LINEAR QUADRATIC REGULATOR (LQR) CONTROLLER DESIGN FOR INVERTED PENDULUM

NOR AKMAL BINTI ALIAS

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University Tun Hussein Onn Malaysia

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ABSTRACT

The Inverted Pendulum System is an under actuated, unstable and nonlinear system. The challenge of this project is to keep the inverted pendulum balanced and track the linear cart to a commanded position. This project presents investigations of performance comparison between conventional Proportional Integral Derivatives (PID) and modern control Linear Quadratic Regulator (LQR) schemes for an inverted pendulum system. The goal is to determine which control strategy delivers better performance with respect to pendulum's angle and cart's position. LQR algorithm needed to compute what the steady-state value of the states should be by multiply that by the chosen gain K, and a new value as the reference is used for computing the input. Gain k is the state feedback gain matrix, so the system becomes a closed loop control system. The LQR should produce a better response compared to PID control strategies in both discrete time and continuous time control.

ABSTRAK

Sistem Bandul Songsang adalah satu sistem di bawah gerakkan, tidak stabil dan tidak linear. Cabaran projek ini adalah untuk memastikan bandul terbalik seimbang dan menjejaki kereta itu dalam keadaan linear kepada kedudukan yang ditetapkan. Projek ini membentangkan siasatan perbandingan prestasi antara *Proportional Integral Derivatives* (PID) dan kawalan moden *Linear Quadratic Regulator* (LQR) skim untuk sistem bandul terbalik. Matlamatnya adalah untuk menentukan strategi kawalan menyampaikan prestasi yang lebih baik berkenaan dengan sudut bandul dan kedudukan kereta ini. Algoritma LQR yang diperlukan untuk mengira nilai keadaan mantap suatu keadaan hendaklah dengan meggandakan gandaan K yang dipilih, dan nilai baru sebagai rujukan yang digunakan untuk mengira input. Gandaan K adalah gandaan matriks maklum balas suatu keadaan, oleh itu sistem akan menjadi satu sistem kawalan gelung tertutup. LQR akan menghasilkan tindak balas yang lebih baik berbanding dengan strategi kawalan PID dalam kedua-dua masa yang berasingan dan kawalan masa yang berterusan.

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LIST OF SYMBOLS AND ABBREVIATION

PID	-	Proportional-Integral-Derivative
LQR	-	Linear Quadratic Regulator
Kp	-	Proportional gain
Ki	-	Integral gain
Kd	-	Derivative gain
V	-	Volt
PWM	-	Pulse Width Modulator
PIC	-	Programmable Integrated Circuit
MATLAB	-	Matrix Laboratory
GUI	-	Graphical User Interface
AC	-	Alternating Current
DC	-	Direct Current
V	-	Voltage
m	-	meter
b	-	friction
l	-	length
F	-	force
x	-	cart position
θ	-	angle

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CHAPTER 1

INTRODUCTION

1.1 Project Background

One-dimensional inverted pendulum is a nonlinear problem, which has been considered by many researchers, Omatu and Yashioka, 1998 [1] and Magana and Holzapfel, 1998 [2], most of which have used linearization theory in their control schemes. In general, the control of this system by classical methods is a difficult task. This is mainly because this is a nonlinear problem with two degrees of freedom (the angle of the inverted pendulum and the position of the cart), and only one control input. The inverted pendulum is used for control engineers to verify a modern control theory since its characteristics as marginally stable as a control. This system is popularly known as a model for the attitude control especially in aerospace field. However, it also has its own deficiency due to its principles, highly non linear and open-loop unstable system. Thus, causing the pendulum falls over quickly whenever the system is simulated due to the failure of standard linear techniques to model the non-linear dynamics of the system. Moreover, it makes the identification and control become more challenging. The common control approaches to overcome the problem by this system namely Linear Quadratic Regulator (LQR) control and Proportional-Integral-Derivative (PID) control that require a good knowledge of the system and accurate tuning to obtain good performance. Nevertheless, it attribute to difficulty in specifying an accurate mathematical model of the process. This project presents investigations of performance comparison between conventional (PID) and modern control (LQR) schemes for an inverted pendulum system. The dynamic model and design requirement have been taken from Carnegie Mellon, University of Michigan. Performance of both control strategy with respect to pendulum's angle and cart's position is examined. Comparative assessment of both control schemes to the system performance is presented and discussed.

1.2 Problem Statement

The challenge of this project is to keep the inverted pendulum balanced and track the linear cart to a commanded position. In practice, it is interesting to point out that similar dynamics and control problem apply to rudder roll stabilization of ships. During this project, the controllers that will be used are the PID and the LQR. So the proposed controllers with proper design and tuning will be able to overcome the above mentioned challenge.

1.3 Objectives

Based on above problem statement, the objectives are:

1.3.1 To design LQR and PID controllers for inverted pendulum.

1.3.2 To compare the performance result for both controllers in order to get the best controller design.

1.3.3 To balance the inverted pendulum by applying a force to the cart that the pendulum is attached to.

1.4 Scopes

The scopes for this Linear Quadratic Regulator Controller Design for Inverted Pendulum are as stated below:

1.4.1 This project uses the long single inverted pendulum as a default configuration.

1.4.2 Design a controller using modern technique which is LQR and conventional technique PID.

1.4.3 Simulate the controllers using Matlab and conclude the best controller based on the simulation results.

1.4.4 Design inverted pendulum system using Matlab.

1.4.5 Control the pendulum's position that should be return to the vertical position after given the initial disturbance.

1.5 Report Outline

This report contains five chapters which are Introduction, Literature Review, Methodology, Result and Analysis and lastly Conclusion and Recommendation.

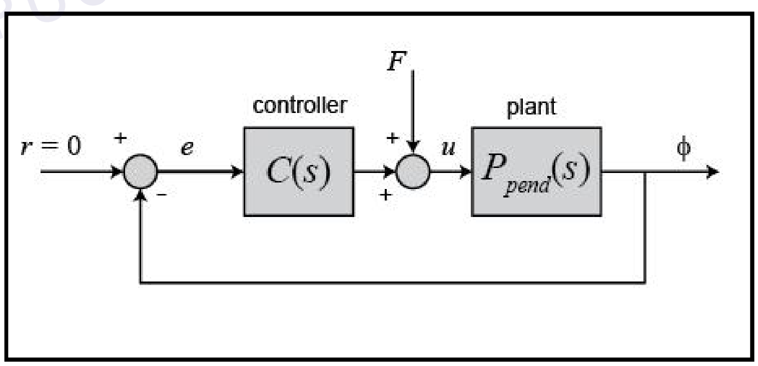
The Introduction chapter is about the project background, problem statement, objectives and scopes. These topics are the basic information about the whole system. The main idea of the system is stated in this chapter.

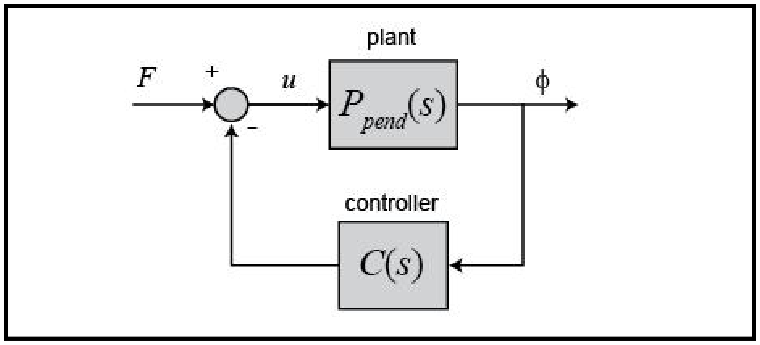
Chapter two contains about the PID and LQR controller, the hardware components as well as the software, and lastly the inverted pendulum example in real life and the previous case study. This chapter is aiming the critical points of knowledge and theoretical to a particular topic. The controllers and the inverted pendulum are the core in this project. The past researches are as the guidance in implementing this system.

Chapter three which is Methodology is about the system overview, hardware design, software parts and system operation. This chapter is concern about the process of designing and implementing this whole system. Every process has the role that have been specify for the system to be function as needed.

Chapter four is basically regarding the results and analysis for this project. The results are displayed and then being analyzed in this chapter so that readers have better understanding about whole process.

The last chapter in this report is Conclusion and Recommendation. This chapter is summary of the whole project. The idea is conclude in this last chapter and the recommendation part is to upgrade the system to a higher level.


F
controller
plant
r = 0
+
e
C(s)
+
u
P_pend(s)
ϕ
-
+



e designed in the PID controller

fer function of the PID controller

$$Gpid = Kp + \frac{Ki}{s} + Kd.s$$

2.2 LQR Controller

In order to overcome some problems that faced by PID controller, the other type of control methods can be developed such as Linear-Quadratic Regulator (LQR) optimal control. LQR is a control scheme that gives the best possible performance with respect to some given measure of performance. The performance measure is a quadratic function composed of state vector and control input.

Linear Quadratic Regulator (LQR) is the optimal theory of pole placement method. LQR algorithm defines the optimal pole location based on two cost function. To find the optimal gains, one should define the optimal performance index firstly and then solve algebraic Riccati equation. LQR does not have any specific solution to define the cost function to obtain the optimal gains and the cost function should be defined in iterative manner.

LQR is a control scheme that provides the best possible performance with respect to some given measure of performance. The LQR design problem is to design a state feedback controller K such that the objective function J is minimized. In this method a feedback gain matrix is designed which minimizes the objective function in order to achieve some compromise between the use of control effort, the magnitude, and the speed of response that will guarantee a stable system. For a continuous-time linear system described by [4]:

$$\dot{x} = Ax + Bu$$

With a cost functional defined as

$$J = \int (x^T Q x + u^T R u) dt$$

Where Q and R are the weight matrices, Q is required to be positive definite or positive semi-definite symmetry matrix. R is required to be positive definite symmetry matrix. One practical method is to Q and R to be diagonal matrix. The value of the elements in Q and R is related to its contribution to the cost function J. The feedback control law that minimizes the value of the cost is:

$$u = -Kx$$

K is given by

$$K = R^{-1} B^T P$$

And P can be found by solving the continuous time algebraic Riccati equation:

$$A^T P + PA - PBR^{-1} B^T P + Q = 0$$

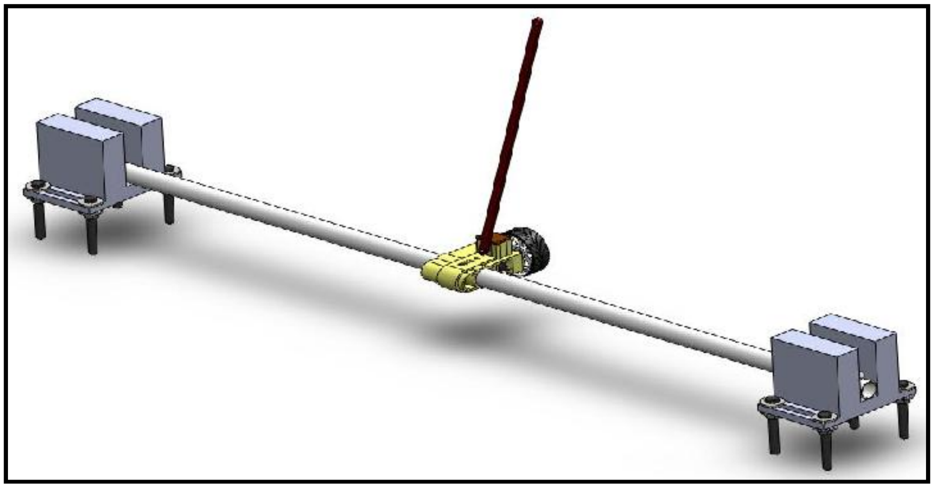
The LQR algorithm is, at its core, just an automated way of finding an appropriate state. As such it is not uncommon to find that control engineers prefer alternative methods like full state feedback (also known as pole placement) to find a controller over the use of the LQR algorithm. With these the engineer has a much clearer linkage between adjusted parameters and the resulting changes in controller behavior. Difficulty in finding the right weighting factors limits the application of the LQR based controller synthesis.

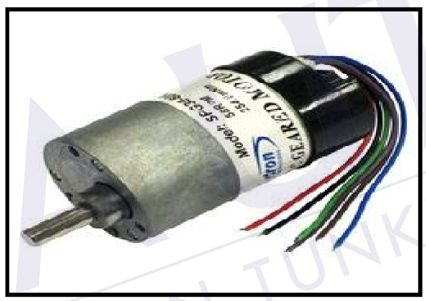
2.3 Inverted Pendulum

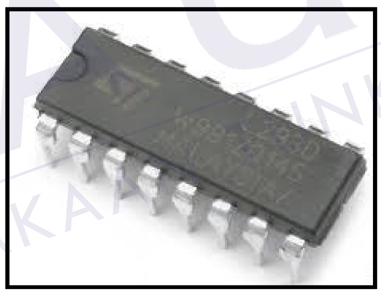
It is virtually impossible to balance a pendulum in the inverted position without applying some external force to the system. The problem involves a cart, able to move backwards and forwards, and a pendulum, hinged to the cart at the bottom of its length such that the pendulum can move in the same plane as the cart. That is, the pendulum mounted on the cart is free to fall along the cart's axis of motion. The system is to be controlled so that the pendulum remains balanced and upright, and is resistant to a step disturbance.

If the pendulum starts off-centre, it will begin to fall. The pendulum is coupled to the cart, and the cart will start to move in the opposite direction, just as moving the cart would cause the pendulum to become off centre. As change to one of parts of the system results in change to the other part, this is a more complicated control system than it appears at first glance. The inverted pendulum cart runs along a track. A rotary position sensor measures the cart position from its rotation and another potentiometer measures the angle of the pendulum.

An inverted pendulum is a pendulum which has its center of mass above its pivot point. It is often implemented with the pivot point mounted on a cart that can move horizontally and may be called a cart and pole. Inverted Pendulum is an inherently unstable system. Force must be properly applied to keep the system intact. To achieve this, proper control theory is required. The Inverted Pendulum is essential in the evaluating and comparing of various control theories.







Darlington driver or n-channel MOSFET, which is why they cannot be used to reverse a motor.

2.4 MATLAB

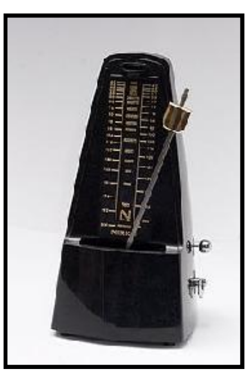
MATLAB (matrix laboratory) is a numerical computing environment and fourth-generation programming language. MATLAB is a weakly typed programming language. It is a weakly typed language because types are implicitly converted. It is a dynamically typed language because variables can be assigned without declaring their type, except if they are to be treated as symbolic objects, and that their type can change. Values can come from constants, from computation involving values of other variables, or from the output of a function.

2.4.1 MATLAB Simulink

Simulink is a block diagram environment for multi domain simulation and Model-Based Design. It supports system-level design, simulation, automatic code generation, and continuous test and verification of embedded systems. Simulink is an input/output device GUI block diagram simulator which contains a Library Editor of tools from which we can build input/output devices and continuous and discrete time model simulations. It contains continuous and discontinuous system model elements.

Simulink, developed by MathWorks, is a commercial tool for modeling, simulating and analyzing multi domain dynamic systems. Its primary interface is a graphical block diagramming tool and a customizable set of block libraries. It offers tight integration with the rest of the MATLAB environment and can either drive matlab or be scripted from it. Simulink is widely used in control theory and digital signal processing for multi domain simulation and Model-Based Design.


MADE IN ITALY
COMMUNICATION
ARDUINO
MEGA
2560
POWER
ANALOG IN
DIGITAL

Types	Description
Mechanical metronome 	• A metronome is any device that produces regular, metrical ticks settable in beats per minute. • These ticks represent a fixed, regular aural pulse. • Some metronomes also include synchronized visual motion for example the pendulum-swing.

Segway PT

- The Segway PT is a two-wheeled, self-balancing, battery-powered electric vehicle invented by Dean Kamen. It is produced by Segway Inc. of New Hampshire, USA.
- The Segway detects, as it balances, the change in its center of mass, and first establishes and then maintains a corresponding speed, forward or backward. Gyroscopic sensors and fluid-based leveling sensors detect the weight shift.

Loading Crane

- Crane is a type of machine, generally equipped with a hoist, wire ropes or chains, and sheaves, that can be used both to lift and lower materials and to move them horizontally.
- It is mainly used for lifting heavy things and transporting them to other places. It uses one or more simple machines to create mechanical advantage and thus move loads beyond the normal capability of a man.

2.6.2.1 LQG/LTR Controller Design for Rotary Inverted Pendulum Quanser Real-Time Experiment

This experiment consists of a rigid link (pendulum) rotating in a vertical plane. The rigid link is attached to a pivot arm, which is mounted on the load shaft of a DC-motor [6]. The pivot arm can be rotated in the horizontal plane by the DC-motor. The DC-motor is instrumented with a potentiometer. In addition, a potentiometer is mounted on the pivot arm to measure the pendulum angle. The principal objective of this experiment is to balance the pendulum in the vertical upright position and to position the pivot arm. Since the plant has two degrees of freedom but only one actuator, the system is under-actuated and exhibits significant nonlinear behavior for large pendulum excursion. The purpose is to design a robust controller in order to realize a real-time control of the pendulum position using a Quanser PC board and power module and the appropriate WinCon real-time software. For the controller design is used a well-known robust method, called LQG/LTR (Linear Quadratic Gauss Ian/Loop Transfer Recovery) which implements an optimal statefeedback. The real-time experiment is realized in the Automatic Control laboratory.

2.6.2.2 DC Motor Controller Using Linear Quadratic Regulator (LQR) Algorithm Implementation on PIC

Linear Quadratic Regulator (LQR) algorithm is one of the controller methods to control a system [7]. In this project, the LQR was implemented on the PIC microcontroller to control the dc motor. The main objective of this controller is to minimize the deviation of the speed of DC motor. DC motor speed is controlled by its driving voltage. The higher the voltage, the higher the motor speed will be. The speed of the motor is specifying that will be the input voltage of the motor and the output will be compare with the input. As the result, the output must be the same as or approximately the same as the input voltage. In this project, the LQR algorithm was implemented on the PIC microcontroller so the result can be shown. Before the implementation on the PIC, the dc motor state space has to be derived. Then, from the state-space, it can be design the

LQR controller by using the MATLAB software. The stable system is got by tuning the Q and R value that can be seen by the simulation.

2.6.2.3 Modeling and Controller Design for an Inverted Pendulum System

The Inverted Pendulum System is an under actuated, unstable and nonlinear system [8]. Therefore, control system design of such a system is a challenging task. To design a control system, this thesis first obtains the nonlinear modeling of this system. Then, a linearized model is obtained from the nonlinear model about vertical (unstable) equilibrium point. Next, for this linearized system, an LQR controller is designed. Finally, a PID controller is designed via pole placement method where the closed loop poles to be placed at desired locations are obtained through the above LQR technique. The PID controller has been implemented on the experimental set up.

2.6.2.4 Real-Time Optimal Control for Rotary Inverted Pendulum

The rotary inverted pendulum system was a highly nonlinear model, multivariable and absolutely unstable dynamic system. It was used for testing various design control techniques and in teaching modern control [9]. The objectives of this study were to develop a real rotary inverted pendulum which derived the mechanical model by using Euler-Lagrange and design controller algorithm for self-erecting and balancing of a rotary inverted pendulum. Research showed a convenient way to implement a real-time control in self-erecting a pendulum from downward position and balancing the pendulum in vertical-upright position. An Energy based on PD controller was applied in self-erecting of the pendulum while LQR controller was applied to balance the pendulum. Results of both control techniques from computer simulation and experiment were given to show the effectiveness of these controllers. Both simulations and experiments were confirmed the control efficiency of the method.

2.6.2.5 Controller Design of Inverted Pendulum Using Pole Placement and LQR

In this paper modeling of an inverted pendulum is done using Euler – Lagrange energy equation for stabilization of the pendulum [10]. The controller gain is evaluated through state feedback and Linear Quadratic optimal regulator controller techniques and also the results for both the controller are compared. The SFB controller is designed by Pole-Placement technique. An advantage of Quadratic Control method over the pole-placement techniques is that the former provides a systematic way of computing the state feedback control gain matrix. LQR controller is designed by the selection on choosing. The proposed system extends classical inverted pendulum by incorporating two moving masses. The motion of two masses that slide along the horizontal plane is controllable.

2.6.2.6 Stabilization of Real Inverted Pendulum Using Pole Separation Factor

Based on the pole placement design technique, a full state feedback controller using separation factor is proposed to stabilize a real single inverted pendulum [11]. The strategy is to start with selection two dominant poles that achieve a certain desired performance, using a separation factor between the selected dominant poles and the other poles to eliminate their effect on the system performance, and finally Ackermann's formula can be used to calculate the feedback gain matrix to place the system poles at the desired locations. Simulation and experimental results demonstrate the effectiveness of the proposed controller, which offers an excellent stabilizing and also ability to overcome the external resistance acting on the pendulum system.

2.7.7 Modeling and Controller Design of Inverted Pendulum

In this paper modeling of an inverted pendulum has been done and then three different controllers (PID & SFB, LQR) have been used for stabilization of the pendulum [12]. The proposed system extends classical inverted pendulum by incorporating two moving masses. The motion of two masses that slide along the horizontal plane is controllable .The results of computer simulation for the system with Proportional, Integral and Derivative (PID) & State Feedback Controllers are shown. Main focus is to derive the mathematical model and the analysis of its system performance, then design a LQR controller in order to get better control.

CHAPTER 3

METHODOLOGY

This research is adopting methods approach involving development and improvement to enhance the overall process of project implementation. The project's Gantt chart is given in Appendix A. The research is conducting in three phase's basis. This phase will make sure that the work will be organized based on the scheduled task.

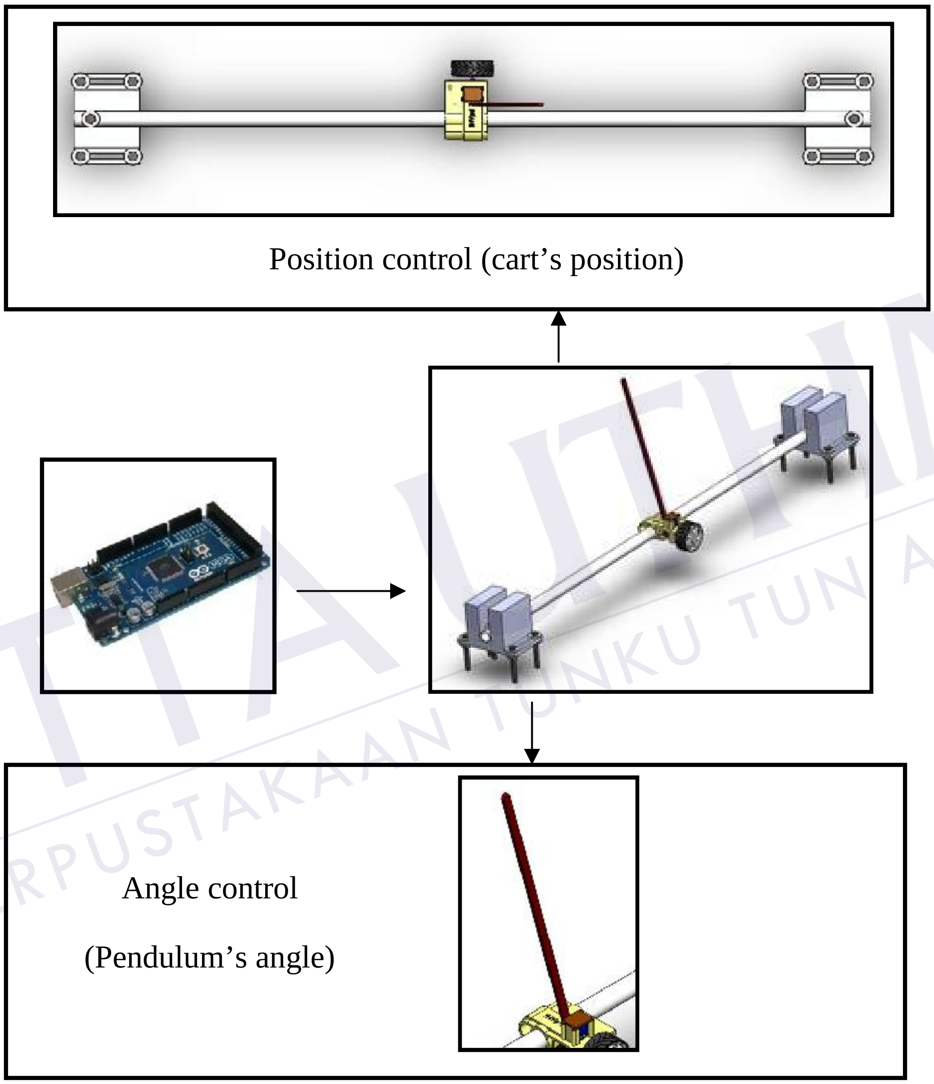
This project is divided into two parts which is hardware and software. Therefore, this chapter will discuss about the process for each parts includes designing, testing, troubleshooting and integrating in order to create one whole system of LQR and PID Controller Design for Inverted Pendulum.

3.1 Familiarization of the System (Phase 1)

In order to implement this project, the first phase that is needed to be concerned is the familiarization of the system. The whole system should be identified and clarified in order to execute the correct process flow of this system.

3.1.1 System Overview

The inverted pendulum is a highly nonlinear and open-loop unstable system. The control of inverted pendulum is to make the cart and swinging rod achieve desired equilibrium position as soon as possible. LQR and PID are the controllers that will be used in order


Position control (cart's position)
Angle control
(Pendulum's angle)

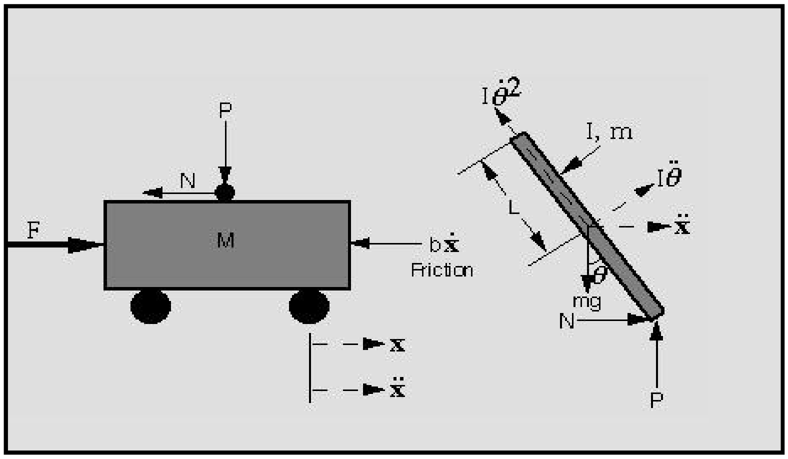
3.2 Project Implementation (Phase 2)

The project implementation will covers all of the controllers that will be done in this project. The mathematical model for each of the controller is very important in order to design the controller. The controller itself plays a big role in order to stabilize the inverted pendulum in upright position.

3.2.1 Inverted Pendulum

In this section, two control methods are proposed and explained in details which are PID and Linear Quadratic Regulator (LQR) controllers. Furthermore, the following design specifications have been made to evaluate the performance of both control schemes. The parameters that are used in designing the inverted pendulum are as below:

M	mass of the cart	0.208 kg
m	mass of the pendulum	0.08 kg
b	friction of the cart	0.16 N/m/sec
l	length to pendulum center of mass	0.382 m
I	inertia of the pendulum	12.5e-6 kg*m^2
L	Length of the rail	0.894m
F	force applied to the cart	
x	cart position coordinate	
theta	pendulum angle from vertical	



natl natical equation for

: cai n the horizontal dire

$$M\ddot{x} + b\dot{x} + N = F$$

rces (n the pendulum in the horiz

$$m\ddot{x} + ml\ddot{\theta}\, cos\theta - ml\grave{a}\theta^{2} \sin\theta = N$$

quat on (2) i equation (1),

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}cos\theta - ml\ddot{\theta}^{2}sin\theta = F$$

e forces along the v rtical direction of the pen lun

$$P\, sin\theta + N\cos\theta - mg\, sin\theta = ml\ddot{\theta} + m\ddot{x}\, c\quad \theta$$

oments about the cent r of g

$$-Pl\, sin\theta - Nl\cos\theta = l\ddot{\theta}$$

on (4) & (5)

$$l + ml^{2})\ddot{\theta} \quad mgl\sin\theta = -ml\ddot{x}\cos\ddot{\theta}$$

The system under consideration is a non-linear system. For ease of modeling and simulation, it has to take a small case approximation such that the system will be a linear one. The linearization point will be θ = Π

$$\theta = \Pi + \phi$$

ϕ is the angle between the pendulum and vertical upward direction. If it is chosen that,

$\phi \approx 0$, then $\cos\theta = -1$, $\sin\theta = -\phi$

So, after linearization equation (6) becomes,

$$(l + ml^2)\phi - mgl\phi = ml\ddot{x} \qquad (7)$$

And equation (3) becomes,

$$(M + m)\ddot{x} + b\dot{x} - ml\phi = F$$

Here, F is the mechanical force to be applied on the moving cart system. But in real time model it needs to input voltage proportional to the force F. If the input voltage is u, then equation (8) becomes,

$$(M + m)\ddot{x} + b\dot{x} - ml\phi = u \qquad (8)$$

3.2.1.2 Transfer Function

The transfer function of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

Transfer function = G(s) = $\frac{L[output]}{L[input]}$

By using the concept of transfer function, it is possible to represent system dynamics by algebraic equation in s. If the highest power of s in the denominator of the transfer function is equal to n, the system is called an nth-order system [11].

Laplace transform of equation (7)

$$(I + ml^2)\phi(s)s^2 - mgL\phi(s) = -mlX(s)s^2 \qquad (9)$$

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