F(q^{-1})}u(k) + \frac{C(q^{-1})}{D(q^{-1})}e(k) \quad (2.21)$$

Where q^{-1} is the backward shift operator, $q^{-1}u(k) = u(k-1)$. The polynomials A , B , C , F and D are thus of the form;

$$M(q^{-1}) = 1 + m_1q^{-1} + \dots + m_nq^{-n} \quad (2.22)$$

Equation (2.21) represents the basic model for a linear system from which all linear models can be derived (L. Ljung, 1983). This basic model structure is, however, seldom used, as it has proven to be too abstract. Instead, it is often preferred to start with the simplest model which has a possibility of describing the system adequately.

The simplest models for linear systems originate from econometrics, where models were built for stochastic time series (O. Nelles, 2001). The common equations featured only white noise as input and included the autoregressive (AR);

$$y(k) = \frac{1}{D(q^{-1})}e(k) \quad (2.23)$$

The moving average (MA);

$$y(k) = C(q^{-1})e(k) \quad (2.24)$$

and the autoregressive moving average (ARMA) forms (combining the first two mentioned).

$$y(k) = \frac{C(q^{-1})}{D(q^{-1})} e(k) \quad (2.25)$$

These models served as building blocks for the subsequent ARX model (autoregressive with exogenous inputs), which contains both an autoregressive and input part. The ARX model is often referred to as the “the mother of all dynamical model structures” (L. Ljung,1983), as it is the most commonly used equation to model linear systems, defined as;

$$A(q^{-1})y(k) = B(q^{-1})u(k) + e(k) \quad (2.26)$$

The model conforms to the basic difference equation form;

$$y(k) + a_1y(k-1) + \dots + a_ny(k-n) = b_1u(k-1) + \dots + b_mu(k-m) + e(k) \quad (2.27)$$

which signifies that the white-noise error term $e(k)$ enters the equation directly. The finite impulse response model (FIR) is a special case of the ARX model, which is better known from filtering theory. The model considers only feedback in the input signal;

$$y(k) = B(q^{-1})u(k) + e(k) \quad (2.28)$$

The use of ARX in identification is justified in many ways, the main reason being that the parameters of the model are easy to estimate (the predictor of the

model defines a linear regression,) (O. Nelles, 2001) (G. D. Nicolao,1997). The parameters are therefore subject to linear optimization techniques such as linear least squares. As a restriction of the model, both the noise $e(k)$ and the input $u(k)$ share the same denominator $A(q)$. This implies that the noise shares the poles (denominator zeros) of the input and enters the process early. Since $e(k)$ is assumed white noise, correlated noise will yield suboptimal estimation with linear regression techniques.

Correlated noise can be modelled as a moving average (2.24), which inserted into (2.26) gives the popular ARMAX form;

$$A(q^{-1})y(k) = B(q^{-1})u(k) + C(q^{-1})e(k) \quad (2.29)$$

A second more realistic structure for the noise is provided by the output error (OE) model, which assumes that the noise enters late in the process without sharing denominator dynamics. The model takes the form;

$$y(k) = \frac{B(q^{-1})}{F(q^{-1})}u(k) + e(k) \quad (2.30)$$

Where the noise is the direct error between the model and the real output. However optimal identification of this model, as opposed to the ARX, requires the simulated outputs, which result in a nonlinear regression (O. Nelles, 2001),(L. Ljung, 1987) Identification of ARMAX and OE model generally involves the use of approximate linear nonlinear optimization method.

An alternative representation for both SISO and MIMO models is provided by state-space models (IIASA's 20th Anniversary, System Identification, 2009). State-space models have the following representation in discrete time

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \quad (2.31)$$

The model parameterization, i.e., the selection of A , B , and C is not unambiguous, which is why state-space models are arguably less preferred for modeling simple relationships (M. Nørgaard, et al, 2000). Furthermore, many identification algorithms, including the one presented here, require strictly systems of input-output form. A system in state-space form can be transformed into an equivalent input-output representation, providing it is a minimal realization. For such systems, the transform can be derived using the forward shift operator q , according to:

$$x(k+1) = qx(k) = qIx(k) = Ax(k) + Bu(k)$$

$$x(k) = (qI - A)^{-1}Bu(k) \quad (2.32)$$

The expression is substituted in (2.29), yielding the relation;

$$\begin{aligned} x(k+1) &= C(qI - A)^{-1}Bu(k) + Du(k) \\ \Rightarrow \frac{y(k)}{u(k)} &= H(q) = C(qI - A)^{-1}B + D \end{aligned} \quad (2.33)$$

If the system is SISO, $H(q)$ is a transfer function, whereas MIMO systems will convert into a matrix of transfer functions. Correspondingly, a model in input-output form can be transformed into various state-space representations, provided it is proper. For a more detailed discussion, refer to (O. Nelles, 2001).

2.6 Nonlinear model structures

The selection of a black-box model structure becomes a more difficult problem when one is dealing with nonlinear systems, although the nomenclature follows that of the linear models. A general model including all nonlinear and linear models can be described as;

$$y(k) = f(\varphi(k), \theta) + e(k) \quad (2.34)$$

$$\varphi(k) = [u(k-1) \dots u(k-n), y(k-1) \dots y(k-n), e(k-1) \dots e(k-n)] \quad (2.35)$$

Where f can represent any function of the regression vector $\varphi(k)$ containing the past inputs, outputs and errors, and a parameter vector. Every special group of linear model can be transformed into their corresponding nonlinear models (ARX-NARX), simply by replacing the linear dependency with a general function. The main challenge arises from estimating the parameters, which essentially becomes a function approximation problem.

The dilemma of choosing the regressor set is discussed further in (J. Sjöberg, H. Hjalmarsson & L. Ljung. *Neural Networks in System Identification*, 1994), from which the following can be concluded, applying both to nonlinear and linear modelling: The regressor can be limited to inputs and no past outputs, if the response time of the system is finite, i.e., only inputs within a finite time frame t affect the output. This construction has the disadvantage of potentially requiring a large number of past inputs. By including past outputs in the regressor, the system can model an infinite and complex response even with a small number of regressors. On the downside, this construction is prone to instability and adds up possible output noise in the model.

2.7 Parameter estimation

When the data and an appropriate model structure have been identified, the following step marks the identification of the parameters. The goal is to estimate the parameters in a way that provides an optimal fit of the model to the data. The majority of the traditional estimation techniques are based on the least-square estimate, which provides a standard method for solving linear regression problems. When the model parameters are constant, a linear estimation can be carried out batch, in a single step. Unfortunately, most real-world engineering problems are intuitively described by nonlinear functions, which require nonlinear optimization methods and a numerical “step-by-step” estimation process.

2.7.1 Linear regression – linear least squares

Linear regression is a common term for approaches where a line or a curve is fit to a set of observations, consisting of one or more independent variables and a (linearly) dependent variable (M. Verhaegen & V. Verdult, 2007). Assuming the dependence is described by a simple model.

$$Y = aX + b \tag{2.36}$$

The regression would imply estimating a and b over the observation set (in this case consisting of the independent variable X and dependent variable Y). In identification, a linear regression arises when the error between the estimated and real output is described by a linear relationship (O. Nelles, 2001). This is a feature of the ARX models, which can be proven by transforming the ARX model (2.26) into regression form;

$$y(k) = \varphi(k)^T \theta + e(k) \quad (2.37)$$

Where, $\varphi(k)^T = [-y(k-1) \dots -y(k-n) u(k-1) \dots u(k-m)]$ is a regression vector and $\theta = [a_1 \dots a_n b_1 \dots b_m]^T$ contains the model parameters. If the noise $e(k)$ is assumed white, the scalar product $\varphi(k)^T \hat{\theta}$ defines the best possible predictor for the output. Notably, the prediction error;

$$\varepsilon(k) = y - \hat{y}(k) = y - \varphi(k)^T \hat{\theta} \quad (2.38)$$

is now of the same form as the equation error $e(k)$. The error for each prediction is hence linearly dependent on the parameter vector φ – exactly as the linear regression requires. The least-squares method is the most well-known way of solving any linear regression (M. Verhaegen & V. Verdult, 2007)(L. Ljung, 1987). Other identification methods for linear regression exist, but are omitted from description here since they are of larger interest in the statistics community. The least-squares method attempts to minimize a squared loss function, in this case the squared sum of residuals;

$$f(\theta) = \frac{1}{N} \sum_{k=1}^N (y(k) - \varphi(k)^T \theta)^2 \quad (2.39)$$

This function can be minimized analytically by setting its derivative (gradient) to zero. The solution for a multi-parameter system is most conveniently obtained by transforming (2.39) into matrix form. This is done by defining a regression matrix, a vector of outputs y , and applying matrix algebra to obtain;

$$(\theta) = \frac{1}{N} (y - \varphi \theta)^T (y - \varphi \theta) = \frac{1}{N} (y^T y - \theta^T \varphi^T y - y^T \varphi \theta - \theta^T \varphi^T \varphi \theta) \quad (2.40)$$

The gradient is given as a vector of partial derivatives of θ ;

$$\frac{\partial}{\partial \theta} f(\theta) = \frac{1}{N} (-2\varphi^T y + 2\varphi^T \varphi \theta) \quad (2.41)$$

and the least-square estimates are determined by setting $\frac{\partial}{\partial \theta} f(\theta)$ to zero, which yields the normal equation;

$$\varphi^T \varphi \theta = \varphi^T y \quad (2.42)$$

The least squares estimate is finally computed by inverting $\varphi^T \varphi$ example with Gaussian elimination, yielding;

$$\hat{\theta}_{LS} = (\varphi^T \varphi)^{-1} \varphi^T y \quad (2.43)$$

The least-squares method can also be used for estimation of certain nonlinear problems, providing the nonlinear function is linearly dependent on the parameters (G. D. Nicolao. System Identification: Problems and Perspectives, 1997). If the measurement samples involve varying degrees of uncertainty or relevance for the estimation, a variant known as weighted least squares can be used. Each measurement is weighted with a factor w_k , which results in the modified final estimate;

$$\hat{\theta} = (\varphi^T W \varphi)^{-1} \varphi^T W y \quad (2.44)$$

Containing a diagonal matrix W with the elements w_k

2.8 Nonlinear methods

Real-world processes often give rise to models that are either nonlinear or include a nonlinearity in the error, rendering the linear least squares method unusable. In accordance with the linear case, the most commonly used criterion function is the sum of the squared residuals, which, in the nonlinear case is;

$$f(\theta) = \frac{1}{N} \sum_{k=1}^N (y(k) - f(\varphi(k), \theta))^2 = \frac{1}{N} \sum_{k=1}^N e(k)^2 \quad (2.45)$$

Whereas linear least squares estimation is straightforward, nonlinear problems lack a corresponding one-step solution. Instead of estimating the parameters directly, the nonlinear methods approximate the function f locally, in a step-by-step process. An initial estimate is provided for the parameters, followed by iterative update through successive approximation, until a satisfying solution is obtained. Nonlinear parameter estimation is generally carried out by nonlinear optimization methods, which are not limited to solving equations of the least-squares form.

2.9 Types of models

To describe a process or a system we need a model of system. This is nothing new, since we use models daily, without paying this any thoughts. For example, when we drive a car and approaching a road bump, we slow down because we fell intuitively that when this speed is too high we will hit the head in the roof. So from experiences we have developed a model of car driving. We have a feeling of how the car will behave when reach the bump and how we will be affected. Here the model of situation can be considered as a mental model. We can also describe the model by linguistic terms. For example if we drive the car faster than 110km/h then we will hit the head at the roof. This is linguistic model, since the model uses words to describe what happens (Bjorn Sohlberg, 2005). A third way of describing the systems is to use scientific relations to make a mathematical model , which describes in what way output signals respond due to changes in input signal. There are different types of models to represent the system.

White Box Modelling; when a model is developed by modelling, we mean that model is constructed completely from mathematical scientific relations, such as differential equations, difference equations, algebraic equations and logical relations. The resulting model is called white box or a simulation model.

For example a model of electrical network using Kirchoff's laws and similar theorems;

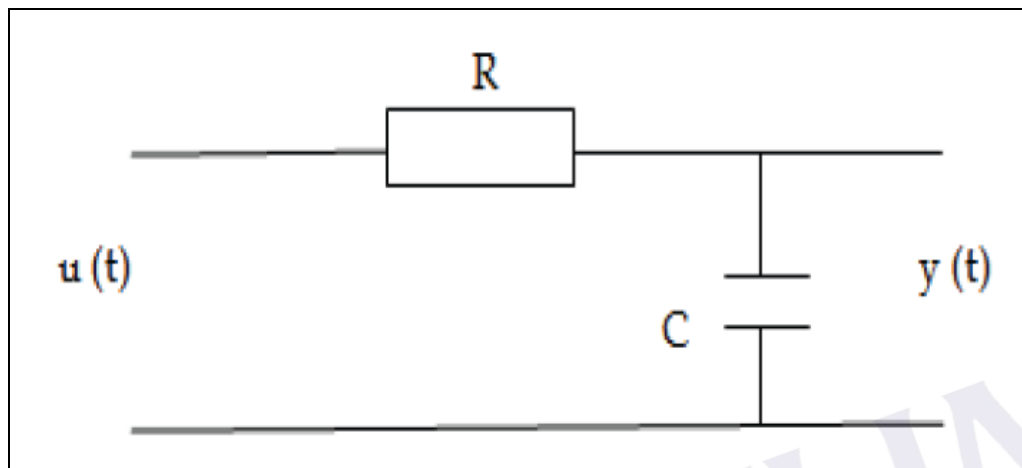


Figure 2.6 RC Circuit

In above RC-circuit where the relation between the input signal $u(t)$ and output signal $y(t)$ is given by Ohm's law. The resulting model would be a linear differential equation with the unknown parameter $M=RC$, which can be estimated from an experiment with the circuit or formal nominal values of the resistor and the capacitor. A mathematical model is given by;

$$M \cdot y'(t) + y(t) = u(t) \quad (2.46)$$

Similarly other processes can be modelled using scientific relations.

Black Box Modelling; when a model is formed by means of identification, we consider the process completely unknown. The process is considered black box with inputs and outputs. Thus it is not necessary to use any particular model structure which reflects the physical characteristics of the system. Normally we use a model which given from a group of standard models. Unknown model parameters are estimated by using measurement data which is achieved from an experiment with the process. In this way model shows input-output relation. Identification using black box models have been used for industrial, economic, ecological and social systems. Within industry, black box models have been used for adaptive control purposes.

REFERENCES

- A. Procházka. Signal analysis and prediction. Birkhäuser, 1998
- Bjorn Sohlberg (2005). Book: Applied Modelling & Identification, Dalarna University, Sweden
- C.T. Sun, "Rule-base structure identification in an adaptive-network based inference system," IEEE Trans. On Systems, vol. 2, no. 1, pp. 64-73, 1994.
- C. J. Park, I-K Jeong, H-K Kim and K. Wohn. Sensor Fusion for Motion Capture System Based on System Identification. Computer Animation 2000, pp.71, 2000.
- D. W. Novotny and T. A. Lipo, "Vector Control and Dynamics of AC Drivers," [Book}, Clarendon press. OXFORD, 1996
- D. C. Karnopp, D. L. Margolis, and R. C. Rosenberg, System Dynamics: A Unified Approach. Wiley & Sons, 1990
- G. Chen, T.T. Pham, J.J. Weiss, "modeling of control systems," IEEE Trans. On Aerospace And Electronic Systems, vol. 31, no. 1, pp. 414-428, 1995
- G.C. Mouzouris, J.M. Mendel, "Dynamic non-singleton logic systems for nonlinear modeling," IEEE Trans. On Systems, vol. 5, no. 2, pp. 199-208, 1997.
- G. D. Nicolao. System Identification: Problems and Perspectives. 12th Workshop on Qualitative Reasoning, June 1997
- I. Eksin, C.T. Ayday, "identification of nonlinear systems," International Conference On Industrial Electronics, Control And Instrumentation, vol. 2/3, pp. 289-293, 1993.
- J. Sjöberg, H. Hjalmarsson and L. Ljung. Neural Networks in System Identification. 10th IFAC symposium on SYSID, pp. 49-71, 1994
- L. Ljung. System Identification: Theory for the User. Prentice-Hall, 1987
- L. Ljung. Theory and Practise of Recursive Identification. The MIT Press, 1983.

- M. Liwshitz-Garik and C. C. Whipple, "Alternating-Current Machines," Van Nostrand., 1961
- M. Nørgaard, O. Ravn, N. K. Poulsen and L. K. Hansen. Neural Networks For Modelling And Control Of Dynamic Systems: A Practitioner's Handbook. Springer, 2000
- M. Verhaegen and V. Verdult. Filtering and System Identification: a Least Squares Approach. Cambridge, 2007
- O. Nelles. Nonlinear System Identification. Springer, 2001
- P. V. d. Bosch, Modelling, identification and simulation of dynamical systems. CRC Press Inc., 1994
- System Identification. Paper Presented on IIASA's 20th Anniversary. 20.9.2009
- T. Kohonen. Self-Organizing Maps. Springer, 1995
- T. Takagi, M. Sugeno, "identification of systems and its applications to modeling and control", IEEE Trans. On Man and Cybernetics, vol. smc-15, no. 1, pp. 116-132, 1985
- Xiangdong Wang, Ming Wei, Jianwen Zhao and Xiang Chen, Parameter Identification Based on SystemIdentification Toolbox in Matlab, Journal of Ordnance Engineering College. 2008, 20(4): 75-78
- Y. Zhu and T. Backx, Identification of multivariable industrial process, for simulation, diagnosis and control. Springer-Verlag, 1993

