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Case Study: Students' Symbolic Manipulation in Calculus Among UTHM Students

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Abstract. Words are symbols representing certain aspects of mathematics. The main purpose of this study is to gain insight into students' symbolic manipulation in calculus among UTHM students. This study make use the various methods in collecting data which are documentation, pilot study, written test and follow up individual interviews. Hence, the results analyzed and interpreted based on action-process-object-schema framework which is based on Piaget's ideas of reflective abstraction, the concept of relational and instrumental understanding and the zone of proximal development idea. The students' reply in the interview session is analyzed and then the overall performance is discussed briefly to relate with the students flexibility in symbolic manipulation in linking to the graphical idea, the students interpretation towards different symbolic structure in calculus and the problem that related to overgeneralization in their calculus problems solving.

1. Introduction

The situation when the students' symbolic manipulation reaches the excellent level cannot be avoided. Symbols are the components of mathematical language that can make a person communicate, manipulate and reflect towards abstract mathematical concepts. [4], [6], [8], [10], [11] and [13] claimed that many difficulties in the process of understanding and learning mathematics, students tend to stuck when they were about to manipulate and understand the algebraic symbols.

Based on references [6] and [10], students' symbols interpretation is based on their previous interactions, successes and failures in their previous lecture time. [1] claimed that researchers identify an understanding of symbolic structure as an instrumental to competent performance in problem solving. In accordance, [7] summarize that items with the same symbolic structure always trigger different interpretations among students.

Calculus plays the important role in mathematical study. The difficulty in understanding calculus concept is the major stumbling block in achieving an excellent level in mathematical study. With the help of multiple representations and accompany with the use of technology can decrease the difficulties.

[3] pointed out the most useful benefits when multiple representations is applied in learning calculus in which students are able to identify and represent the same thing in different representations, flexible in moving from one representations to another, allow students to see the relationships, students

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can develop a better conceptual understanding, broaden and deepen their understanding and strengthen their abilities in solving problems.

Meanwhile, [5] claimed that the use of technology affect students learning and understanding positively where they can access multiple representations concept images freely in order to enrich the conceptual understanding. Thus, students need skills in symbolic manipulation and technology in order to make use the calculus knowledge in effective manner. There are other quite considerable studies were carried out on mathematics education in Malaysia [14, 15].

2. Materials and Method

This qualitative study conducted and analyzed in four stages as listed below:

(i) Documentation
(ii) Pilot study
(iii) Written test
(iv) Individual interview

The data gathered was analyzed, interpreted and recorded by using Microsoft Office Excel 2007. The method in gathering the data is strata sampling for both pilot study and written test session where the corresponding respondents are from UTHM students in FSTPi that majoring in Science (Mathematics Technology) and Science (Industrial Statistics).

3. Result

Students tend to manipulate the calculus problems based on their understanding in the process of solving calculus problems. They also very flexible in solving calculus problems by using graphical idea and at the same time students are able to interpret well when they are given a various different symbolic structure in calculus.

There were several trends in answering the calculus problems that applied by the students. Referring to [2], an action-process-object-schema framework that is based on Piaget's idea of reflective abstraction is one of the analyzation materials in this study. An action is an externally driven transformation of a mathematical object(s) where the action can be in many methods such as memorization or using a step by step algorithm or formula to follow. After the actions taken are interiorized and reflected upon then the process will take place where the process is internally driven. At this rate, students are able to reflect and describe all the steps in a transformation without having to actually perform them. In solving the very first written test question students answer are variety. The question is to find the derivative of $y = \frac{x^4 - 3x^3 - 4x^2}{3x^2}$.

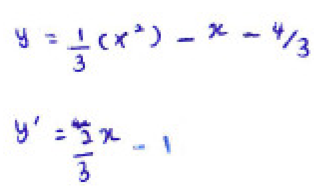

Figure 1. Simplification procedure.

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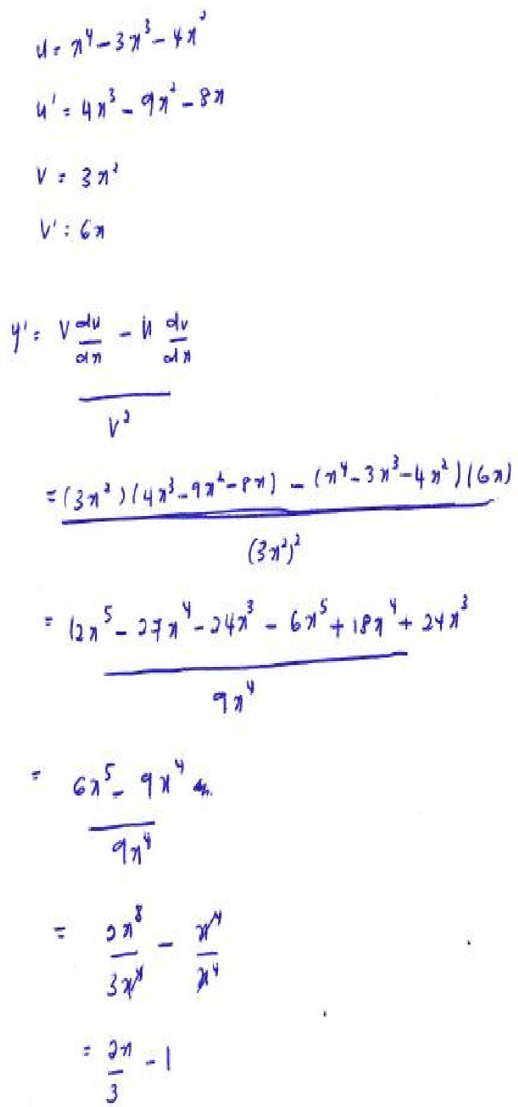

Figure 2. Quotient Rule Procedure.

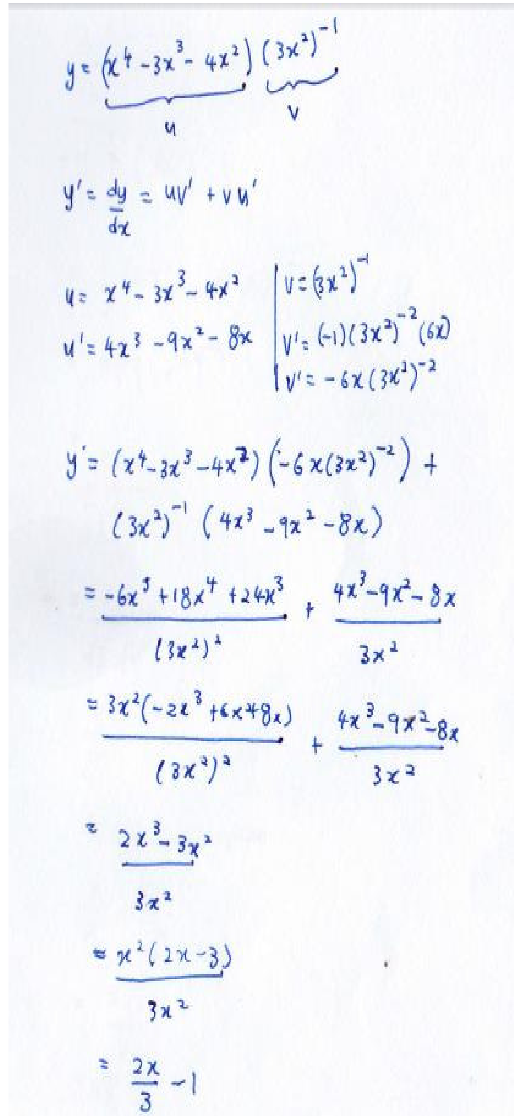

Figure 3. Product Rule Procedure.

Based on the Figure 1, 2 and 3, the algorithm applied using three different procedures. One can do a specific manipulation by identifying and knowing a specific procedure. Having one or more alternatives solution will allow a greater flexibility and efficiency to choose the best option for a given problem.

Meanwhile, the second problem is to evaluate $\int_0 \sqrt{4 - x^2}\, dx$ which shows that there are two different procedures in students work out.

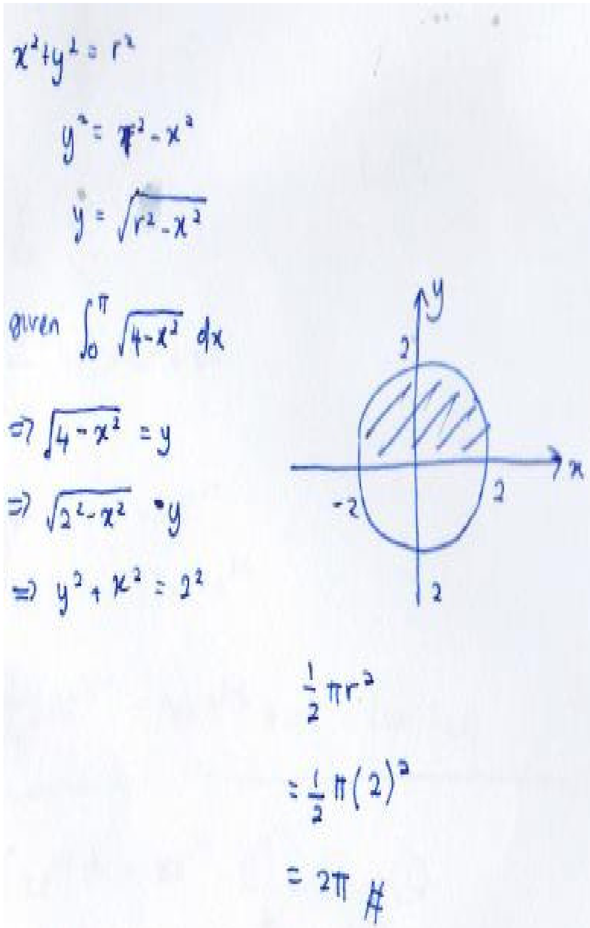

Figure 4. Solution by using circle equation formula procedure.

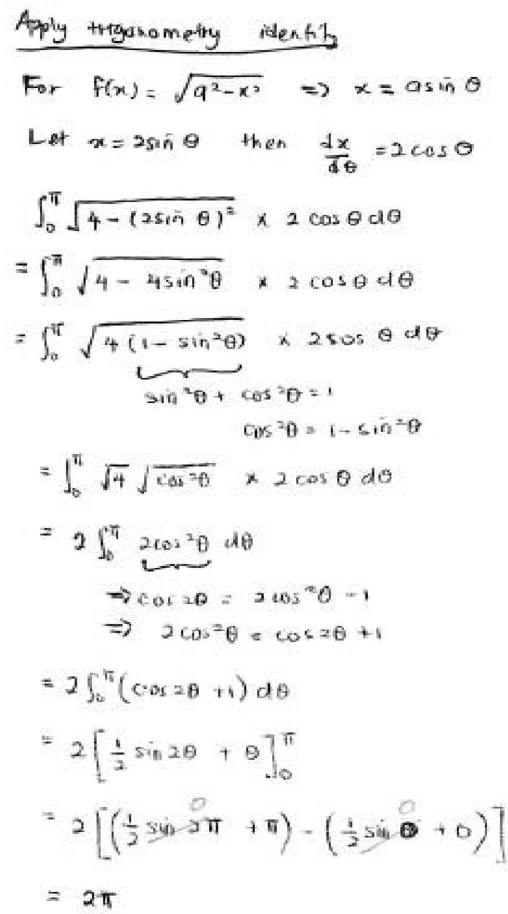

Figure 5. Solution by using trigonometry identity substitution procedure.

Based on Figure 4 and 5, students are flexible in solving problems with linked it with graphical idea and manipulate the trigonometry identity. The relational and instrumental understanding phenomenon was referred to [9]. The phenomena can be seen when the students are given the task in evaluating $\frac{d}{dx}\left(\sqrt{3x^2}\right)$.

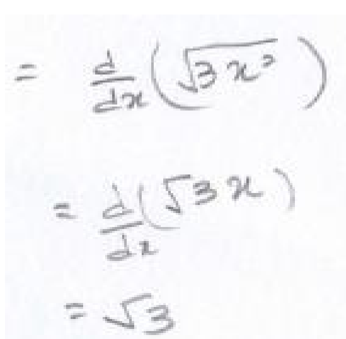

Figure 6. Algorithm with relational understanding.

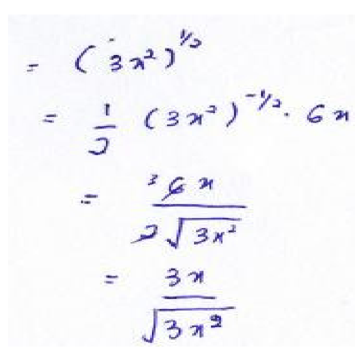

Figure 7. Algorithm with instrumental understanding.

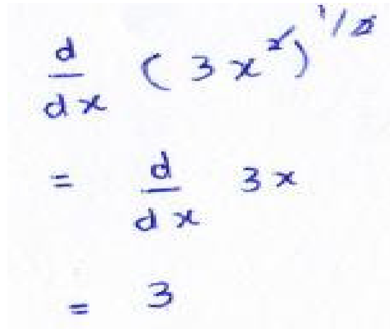

Figure 8. An erroneous answer.

The reference [9] claimed that relational understanding is "knowing both what to do and why" (p. 2). While the instrumental understanding knows what to do but never know the reasons behind the action taken. Figure 7 shows the correct answer without simplification but it is not accepted in this study since it has no symbol sense in the algorithm. The problem that related to overgeneralization in solving calculus problem is figured out in Figure 8 where students are over confident in answering the question.

Explaining zone of proximal development with referring to [12] idea, the very last written question shows how students replied and reacted in verifying $\frac{d}{dx}\left(xe^{3x}\right) = 3xe^{3x} + e^{3x}$ and hence they are told to show that $\int xe^{3x}\, dx = \frac{xe^{3x}}{3} - \frac{e^{3x}}{9} + C$. Figure 9 show that the students achieved the zone of proximal development where the learners can solve the problem.

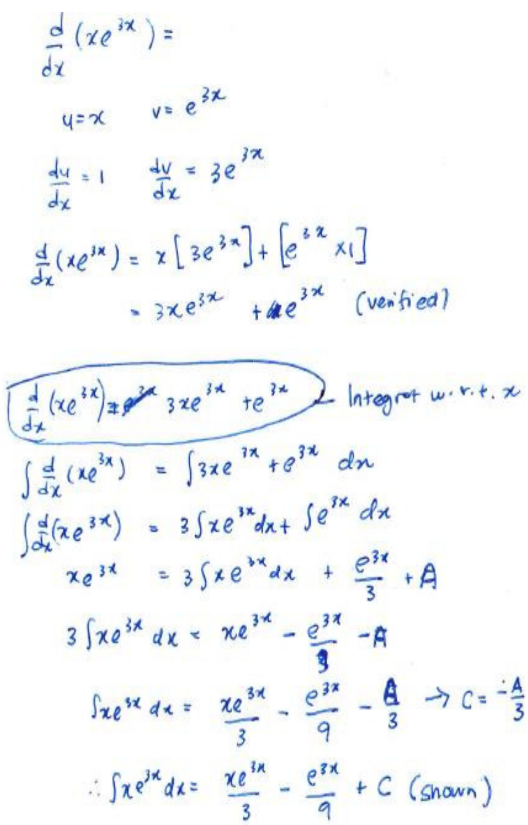

Figure 9. The students' work out when the guidance exists.

From the overall results, different students have different abilities in answering calculus problems. The step by steps taken is depending on their conceptual understanding. The problem regarding with overgeneralization also detected in students answer. Besides, the acronym 'LoPET' used by students in solving integration problem. The most important thing is students demand in using technology in solving calculus problem.

4. Conclusion

In conclusion, the student's reply in the problem solving is always inconsistence. This is because of the lack of basic root learning in calculus subject and the problem that related to overgeneralization. More able students can manipulate and interpret the symbols structure effectively. Some students have the ability in applying mathematical rules in effective way and some of them also have a good memorization skill in order to solve the calculus problems.

It is recommended that the students need to master in basic root learning and do a lot of calculus exercises so that they can answer calculus questions effectively. Mastering in symbolic manipulation also can be a good method in making the complex question become easier to solve. The dependence on technology also should be considered so that the students can practice the correct ways in simplifying equation manually. The involvement of more participants with the other university students in the future research can give a wide spectrum of results in symbolic manipulation. The most importance is students are allowed to use technology as requested to observe whether the results will be better or vice versa.

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