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Hydromagnetic Flow and Heat Transfer Adjacent to an Unsteady Stretching Vertical Sheet with Prescribed Surface Heat Flux

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Abstract. The unsteady hydromagnetic flow adjacent to a stretching vertical sheet is studied. The unsteadiness in the flow and temperature fields is caused by the time dependence of the stretching velocity and the surface heat flux. The governing partial differential equations are reduced to nonlinear ordinary differential equations, before being solved numerically. Comparison with previously published results as well as the exact solution for the steady-state case of the present problem is made, and the results are found to be in good agreement. Effects of the unsteadiness parameter, magnetic parameter, and Prandtl number on the flow and heat transfer are fully examined.

Keywords: Magnetic field; Stretching sheet; Surface heat flux; Unsteady flow

INTRODUCTION

The study of flow over a stretching sheet has generated much interest in recent years in view of its numerous industrial applications in industrial manufacturing processes and practical applications. Some of the applications are aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film, condensation process of metallic plate in a cooling bath and glass, and also in polymer industries. Micheal Faraday was the first person studied the problem of magnetohydrodynamics (MHD). However, study of the two-dimensional steady flow of an incompressible viscous fluid caused by a linearly stretching plate was pioneered by [1]. This problem has later been extensively studied in Newtonian and non-Newtonian fluids, steady and unsteady flows, hydrodynamic and hydromagnetic fluids in many other situations. For example, steady boundary layer flow over a vertical stretching surface in incompressible viscous fluid by considering the buoyancy effect has been studied by [2-7]. [8] examined the heat and mass transfer characteristics of the boundary layer flows induced by continuous surfaces with rapidly decreasing velocities and this problem was then extended by [9] to a vertical surface, where the effect of buoyancy forces was taken into consideration.

However, all these papers are for the problems with steady flow. The transient or unsteady flow becomes more interesting in engineering and technological problems because the stretching may start impulsively from rest. [10] was the first who investigated the boundary layer flow of a finite liquid film over an unsteady

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stretching surface by reducing the time. MHD boundary layer flow problem over a heated stretching sheet with variable viscosity has been discussed by [11], while [12] investigated the radiation and mass transfer effects of the same problem. [13] presented the heat transfer over an unsteady stretching permeable surface with prescribed wall temperature. They discussed the effect of unsteadiness in the flow and temperature fields caused by the time-dependent of the stretching velocity and surface temperature. Hydromagnetic flow and heat transfer adjacent to a stretching sheet with prescribed surface heat flux had been studied by [14], while [15] investigated the unsteady laminar MHD boundary layer flow and heat transfer of incompressible, viscous and electrically conducting fluid over a stretching sheet in the presence of transverse magnetic field with heat source/sink.

In view of the above mentioned literatures, the objective of this paper is to study the mixed convection hydromagnetic flow and heat transfer adjacent to an unsteady stretching vertical sheet with prescribed surface heat flux. To the best of our knowledge, this problem has not been studied before.

PROBLEM FORMULATION

Consider the unsteady, two-dimensional mixed convection flow adjacent to a stretching vertical sheet immersed in an incompressible electrically conducting fluid in the presence of a transverse magnetic field $B(x)$ as shown in Figure 1.

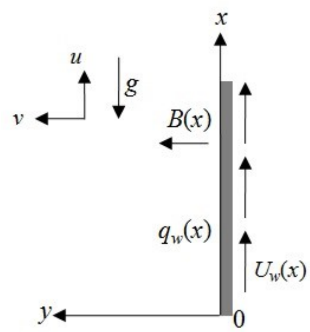

FIGURE. 1. Physical diagram and coordinate system

The stretching velocity and the surface heat flux are assumed to be of the forms $U_w(x,t) = \dfrac{ax}{1-\gamma t}$ and $q_w = \dfrac{bx}{(1-\gamma t)^{\frac{5}{2}}}$, respectively, where a, b and γ are constants with $a > 0$ and $b,\ \gamma \geq 0,$ and both a and γ have dimension time^{-1}. The magnetic Reynolds number is assumed to be small, and thus the induced magnetic field is negligible. Under the foregoing assumptions and applying the usual boundary layer approximations, the governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u + g\beta(T - T_\infty), \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are the velocity components along the x and y axes, respectively. Meanwhile $t,\ \nu,\ \rho,\ \alpha, \beta,\ \sigma,\ T,\ T_\infty$ and g are the time, kinematic viscosity, fluid density, thermal diffusivity, thermal expansion coefficient, electrical conductivity, fluid temperature, ambient temperature and acceleration due to gravity, respectively. The boundary conditions are:

$$u = U_w,\ v = 0,\ \frac{\partial T}{\partial y} = -\frac{q_w}{k} \text{ at } y = 0,$$
$$u \to 0,\ T \to T_\infty \text{ as } y \to \infty, \tag{4}$$

where U_w, q_w and k are the velocity of the stretching sheet, the surface heat flux and thermal conductivity, respectively. It should be noted that at $t = 0$ (initial motion), Eqs. (1)-(3) describe the steady-state flow past a stretching surface with a velocity $U_{ws}(x) = ax$ and subject to a heat flux $q_{ws}(x) = bx$ [16]. This particular form of $U_w(x,t)$ and $q_w(x,t)$ has been chosen to be able to devise a new similarity transformation, which transforms the governing partial differential (1)-(3) into a set of ordinary differential equations, thereby facilitating the exploration of the effects of the controlling parameters [17].

In order to obtain similarity solutions of Eqs. (1)-(4), we assume that the unsteady magnetic field B is of the form $B = B_0/\sqrt{1-\gamma t}$, where B_0 is a constant [18]. The continuity equation (1) is satisfied by introducing a stream function ψ such that $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. The momentum and energy equations can be transformed into the corresponding nonlinear ordinary differential equations by the following transformation [19-20].

$$\eta = \left(\frac{U_w}{\nu x}\right)^{\frac{1}{2}} y, \qquad f(\eta) = \frac{\psi}{\left(\nu x U_w\right)^{\frac{1}{2}}}, \qquad \theta(\eta) = \frac{k(T - T_\infty)}{q_w}\left(\frac{U_w}{\nu x}\right)^{\frac{1}{2}}. \tag{5}$$

The transformed nonlinear ordinary differential equations are:

$$f''' + ff'' - f'^2 - A\left(f' + \frac{1}{2}\eta f''\right) - M f' + \lambda\theta = 0, \tag{6}$$
$$\frac{1}{\Pr}\theta'' + f\theta' - f'\theta - A\left(2\theta + \frac{1}{2}\eta\theta'\right) = 0, \tag{7}$$

subject to boundary conditions (4) which become

$$\begin{aligned} &f(0) = 0,\ f'(0) = 1,\ \theta'(0) = -1, \\ &f'(\eta) \to 0,\ \theta(\eta) \to 0 \quad \text{as } \eta \to \infty, \end{aligned} \tag{8}$$

where primes denote differentiation with respect to η, $A = \gamma / a$ is a parameter that measures the unsteadiness, $M = \sigma B_0^2 / (\rho a)$ is the magnetic parameter and $\Pr = \nu / \alpha$ is the Prandtl number. In Eq. (6), λ is a constant and is called the mixed convection parameter, which is defined as $\lambda = Gr_x / \mathrm{Re}_x^{5/2}$, where $Gr_x = g\beta q_w x^4 / (k\nu^2)$ is the local Grashof number and $\mathrm{Re}_x = U_w x / \nu$ is the local Reynolds number.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x which are defined as

$$C_f = \frac{2\tau_w}{\rho U_w^2}, \qquad Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, \tag{9}$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \qquad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{10}$$

with μ being the dynamic viscosity. Using similarity variables (5), we obtain

$$\frac{1}{2} C_f \mathrm{Re}_x^{1/2} = f''(0), \qquad Nu_x / \mathrm{Re}_x^{1/2} = 1/\theta(0). \tag{11}$$

We note that when $A = 0$ (steady-state flow) and $\lambda = 0$ (forced convection flow), Eq. (6) has an exact solution [21].

$$f(\eta) = \frac{1}{\sqrt{1+M}} \left(1 - e^{-\sqrt{1+M}\eta}\right), \tag{12}$$

and the solution for the energy equations (7) is given by

$$\theta(\eta) = \frac{1}{\gamma} e^{-\frac{\mathrm{Pr}}{\sqrt{1+M}}\eta} \frac{F(\gamma-1, \gamma+1, -\gamma e^{-\sqrt{1+M}\eta})}{F(\gamma-1, \gamma, -\gamma)}, \tag{13}$$

where $\gamma = \mathrm{Pr}/(1+M)$, and $F(a,b,z)$ denotes the confluent hypergeometric function [22] with

$$F(a,b,z) = 1 + \sum_{k=1}^{\infty} \frac{a_k}{b_k} \frac{z^k}{k!},$$

$$a_k = a(a+1)(a+2)...(a+k-1),$$
$$b_k = b(b+1)(b+2)...(b+k-1).$$

In addition, from Eqs. (12) and (13), the skin friction coefficient $f''(0)$ and the surface temperature $\theta(0)$ can be shown to be given by

$$f''(0) = -\sqrt{1+M},$$
$$\theta(0) = \frac{\sqrt{1+M}}{\mathrm{Pr}} \frac{F(\gamma-1, \gamma+1, -\gamma)}{F(\gamma-1, \gamma, -\gamma)}. \tag{14}$$

RESULTS AND DISCUSSION

Hydromagnetic flow and heat transfer adjacent to an unsteady stretching vertical sheet with prescribed surface heat flux is studied by considering two-dimensional mixed convection. The transformed Eqs. (6) and (7) subjected to boundary conditions (8) were solved numerically for some values of the governing parameters such as the unsteadiness parameter (A), the magnetic parameter (M), Prandtl number (Pr) and mixed convection parameter (λ) on the skin friction coefficient and the heat transfer rate of the surface by using bvp4c solver in MATLAB. Comparison values of the surface temperature $\theta(0)$ with those of previous studies as well as the series solution for several values of parameters have been done to assess the accuracy of the present method. The comparison of the obtained results with those reported by [23], [24] and [14] is presented in Table 1. It is observed that the results show a favorable agreement.

TABLE 1. Comparison with previously published data for the values of surface temperature $\theta(0)$.

A	λ	Pr	M	[23]	[24]	[14]	Present results	Eq. (14)
0	0	0.72	0	1.2253		1.2367	1.23666	1.236657472
		1		1		1	1	1
		10		0.2688		0.2688	0.26877	0.2687685151
		6.7			0.333303	0.3333	0.33330	0.3333030612
			0.5		0.339715	0.3398	0.33971	0.3397152199
			1		0.345377	0.3454	0.34538	0.3453771711
			5		0.380930	0.3809	0.38093	0.3809302053
	1	1	0			0.9240	0.92398	
			2			1.0665	1.06654	
0.5							1.56627	
1			5				1.87377	

Values of the skin friction coefficient and the local Nusselt number on some values of magnetic parameter, M when the parameters of $\Pr$, λ and A are fixed to unity are depicted in Table 2. The velocity and temperature profiles for different values of magnetic parameter are shown in Figs. 2 and 3, respectively. It can be seen that an increase in transverse magnetic field tends to reduce the boundary layer thickness, hence causes the fluid motion to slow down, which implies an increase in the velocity gradient at the surface as shown in Fig. 2. We can say that the skin friction coefficient $f''(0)$ increases as the magnetic parameter M increases. Contrarily with Fig. 2, Fig. 3 shows that the effect of magnetic field is to enhance the fluid temperature in the boundary layer, which in turn increases the surface temperature $\theta(0)$. Thus, the heat transfer rate at the surface $1/\theta(0)$ decreases as M increases.

TABLE 2. Values of $f''(0)$ and $1/\theta(0)$ for different values of M when $\Pr = 1$, $\lambda = 1$, $A = 1$.

M	$\lvert f''(0) \rvert$	$1/\theta(0)$
0.5	0.15750	0.55166
1	0.24325	0.54926
1.5	0.32670	0.54699
2	0.40803	0.54482
5	0.85876	0.53368

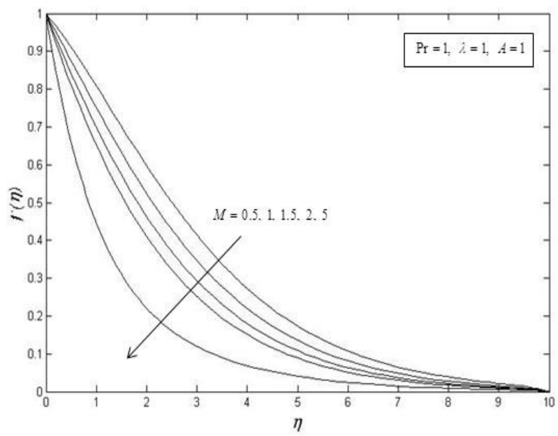

FIGURE. 2. Velocity profiles $f'(\eta)$ for some values of magnetic parameter M

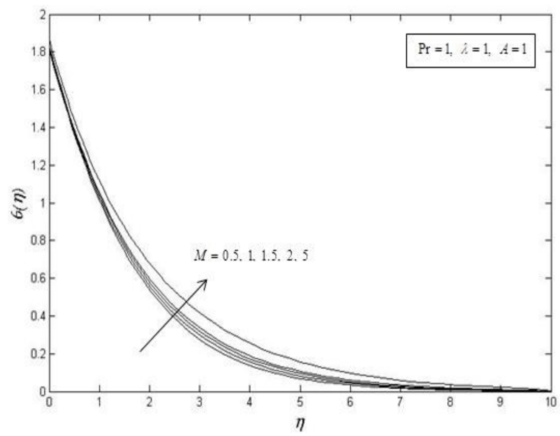

FIGURE. 3. Temperature profiles $\theta(\eta)$ for some values of magnetic parameter M

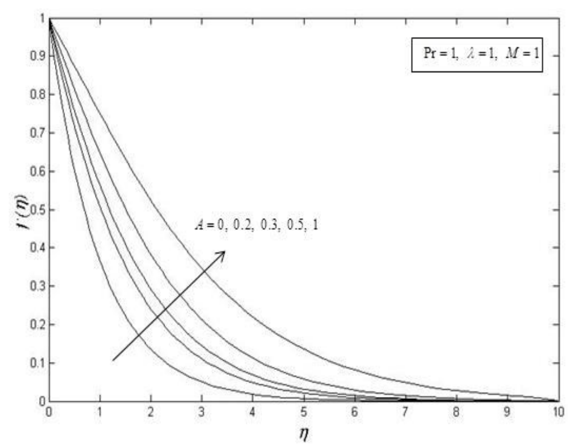

FIGURE. 4. Velocity profiles $f'(\eta)$ for some values of unsteadiness parameter A

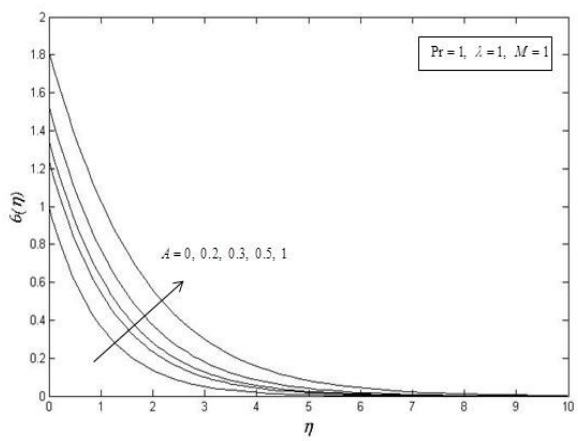

FIGURE. 5. Temperature profiles $\theta(\eta)$ for some values of unsteadiness parameter A

Figures 4 and 5, respectively, show the samples of velocity and temperature profiles for some values of unsteadiness parameter with fixed values of $\Pr$, λ and M. From Fig. 4, it can be clearly seen that an increase of the unsteadiness parameter tends to increase the boundary layer thickness, which causes the fluid motion to speed up which implies the decrease of velocity gradient at the surface. The skin friction coefficient $f''(0)$ decreases when the unsteadiness parameter increases, hence the velocity distribution increases. The same behaviour is observed for the temperature distribution as presented in Fig. 5, which implies the heat transfer rate at the surface to decreases as the unsteadiness parameter increases.

Velocity and temperature profiles for some values of Prandtl number with fixed values of A, λ and M are shown in Figs. 6 and 7, respectively. From Fig. 6, we can see that there is a slight decrease of the boundary layer thickness, thus slow down the fluid motion and consequently increase the velocity gradient at the surface. Therefore, the skin friction coefficient $f''(0)$ increases as the Prandtl number increases. From Fig. 7, we can conclude that the temperature distribution decreases due to an increase of Prandtl number. The boundary layer thickness decreases which implies an increase in temperature gradient. Hence, the local Nusselt number increases with increasing value of Prandtl number.

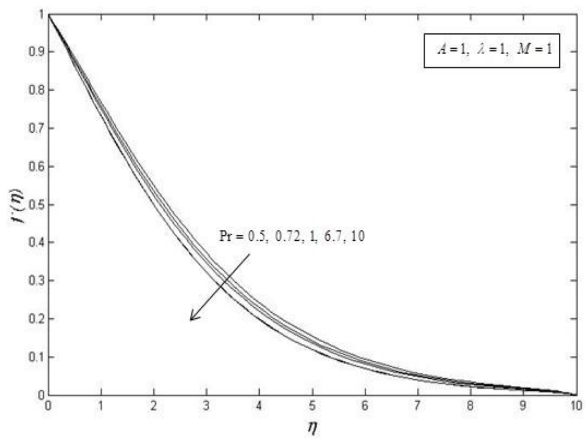

FIGURE. 6. Velocity profiles $f'(\eta)$ for some values of Prandtl number Pr

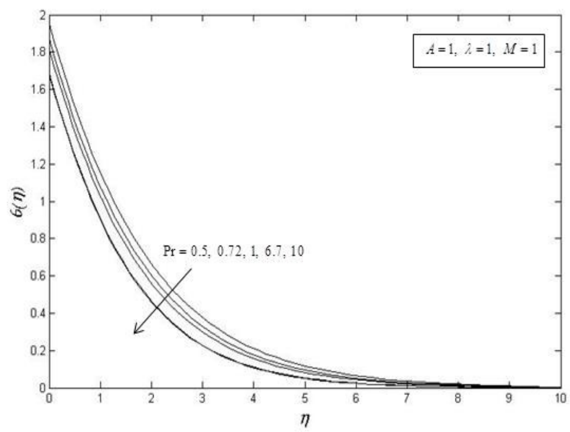

FIGURE. 7. Temperature profiles $\theta(\eta)$ for some values of Prandtl number Pr

CONCLUSION

The problem of hydromagnetic flow and heat transfer adjacent to an unsteady stretching vertical sheet with prescribed surface heat flux has been investigated theoretically. The governing partial differential equations are first transformed into ordinary differential equations using similarity transformation. Numerical solutions were obtained using bvp4c in MATLAB. The effects of several parameters such as parameter that measures the unsteadiness (A), the magnetic parameter (M), and Prandtl number (Pr) on the skin friction coefficient, the heat transfer rate of the surface, velocity and temperature profiles have been analysed and discussed. The comparison of the numerical results obtained for certain values of parameters showed a very good agreement with the existing results from open literatures. It is found that the skin friction coefficient increases while the local Nusselt number decreases in the presence of magnetic field. The effect of the unsteadiness parameter is to raise the fluid motion, and in consequence decreases the skin friction coefficient and the local Nusselt number. Increasing the value of Prandtl number is to increase both physical quantities of interest.

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