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Unsteady Blood Flow with Nanoparticles Through Stenosed Arteries in the Presence of Periodic Body Acceleration

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Abstract. The effects of nanoparticles such as Fe_30_4,$Ti0_2$, and Cu on blood flow inside a stenosed artery are studied. In this study, blood was modelled as non-Newtonian Bingham plastic fluid subjected to periodic body acceleration and slip velocity. The flow governing equations were solved analytically by using the perturbation method. By using the numerical approaches, the physiological parameters were analyzed, and the blood flow velocity distributions were generated graphically and discussed. From the flow results, the flow speed increases as slip velocity increases and decreases as the values of yield stress increases.

1. Introduction

Artery stenosis (or atherosclerosis) is an arterial disease caused by accumulation of fats and fibrous tissue in the lumen of arterial wall [1]. The presence of stenosis is detrimental to a blood circulatory system. The study of blood flow in stenosed arteries has been reported by many researchers. Chakravarty and Mandal [2] studied the two-dimensional blood flow through tapered arteries with stenosis. Liu et al. [3] reported that stenosis restricted the flow field at the end of stenotic and tapered arteries. Sankar and Lee [4] investigated analytically the pulsatile flow model of Herschel Bulkely fluid in stenosed arteries with the help of regular perturbation method. Regular perturbation method was used by Akbar et al. [5] in order to study the characteristics of Reiner-Rivlin model in tapered arteries with stenosis. Later, Akbar [6] analyzed the tangent hyperbolic blood flow through tapered artery with mild stenosis. Recently, the slip effects on the wall of tapered artery with mild stenosis have been studied by Ijaz and Nadeem [7] by treating the working fluid as Newtonian.

Flow cases involving different stenosed arteries have been studied by several researchers. Mandal et al. [8] investigated the effects of periodic body acceleration on blood flow. They considered the arterial wall as an elastic cylindrical tube containing a stenosis on the lumen. Mishra et al. [9] modelled the blood flow through a composite stenosis in an artery with permeable wall. Yadav and Kumar [10] studied the effects of length of stenosis on the resistance of blood flow (Bingham plastic model) through a generalized artery with multiple stenosis. They found that the flow resistance decreased as the shape parameter increased and the size of stenosis decreased. The study of blood flow subjected to magnetic field in arterial stenosis (with slip effect) has been investigated by Hazarika and Sharma [11]. They observed that as the Hartman number increased, both fluid velocity and wall shear stress would decrease. Also, they claimed that increased Reynolds number would lead to the increases of fluid velocity and wall shear stress. Siddiqui et al. [12] have modelled the blood as non-Newtonian Bingham plastic fluid model. The model was used to study the flow through stenosed artery in the presences of slip velocity and body acceleration.

The impacts of nanoparticles such as copper, titanium and aluminum on mild stenosis were studied by Ahmed and Nadeem [13]. Rahbari et al. [14], investigated the magnetic nonNewtonian blood flow (consisting of nanoparticles) in the permeable arteries. Their result showed that the decrease of Brownian motion parameter would increase the thermophoresis rate as well as the blood velocity. The impact of nanoparticles on the stenosed blood flow subjected to magnetic field has been studied by Shahzadi and Nadeem [15]. By varying the flow parameters, flow parameters such as velocity, streamlines, pressure rise and pressure gradient have been examined. Hayat et al. [16] reported the impact of external magnetic field on the entropy generation of nanoparticles in peristaltic flow (in rotating frame of reference). The associated heat transfer process due to radiation has been studied as well. The dispersion of titanium magneto-nanoparticles in peristaltic blood flow through a uniform tube has been reported by Bhatti et al. [17]. The Sisko fluid model has been considered in representing the blood.

As highlighted by Siddiqui et al. [12], modelling of nanoparticles in blood flow is gaining popularity nowadays. In the current work, we have studied the blood flow through an arterial stenosis (with nanoparticles) in the presences of periodic body acceleration and oscillating pressure gradient acting in the axial direction. The effects of nanoparticles on the blood velocity have been analyzed numerically using Mathcad software. The proposed mathematical model could serve as a promising diagnostic tool.

2. Formulation of the problem

A stenosis is generated in the arterial wall of radius $\overline{R}(\overline{z})$ as shown in Figures 1. The size of stenosis is expressed in axial direction 'z' and height in which its growth model (see Eq. (1)) follows that reported by Siddiqui and Shah [12]. We assumed that the blood flowed in the axial direction and nanoparticles were uniformly distributed throughout the blood.

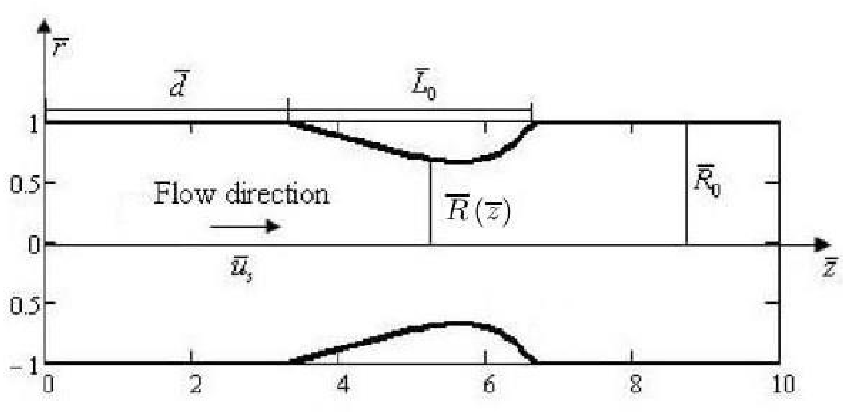

Figure 1. Geometry of stenosis.

$$\frac{\overline{R}(\overline{z})}{\overline{R}_0} = \begin{cases} 1 - \overline{\psi}\left[\overline{L}_0^{m-1}\left(\overline{z}-\overline{d}\right) - \left(\overline{z}-\overline{d}\right)^m\right] & , \quad \overline{d} \le \overline{z} \le \overline{d}+\overline{L}_0 \\ 1 & , \quad \text{otherwise} \end{cases} \tag{1}$$

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where $\overline{R}(\overline{z})$ is the radius of the artery in the stenosed region, R_0 is the radius of the normal artery, L_0 is the length of the stenosis, d is the region of the stenosis and $\overline{\psi} = \frac{\overline{\xi}_s}{R_0 L_0^m} \frac{m^{m/(m-1)}}{(m-1)}$ where $\overline{\xi}_s$ is the maximum height of the stenosis at $\overline{z} = \left(\overline{d} + \overline{L}_0\right)/m^{m/(m-1)}$ such that $\xi_s/R_0 < 1$.

For an unsteady blood flow in artery shown in Fig. 1, it is assumed that; (1) there is a force due to the relative motion between fluid and nanoparticles; (2) the motion of nano particles is governed by Newton's second law; and (5) the driving force added to the fluid flow is a pulsating pressure gradient, i.e.,

$$\rho_{nf} \frac{\partial \overline{u}}{\partial \overline{t}} = -\frac{\partial \overline{p}}{\partial \overline{z}} - \frac{1}{\overline{r}} \frac{\partial}{\partial \overline{r}} (\overline{r} \overline{\tau}) + \overline{F}(\overline{t}), \tag{2}$$

where ρ_{nf} is the effective density of nanofluid, $\overline{u}$ is the axial velocity along the z-direction, $\overline{t}$ is the time and τ is the shear stress.

According to Siddiqui et al. [12] and Akbarzadeh [17], the flow of blood is generated by the pressure gradient $\frac{\partial \overline{p}}{\partial \overline{z}}$ due to the heart pumping action and the body acceleration $\overline{F}(\overline{t})$, is assumed to be given by a harmonic formula as follows :

$$-\frac{\partial \overline{p}}{\partial \overline{z}}(\overline{z}, \overline{t}) = A_0 + A_1 \cos(\overline{\omega}_p \overline{t}), \tag{3}$$

where A_0 is the steady-state part of the pressure gradient, $\overline{A_1}$ is the amplitude of the pressure fluctuation giving rise to the systolic and diastolic pressures, $\omega_p = 2\pi f_p$ is the heart pressure frequency, where f_p is the pulse rate frequency. While

$$\overline{F}(\overline{t}) = A_g \cos(\overline{\omega}_g \overline{t} + \varphi), \tag{4}$$

where A_g is the amplitude of the acceleration, $\omega_g = 2\pi f_g$ is the frequency, where f_g is the pulse rate frequency and φ is the lead angle of the body acceleration with respect to the pressure gradient. Must be noted that, the effect of gravity in radial direction is neglected. For Bingham plastic fluid, the stress tensor τ is defined as:

$$\begin{aligned} \overline{\tau} = \overline{\tau}_y - \mu_{nf} \frac{\partial \overline{u}}{\partial \overline{r}} \quad & \text{for} \quad \overline{\tau} \geq \overline{\tau}_y, \\ \frac{\partial \overline{u}}{\partial \overline{r}} = 0 \quad & \text{for} \quad \overline{\tau} < \overline{\tau}_y, \end{aligned} \tag{6}$$

where μ_{nf} is the effective dynamic viscosity of the nanofluid and τ_y is the yield stress. The boundary conditions are:

$$\begin{aligned} \overline{u} = \overline{u}_s \quad & \text{at} \quad \overline{r} = \overline{R}(\overline{z}), \\ \overline{r} \text{ is finite} \quad & \text{at} \quad \overline{r} = 0, \end{aligned} \tag{7}$$

where $\overline{u_s}$ is the slip velocity at the stenotic walls.

The dimensionless form of the governing equations can be obtained by introducing the following dimensionless variables:

$$R(z) = \frac{\overline{R}(\overline{z})}{\overline{R}_0}, \quad u = \frac{\overline{u}}{A_0\overline{R}_0^2/4\mu_f}, \quad \theta = \frac{\overline{\tau}_y}{A_0\overline{R}_0/2}, \quad r = \frac{\overline{r}}{\overline{R}_0}, \quad t = \overline{t}\overline{\omega}_p \quad ,$$

$$A = \frac{A_1}{A_0}, \quad B = \frac{A_g}{A_0}, \quad \omega = \frac{\overline{\omega}_g}{\overline{\omega}_p}, \quad \xi_s = \frac{\overline{\xi}_s}{\overline{R}_0}, \quad \tau = \frac{\overline{\tau}}{A_0\overline{R}_0/2}, \tag{8}$$

Using the above dimensionless variables, Eqs. (2)-(7) can be written in non-dimensional form:

$$\alpha^2\frac{\rho_{nf}}{\rho_f}\frac{\partial u}{\partial t} = 4\left[(1+A\cos t)+B\cos(\omega t+\varphi)\right]-\frac{2}{r}\frac{\partial}{\partial r}(r\tau), \tag{9}$$

$$\tau = \theta-\frac{1}{2}\frac{\mu_{nf}}{\mu_f}\frac{\partial u}{\partial r}; \text{ for } \tau \geq \tau_y, \tag{10}$$

$$\frac{\partial u}{\partial r} = 0, \qquad \text{for } \tau < \theta, \tag{11}$$

where $\alpha^2 = \frac{\overline{\omega}_p\overline{R}_0^2\rho_f}{\mu_f}$, α is the Womersley frequency parameter and μ_f is the dynamic viscosity of blood.

The boundary conditions are:

$$u = u_s, \text{ at } r = R(z), \tag{12}$$

$$\tau \text{ is finite at } r = 0. \tag{13}$$

The effective nanofluid particles ρ_{nf}, is:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \tag{14}$$

where ϕ is the solid volume fraction and ρ_f is the density of blood, ρ_s is the density of dispersed copper nanoparticles.

In view of Eqs. (14), Eqs. (9)-(11) yields

$$\alpha^2\frac{\partial u}{\partial t} = 4D_2\left[(1+A\cos t)+B\cos(\omega t+\varphi)\right]-D_2\frac{2}{r}\frac{\partial}{\partial r}(r\tau), \tag{15}$$

$$\tau = \theta-\frac{1}{2(1-\phi)^{2.5}}\frac{\partial u}{\partial r} \quad \text{for } \tau \geq \tau_y, \tag{16}$$

$$\frac{\partial u}{\partial r} = 0 \quad \text{for } \tau < \theta, \tag{17}$$

where $D_1 = (1-\phi)+\phi\frac{\rho_s}{\rho_f}$ and $D_2 = \frac{1}{D_1}$.

The thermophysical properties of bio-fluid (blood) and nanoparticles [13] at 25^0C are presented in Table 1.

Physical properties	Based fluid (Blood)	Fe_3O_4	TiO_2	Cu
ρ(kg/m^3)	1063	5200	4250	8933

Table 1.

3. Solution technique

Let us consider the following perturbed solutions for velocity u and shear stress τ in the following form:

$$u(z,r,t) = u_0(r,t) + \alpha^2 u_1(r,t) + ... \tag{18}$$

$$\tau(z,r,t) = \tau_0(r,t) + \alpha^2 \tau_1(r,t) + ... \tag{19}$$

Substituting u and τ into Eqs. (15)-(17) and collecting the coefficients of equal power of α^2 yields:

$$, \quad \frac{\partial}{\partial r}(r\tau_0) = 2r\left[(1 + A\cos t) + B\cos(\omega t + \varphi)\right] \tag{20}$$

$$\frac{\partial}{\partial r}(r\tau_1) = -\frac{r}{2D_2}\frac{\partial u_0}{\partial t}. \tag{21}$$

Integrating Eq. (20) and using the boundary condition (13), we obtain

$$\tau_0 = g(t)r, \tag{22}$$

where $g(t) = [(1 + A\cos t) + B\cos(\omega t + \varphi)]$.

Substituting Eqs. (18) and (19) into Eq. (16) and collecting the coefficients of equal power of α^2, we obtain

$$\frac{\partial u_0}{\partial r} = 2(1-\phi)^{2.5}(\theta - \tau_0) \tag{23}$$

$$\frac{\partial u_1}{\partial r} = -2(1-\phi)^{2.5}\tau_1. \tag{24}$$

Integrating Eq. (23), and using Eq. (22) and boundary conditions (12), we obtain

$$u_0 = u_s + (1-\phi)^{2.5}\left\{(1-R)\left[2\theta - g(t)(r+R)\right]\right\}. \tag{25}$$

Similarly, the solutions of u_1 and τ_1 can be obtained by using Eqs. (24) and (21)

$$u_1 = -\frac{1}{16D_2}\left\{(1-\phi)^{6.25} g'(t)\left[r^4 - 4r^2R^2 + 3R^4\right]\right\} \tag{26}$$

$$\tau_1 = \frac{1}{8D_2}(1-\phi)^{2.5} g'(t)\, r\left(r^2 - 2R^2\right). \tag{27}$$

Hence

$$\begin{aligned} u(z,r,t) &= u_s + (1-\phi)^{2.5}\left\{(1-R)\left[2\theta - g(t)(r+R)\right]\right\} \\ &\quad -\alpha^2\frac{1}{16D_2}\left\{(1-\phi)^{6.25} g'(t)\left[r^4 - 4r^2R^2 + 3R^4\right]\right\} + ... \end{aligned} \tag{28}$$

$$\tau(z,r,t) = g(t)r + \alpha^2\frac{1}{8D_2}(1-\phi)^{2.5} g'(t)\, r\left(r^2 - 2R^2\right) + ... \tag{29}$$

4. Results and discussions

The obtained results have been compared with those generated from the physiological parameters reported in [9]. As shown in Figure 2, Equation (30) reported by Siddiqui et al. [9] has been used to describe the motion of blood with nanoparticles. In this work, we have developed a mathematical model that represents the non-Newtonian blood flow through a stenosed artery.

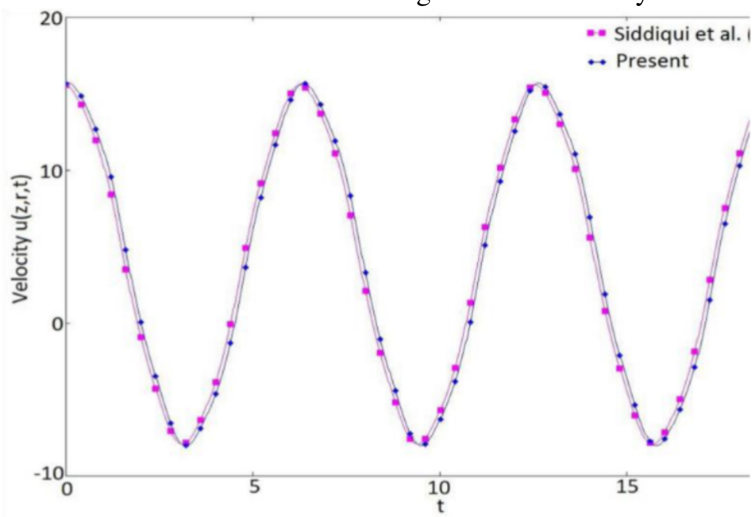

Figure 2. Comparison of blood velocity profiles in the stenosed artery.

The influences of nanoparticles (Fe_30_4, $Ti0_2$, Cu) as drug carriers in the blood flow stream have been incorporated in the model as well. By assuming that the Womersley number is small, the approximate solutions of axial velocity can be determined. Figure 3 shows the distribution of axial velocity for different values of slip velocity while fixing the other parameters. As the slip velocity increase, the blood velocity increases as well. The axial velocity distribution $u(z,r,t)$ is reported in Figure 4 for different values of yield stress. For $\theta=0$, the velocity profiles of all fluids have been plotted. As theta increases, the axial velocity decreases due to the existence of nanoparticles.

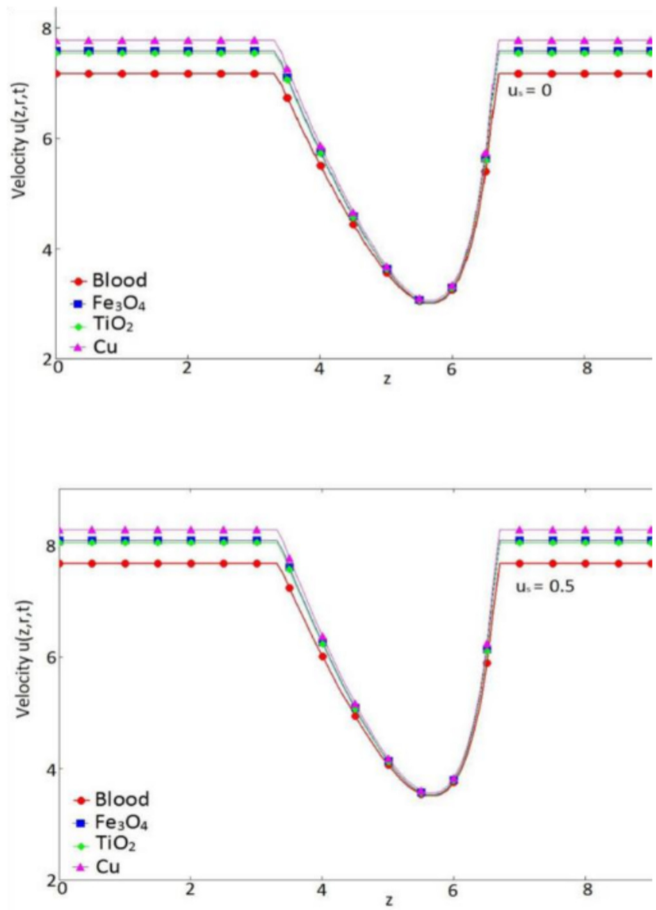

Figure 3. Variation of velocity distribution $u(z,r,t)$ with z for different values of slip velocity.

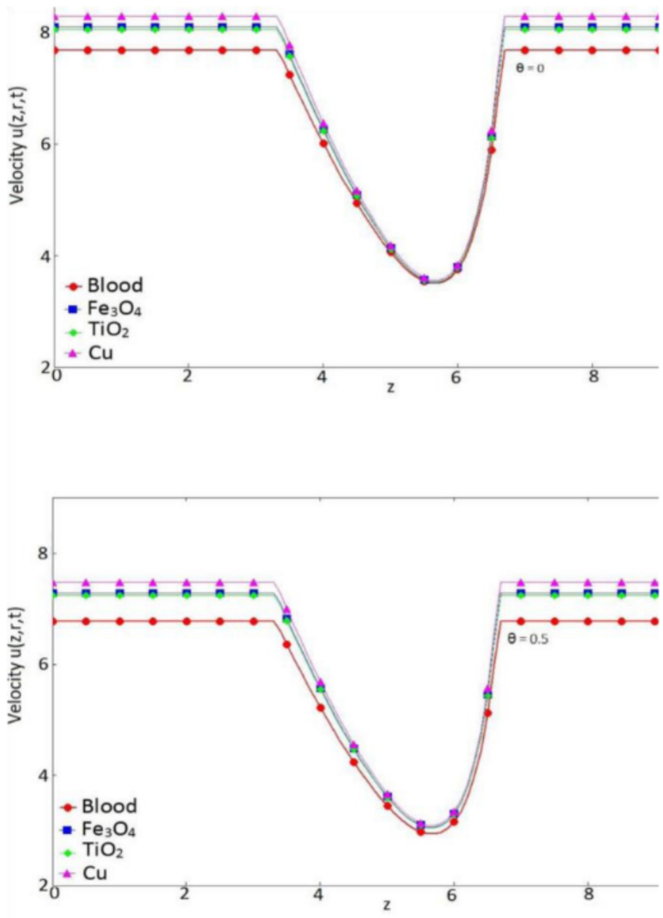

Figure 4. Variation of axial velocity $u(z,r,t)$ for different values of yield stress.

5. Conclusions

In this research, we have developed a mathematical model to describe the flow and velocity distribution of blood in a stenosed artery. The effects of nanoparticles ($Fe_3 0_4$, $Ti0_2$, Cu), oscillating pressure gradient, periodic body acceleration, yield stress and slip velocity have been incorporated in the model as well. The flow governing equations have been solved numerically by using MATHCAD software. The findings of the current work are:

- The blood flow velocity in stenosed artery can be controlled via nanoparticles.
- Blood velocity increases with respect to the slip velocity.
- The stenosed blood flow velocity decreases as yield stress increases.

6. Acknowledgement

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References

[1] Xiao H, Avolio A and Huang D 2016 A novel method of artery stenosis diagnosis using transfer function and support vector machine based on transmission line model: A numerical simulation and validation study *Computer Methods and Programs in Biomedicine* **129** 71-81.

[2] Chakravarty S and Mandal P K 2000 Two-dimensional blood flow through tapered arteries under stenotic conditions *International Journal of Non-Liner Mechanics* **35** 779-793.

[3] Liu G T, Wang X J, Ai B Q and Liu L G 2004 Numerical study of pulsatile flow through a tapered artery with stenosis *China Journal Physics* **42** 401-409.

[4] Sankar D S and Lee U Two-fluid Herschel-Bulkley model for blood flow in catheterized arteries *Journal of Mechanical Science Technology* **22** 1008-1018.

[5] Akbar N S, Nadeem S and Mekheimer Kh S Rheological properties of Reiner-Rivilin fluid model for blood flow through tapered artery with stenosis *Journal of Egyptian Mathematical Society* **24** 138-142.

[6] Akbar N S 2016 Non-Newtonian model study for blood flow through a tapered study with a stenosis *Alexandria Engineering Journal* **55** 321-329.

[7] Ijaz S and Nadeem S 2016 Slip examination on the wall of stenosed artery with emerging application of nanoparticles *International Journal of Thermal Science* **109** 401-412.

[8] Mandal P K, Chakravarty S, Mandal A and Amin N Effect of body acceleration on unsteady pulsatile flow of non-Newtonian fluid through a stenosed artery *Applied Mathematics and Computation* **189** 766-779.

[9] Mishra S, Siddiqui S U, Medhavi A A 2011 Blood flow through a composite stenosis in an artery with permeable wall *Application Applied Mathematics: An International Journal 6* 1798-1813.

[10] S.S.Yadav, K.Kumar, Bingham plastic characteristic of blood flow through a generalized atherosclerotic artery with multiple stenosis. Advance in Applied Scicience Research 3 (2012) 3551-3557.

[11] Hazarika G C and Sharma B 2014 Blood flow with effects of slip in arterial stenosis due to presence of applied magnetic field *International Journal of Modern Sciences and Engineering Technology 1* 45-54.

[12] Siddiqui S U, Shah S R and Geeta 2015 A biochemical approach to study the effect of body acceleration and slip velocity through stenotic artery *Applied Mathematics and Computation* **261** 145-155.

[13] Ahmed A and Nadeem S 2016 The study of (Cu,TiO2,Al2O3) nanoparticles as antimicrobials of blood flow through diseased arteries *Journal of Molecular Liquids* **216** 615-623.

[14] Rahbari A, Fakour M, Hamzehnezhad A, Vakilabadi M A, Ganji D D 2017 Heat transfer and fluid flow of blood with nanoparticles through porous vessels in a magnetic field: A quasi-one dimensional analytical approach *Mathematical Biosciences* **283** 38-47.

[15] Shahzadi I and Nadeem S 2017 Inclined magnetic field analysis for metallic nanoparticles submerged in blood with convective boundary condition *Journal of Molecular Liquids* **230** 61-73.

[16] Hayat T, Rafiq M, Ahmad B, Asghar S 2017 Entropy generation analysis for peristaltic flow of nanoparticles in a rotating frame. *Int. J. of Heat and Mass Transfer***108** 1775-1786.

[17] Bhatti M M, Zeeshan A and Ellahi R 2016 Endoscope analysis on peristaltic blood flow of Sisko fluid with Titanium magneto-nanoparticles *Comp. in Biology and Medicine* **78** 29-41.

[18] Akbarzadeh P (2016) Pulsatile magneto-hydrodynamic blood flows through porous blood vessels using a third grade non-Newtonian fluids model *Computer Methods and Programs in Biomedicine* **126** 3-19.