

The Development of Proofs in Analytical Mathematics for Undergraduate Students

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Abstract. Proofs in analytical mathematics are essential parts of mathematics, difficult to learn because its underlying concepts are not visible. This research consists of problems involving logic and proofs. In this study, a short overview was provided on how proofs in analytical mathematics were used by university students. From the results obtained, excellent students obtained better scores compared to average and poor students. The research instruments used in this study consisted of two parts: test and interview. In this way, analysis of students' actual performances can be obtained. The result of this study showed that the less able students have fragile conceptual and cognitive linkages but the more able students use their strong conceptual linkages to produce effective solutions

1. Introduction

This study examines the proofs in analytic mathematics and the concepts of learning mathematical proof for undergraduate students at the university level. It focuses more on the basis of proofs and looks deeper into the history of proof. Proofs are central to mathematics as the level of mathematics increase, and then it becomes more and more complicated [1]. Following are the theoretical discussions of mathematical proof among undergraduate students in the methodology described in Section 2. The methodology on the mistake made by students while attempting to construct is a valid argument. The results from each data instrument used in methodology are presented: test and interview. The results are presented in two ways: The ability of students to give correct answer by categories of excellent, average and poor and common mistakes for each question. The data were collected during the study and reported. As a part of the study, a question is stated, if studied further, the researcher would have a better understanding of students' proof performance and the comprehension of their proving methods [5, 6].

Students viewed mathematics as something bored due to the approach that too technical and mathematics presented to them contain many symbols which do not have meaning. This is because; students have not been introduced on how the properties or theorem is obtained. Many students have trouble the first time they take a mathematics course in which proofs play a significant role. The inability to communicate proofs in an understandable manner has troubled students and teachers in all branches of mathematics. There is a lack of proper method for explaining theoretical mathematics. The objectives of this study is to identify students' performance in problem solving skills for the development of proofs

and to recognize common mistakes with difficulties students encounter in solving the proving problems. There are other quite considerable studies were carried out on mathematics education in Malaysia [8, 9].

2. Methodology

The methodology, as the central part of this study, was conducted at Universiti Tun Hussein Onn Malaysia, during semester 1 of 2015/2016. Subjects of the study were undergraduate Mathematics Technology students who were enrolled in the Discrete Mathematics course during the previous semester.

2.1 Selection of sample

The students chosen are those second year undergraduates who had undergone the same Discrete Mathematics course in the first year. They were from group BWA program (Mathematics Technology). Each group of students was divided into three categories according to their grades A, B and C. Students having grade E and some with grade D were considered not to reach the minimum requirement in the first year examination.

2.2 Research instrument

The research instrument consists of two section which are: test and interview.

2.2.1 Test (Questionnaire)

The test consists of four questions. All four are related to mathematical proof and understanding of the same. In the questionnaire of the test, the problems were more direct and formal proofs. It consists of four questions that ask students to show their solutions. The questionnaire contain the different type of proof which are direct proof, proof by contradiction, proof by contrapositive and proof by counter example. Students are given the opportunities to display each steps of proving. Two different approaches are used to the questions, one is to evaluate if students answer correctly, and the other is to see their reason behind the answer. Students' process of solving the given problems is interpreted by using the four-stage process as explained in Rodgers [2] as follows:

- 1) Analyze the structure
- 2) Write the end and beginning
- 3) Connect the beginning with the end
- 4) Polish the proof

2.2.2 Interview

In this study, students will be given questionnaire administered in interviews. This combined approach is used to take into account of individual strengths and limitations of the two methods. According to Amit and Vinner [3], the common belief is that an interview is a better instrument than the questionnaire. This is because many ambiguities can be resolved in an interview that cannot be resolved in a questionnaire. Also, some spontaneous reactions in an interview can be extremely illuminating, much more than the controlled or even inhibited reactions one can obtain in a questionnaire. Further study were done through statistical analysis [4, 5, 7].

2.3 Methods of analyzing data

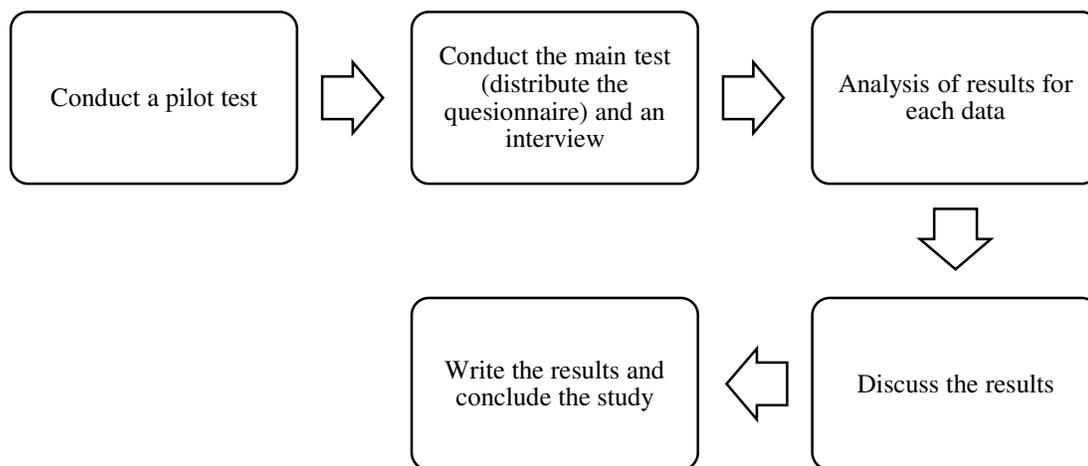


Figure 1. Flowchart of data collection process.

3. Result

The collected data from the research were reported and discussed. First, the discussion for overall percentage of the correct answer was presented. Lastly, the researcher presented an extended table of common mistakes students commit.

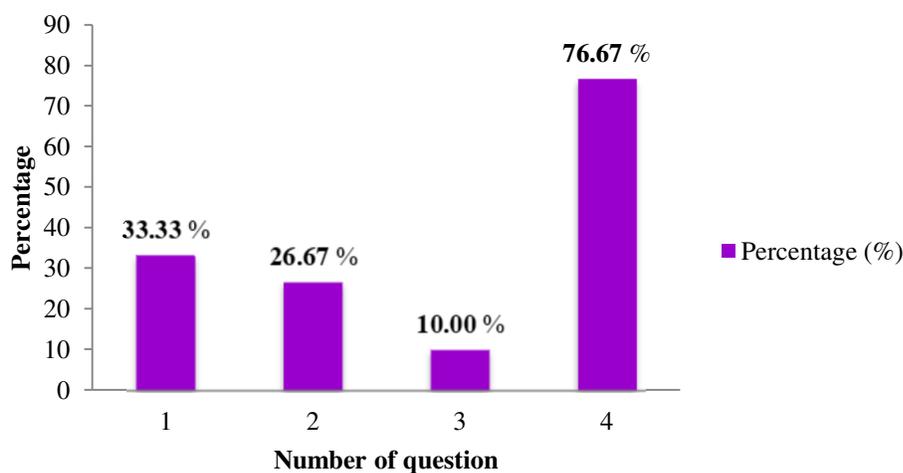


Figure 2. Percentage of correct answers.

Figure 2 shows that none of the questions can be answered 100% correct by all students. Based on the graph, it demonstrated students having trouble in solving question 3 and question 4 showed highest percentages. This discussion was based on their discrete mathematics results which were excellent, average, and poor performance.

3.1 Question 1

Question 1 for the test is as follows:

Proof. (Direct) The sum of any two even integers x and y is even.

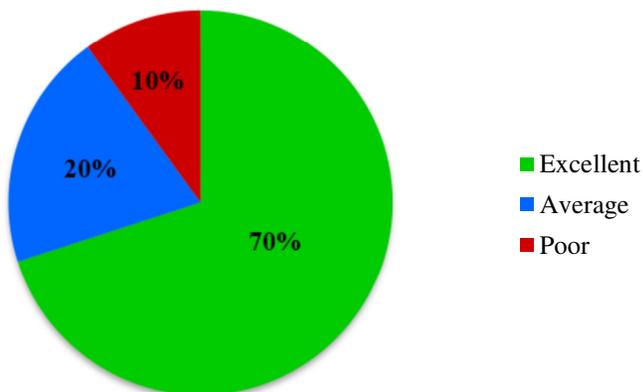


Figure 3. Pie chart for percentage of students answered correctly Question 1.

From 30 students, only 10 students were able to proof the statement in question 1 correctly which are 7 excellent students, 2 average students and 1 poor student (refer Figure 3).

3.2 Question 2

Question 2 for the test is one of the best known examples of proof by contradiction. The question is as follows:

Proof. (Contradiction) $\sqrt{2}$ is irrational.

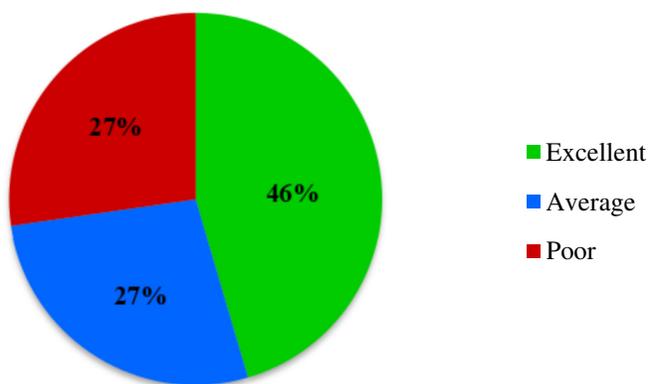


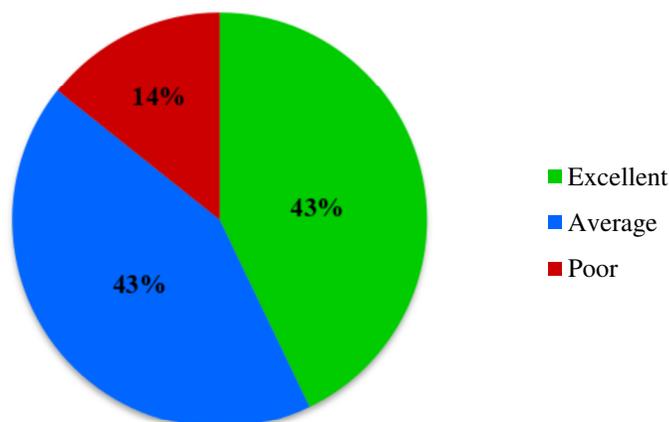
Figure 4. Pie chart for percentage of students answered correctly Question 2.

From overall 30 students, only 11 students able to answer this question which composed of 5 excellent students, 3 average students and 3 poor students. Generally, all students can answer question 2, but most of them did mistakes (refer Figure 4).

3.3 Question 3

The Question 3 asked students to proof by contrapositive. The question is as follow:

Proof. (Contrapositive) Suppose $\text{p} \in \mathbb{Z}$. If p^2 is even, then p is even.



The **Figure 5.** Pie chart for percentage of students answered correctly Question 3. The pie chart showed the percentage of students in category 3 which is poor students is 14% while percentage of students in category 1 and 2 is 43%. Most of the students were unable to answer this question (refer Figure 5)

3.4 Question 4

The last question in the test is proof by counterexample. The question is as follows:

Proof. (Counterexample) The statement “all odd numbers are prime” is false.

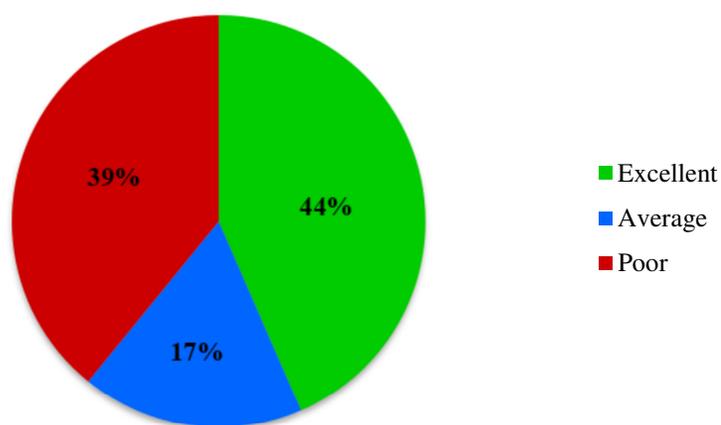


Figure 6. Pie chart for percentage of students answered correctly Question 4.

Most of 30 students can answer this question correctly. There was 23 students that give correct answer and able to explain it mathematically (refer Figure 6).

3.5 Common mistakes

Table 1 is a summary of common mistakes discussed previously and it consists of two categories which are; Mistakes and Difficulties.

Table 1. Extended list of common mistakes in constructing mathematical proof.

Mistake	Difficulties
Arguing from example	Inability to generalize Uncomfortable with symbolic notations Difficulty to employ abstract reasoning
Same identifier	Uncomfortable with symbolic notations Uncertain of general definitions Doesn't understand the relation between symbols and numbers they replace
Jumping to conclusion	Inability to think abstractly Doesn't understand process of justification Unsure of what needs to be proved Disregarding some cases
Begging the question	Unsure of what needs to be proved Inability to see the difference of what is given and what asked
Misuse of the word "if"	Doesn't understand process of justification
Intuitive proof	Insufficient knowledge about the topic Inability to think abstractly Doesn't understand process of justification Inability to manipulate with the symbolic notation
Solving for unknown	Doesn't understand process of justification Inability to generalize Unsure of what needs to be proved
Wrong conclusion or no conclusion (no justification)	Unsure of what needs to be proved Doesn't understand the claim Doesn't understand the conditions Inability to make a connection between the conditions and claim Proving irrelevant claim
Computational mistakes	Mindless mistakes Disregarding conditions

4. Conclusion

In most problems discussed, students preferred to provide a direct proof and the instructions are centered on the method, but it have been noticed that even when students had an opportunity to prove the claim using another proving method most of them chose for the direct proof. The most important difficulty students' face is the inability to make a connection between the conditions and claim. Another troublesome point is students' ambiguity in their mathematical knowledge from earlier education. The third most major difficulty students faced during the study was unsure of what needs to be proved. Finally, from the difficulties we noticed that students do not realize the importance of the definitions and axioms. Therefore, students start to think about definitions and axioms as a new part of mathematics instead of looking at them as the basis for any mathematical activity.

Acknowledgments

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