

IDENTIFICATION OF A PROBABILITY DISTRIBUTION FOR EXTREME RAINFALL SERIES IN EAST MALAYSIA

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**Laporan Projek ini dikemukakan
sebagai memenuhi sebahagian daripada syarat
penganugerahan Ijazah Sarjana Kejuruteraan Awam**

**Fakulti Kejuruteraan Awam & Alam Sekitar
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**For my beloved mother and my late father who never had a chance
to see his son's success**

and

For my dearest wife, Ayni, and two children, Mohd Izzuan and Nur Izni

May ALLAH protect and bless us all.



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ABSTRACT

The goal of this study was to evaluate the goodness-of-fit of the alternate probability distributions to sequences of the annual maximum stream flows in the East Malaysian states of Sabah and Sarawak. We will never know with certainty, the actual amount of rainfall that will occur in the future. So a statistical analysis of this nature can provide guidance on which probability distributions can give reasonable approximation. Basically, this study is a statistical analysis on extreme annual rainfall series in East Malaysia. It will discuss the comparative assessment of eight candidate distributions in providing accurate and reliable maximum rainfall estimates for East Malaysia. The models considered were the Exponential (EXP), Gamma (GAM), Generalized Extreme Value (GEV), Generalized Logistic (GLO), Generalized Pareto (GPA), Gumbel (GUM), Pearson Type III (PE3) and Wakeby (WAK). Annual maximum rainfall series for one-hour resolution from a network of ten Principal Gauging Stations located five each in Sabah and Sarawak were selected for this study. On top of that, data for the fifteen-minutes were also taken for analysis to act as a check to the result. The length of rainfall records varies from seventeen to twenty-one years. Model parameters were estimated using the L-moment method. The quantitative assessment of the descriptive ability of each model was based on using the Probability Plot Correlation Coefficient (PPCC) test combined with Relative Root Mean Squared Error (RRMSE), Root Mean Squared Error (RMSE) and Maximum Absolute Error (MAE). Ranking of PPCC in descending order and the other three criteria on ascending orders were taken and the top three distributions from the ranking for each station were chosen. The GEV distribution came out on top that occurs frequently on most of the stations is selected as the best fitting distribution to describe the extreme rainfall series for East Malaysia.

ABSTRAK

Tujuan utama kajian ini adalah untuk menilai ujian cocokan kuantitatif bagi setiap taburan kebarangkalian yang terjadi bagi taburan hujan maksimum di Sabah dan Sarawak. Kita tidak akan mengetahui dengan tepat berapa amaun hujan yang akan turun di masa akan datang jadi kajian statistik seperti ini perlu dijalankan untuk memberi sedikit sebanyak panduan tentang taburan kebarangkalian yang mana sesuai digunakan. Ini adalah merupakan kajian statistik untuk taburan hujan maksima di Malaysia Timur dan akan membincangkan mengenai penilaian ke atas lapan calon taburan frekuensi didalam memberikan anggaran yang tepat. Calon-calon untuk model taburan frekuensi tersebut adalah terdiri dari *Exponential (EXP)*, *Gamma (GAM)*, *Generalized Extreme Value (GEV)*, *Generalized Logistic (GLO)*, *Generalized Pareto (GPA)*, *Gumbel (GUM)*, *Pearson Type III (PE3)* and *Wakeby (WAK)*. Siri taburan maksima hujan tahunan untuk sela satu jam dari sepuluh tolok rakaman hujan automatik yang terletak lima di Sabah dan lima di Sarawak digunakan untuk kajian ini. Disamping itu dat untuk sela lima belas minit juga digunakan sebagai semakan. Rekod untuk taburan hujan adalah selama antara tujuh belas hingga dua puluh satu tahun. Anggaran parameter model adalah berasaskan Kaedah momen-L manakala ujian cocokan kuantitatif untuk menilai keupayaan diskriptif setiap model adalah berasaskan ujian Koefisien Korelasi Plot Kebarangkalian (KKPK), dan tiga kriteria kejituan yang lain iaitu ralat Relatif Punca Min Kuasa Dua (RRPMKD), ralat Punca Min Kuasa Dua (RPMKD) dan Sisihan Mutlak Maksimum (SMM). KKPK diatur dalam susunan menurun manakala ketiga kriteria yang lain diatur dalam aturan meninggi, dan tiga taburan frekuensi yang teratas bagi setiap stesyen akan diambilkira sebagai calon terbaik. Dari analisa yang dijalankan bagi kajian ini, didapati bahawa taburan GEV adalah lebih sesuai dipilih sebagai taburan frekuensi untuk siri hujan ekstrim bagi Malaysia Timur.

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ABBREVIATIONS AND SYMBOLS

A^2	Anderson-Darling test statistic
AMS	Annual Maximum Series
C_s	Coefficient of Skewness
cdf	Cumulative distribution function
D	Distribution / Kolmogorov-Smirnov test statistic
DWD	Deutscher Wetterdienst (German Weather Bureau)
E	Estimation method
E_i	Expected frequency for bin i (Chi-squared test)
EDF	Empirical distribution function
ENT	maximum Entropy method
F	Cumulative probability of non-exceedance
$F(x)$	Cumulative distribution function (<i>cdf</i>)
$f(x)$	Probability distribution function (<i>pdf</i>)
G	Skewness for theoretical sample
i	Rank of observation in ascending order
IEA	Institution of Engineers, Australia.
k	Kurtosis for theoretical sample
L-CV	L-coefficient of variation
LS	Least Square method
m	Rank of observation in descending order
MAE	Maximum Absolute Error
MLX	Mixed Moments method

MLE	Maximum Likelihood Estimator
MMS	Department of Malaysia Meteorological Services
MOM	Method of Moments
n	Sample size
NERC	National Environmental Research Council
h	(Reserved frequency for bin i (Chi-squared test)
P	Probability of exceedance events
PDN	Probability Distributions
PDS	Partial duration series
PFT	Peak over threshold series
PPCC	Probability plot correlation coefficient
PWM	Probability weighted moments
Q	Flood magnitude
Q_T	Estimated flood magnitude in T-years
Q_i	(Reserved magnitude of Q)
q	ith plotting position
r	Correlation coefficient, probability plot correlation coefficient value
RMSE	Root mean squared error
RRMSE	Relative root mean squared error
s	Sample standard deviation
s^2	Variance for theoretical moment
T	Return period for AMS
T_1	Return period for PDS
w	ith quantile
\bar{w}	Average value of fitted quantile

MLE	Maximum Likelihood Estimator
MMS	Department of Malaysia Meteorological Services
MOM	Method of Moments
N	Sample size
NERC	National Environmental Research Council
O_i	Observed frequency for bin i (Chi-squared test)
P	Probability of exceedance events
PDs	Probability Distributions
PDS	Partial duration series
POT	Peak over threshold series
PPCC	Probability plot correlation coefficient
PWM	Probability weighted moments
Q, q	Flood magnitude
Q_T	Estimated flood magnitude in T-years
\hat{Q}	Observed magnitude of Q
q_i	ith plotting position
r	Correlation coefficient, probability plot correlation coefficient value
RMSE	Root mean squared error
RRMSE	Relative root mean squared error
s	Sample standard deviation
s^2	Variance for theoretical moment
T	Return period for AMS
T_E	Return period for PDS
w_i	ith quantile
\bar{w}	Average value of fitted quantile

x_i	i th ordered observation
x_p	p th quantile or 100 p percentile
\bar{x}	Average value of the observation / mean for theoretical moment
$x(F)$	Quantile function
Y_i	Lower limit for class i (Chi-squared test)
Y_u	Upper limit for class i (Chi-squared test)
α	Scale parameter
β, ξ	Location parameter
β_r	r th probability weighted moment
χ^2	Chi-squared test
δ	Parameter for Wakeby distribution
γ	Parameter for Wakeby distribution / Skewness for sample moment
κ	Shape parameter / kurtosis for sample moment
λ	Mean number of peaks per year
λ_r	r th L-moment
μ	Mean for sample moment
σ^2	Variance for sample moment
τ_2	L-coefficient of variation
τ_3	L-skewness
τ_4	L-kurtosis
Γ	Gamma function
Σ	Summation

PROBABILITY DISTRIBUTIONS

EV1	Extreme Value Type 1 distribution
EV2	Extreme Value Type 2 distribution
EV3	Extreme Value Type 3 distribution
GAM	Gamma distribution
GEV	Generalized Extreme Value distribution
GUM	Gumbel distribution
GLO	Generalized Logistic distribution
GPA	Generalized Pareto distribution
LP3	Log Pearson Type 3 distribution
PE3	Pearson Type 3 distribution
WAK	Wakeby distribution



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CHAPTER I



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CHAPTER I

INTRODUCTION

1.1 Background

After five decades, the field of statistical hydrology continues to evolve and remains a very active area of investigation. Researchers continue to examine various distributions, methods of estimation of parameters, and problems related to regionalization. However, much of this material appears in journals and reports and usually in a form which is not easily accessible to practitioners and students-and hence producing a bigger gap between research and practice.

The eighties proved to be important years with many significant contributions. Due to its large economical and environmental impact, flood frequency analysis remains a subject of great important and interest, and research on improved methods for obtaining reliable flood estimates has continued into the nineties, although with different emphasis. In the seventies and eighties much effort was spent on developing efficient at-site flood frequency procedures. New distributions and estimation methods were introduced in the hydrologic journals, some of them developed specifically for flood frequency analysis. It seems that this tendency has decelerated somewhat at the beginning of the nineties. Researchers are gradually realizing that the lack of sufficiently long data series imposes an upper limit on the degree of sophistication that can reasonably be justified in at-site flood frequency analysis. It has been emphasized by many that instead of developing new methodologies for flood frequency analysis, effort should be spent on comparing existing ones and on looking for other sources of information (Potter, 1987; Bobée

et al., 1993). Regionalization is probably the most viable avenue for improving flood estimates, and fortunately this seems to be the direction that the researches have taken in the nineties.

Before designing a variety of engineering works in water resources planning and other water related projects, engineers often require flood estimates at a particular proposed site or project location. Flood volume estimate is very important in predicting or estimating the return period of rare events such as extreme rainfalls or precipitation for a site or a group of sites.

The general purpose of frequency analysis is to relate the magnitude of extreme events to their frequency of occurrence through the use of probability distributions (Chow et al., 1988). The data observed over an extended period of time in a hydrologic system are analyzed in frequency analysis and are assumed to be independent and identically distributed. Further, it is assumed that the floods have not been affected by natural or manmade changes in the hydrological regime in the system.

In practice, the actual probability distribution for both at-site and a regional data is quite unknown. For this study of the East Malaysian states of Sabah and Sarawak, the data collected is quite short in relation to other countries and began in the year 1951 for monthly rainfall and only from 1979/1981 for the detailed hourly duration rainfalls. So due to the very limited short data available, it is quite possible that some output from the analysis might not be that accurate. Hence it is quite necessary that a more detailed study should be done for better estimates of design storms based on a more reliable and longer period of data.

With the advancement of computer software, the efficiently quantitative method for goodness-of-fit such as the probability plot correlation coefficient test will provide a more reliable output of the existing techniques. And also with the introduction of the L-moments (Hosking, 1990) numerous researchers have recommended them to assess the goodness-of-fit of various probability distributions to data samples of stream flow and precipitation (Chowdhury et al., 1991, Hosking and Wallis, 1993, Stedinger et al., 1993).

1.2 Statement of Problem

The focus of this study is to determine the most appropriate probability distribution of extreme rainfall that are best suited to East Malaysia, which comprises the states of Sabah and Sarawak. For that purpose, the data from the annual maximum series were chosen and more preferred to be used instead of the partial duration series by virtue of its simplicity and easier to extract and analyze.

The process of probability fitting involved several steps. The problems of finding the optimal combination of the best estimation techniques with the most suitable distribution have actively been discussed over the past couple of decades. Hosking and Wallis (1993) organized regional frequency analysis into four stages: (1) Screening of the data; (2) identification of homogeneous region; (3) choice of a regional probability distribution; and (4) estimation of the regional probability distribution. Normally, if the final goal is to estimate a regional flood frequency, the process should proceed from one stage to the next without skipping intermediate stages. The identification of homogeneous region is normally important for identifying a regional shape or skew parameter to be used with the regional estimation procedure. So a comprehensive and global study on this subject matter will be an enormous task to undertake, and let alone the stochastic nature of the empirical rainfall data is a big hurdle to overcome along with the spatial and temporal differences of the region. Because of that, most studies are done locally, using at-site data and also restricted to a few commonly used distributions.

It should be noted also that despite the availability of rainfall data from either the Malaysia Meteorological Services Department or the Department of Drainage and Irrigations Malaysian, a formal study on the probability distribution of annual extreme rainfall series in East Malaysia has never been conducted. Thus there is an urgent need for this research to done immediately so that the existing rainfall atlas can be revised to the latest development.

Regionalization study is also important in determining the spatial and temporal patterns of rainfall in the region which are directly affected by the monsoon seasons. The mechanisms, which bring heavy rains to different parts of

the region, should be incorporated into the homogeneity study, which, at the present moment are very lacking. Unfortunately, this study is not included in the scope of this report.

1.3 Study Objectives

The objectives of this study to perform a detailed assessment of various probability distributions to determine the best distribution model for describing the extreme rainfall series for East Malaysia. This is important because there is no formal study on the subject has ever been done and hence the need for it in order to identify the pattern of distribution for these areas. Apart from that, it is to provide a bit on theoretical knowledge of spatial and temporal pattern of extreme and monthly rainfalls in East Malaysia.

1.4 Scope of Study

The study will be basically a statistical analysis to determine the best probability distribution that can be applied to East Malaysia based on annual extreme rainfall series and not from the partial duration series. All the data was collected from the Malaysia Meteorological Services Department located in Petaling Jaya, Selangor . The data for the annual maximum series collected was for five principal stations in Sabah and another five stations for Sarawak. These stations are located in Labuan, Kota Kinabalu, Kudat, Sandakan and Tawau for Sabah, and Kuching, Sri Aman, Sibul, Bintulu and Miri for Sarawak. The locations and altitudes of each station is as shown in Table 1.1 and Figure 1.1 below.

Table 1.1 : Position and Elevations of Each Principal Stations

<u>Station</u>	<u>Latitude</u>	<u>Longitude</u>	<u>Ht above M.S.L (m)</u>
Kuching	1° 29' N	110° 20'E	21.7
Sri Aman	1° 13' N	111° 27'E	9.6
Sibu	2° 15' N	111° 58'E	30.9
Bintulu	3° 12' N	113° 02'E	3.1
Miri	4° 20' N	113° 59'E	17.0
Labuan	5° 18' N	115° 15'E	29.3
Kota Kinabalu	5° 56' N	116° 03'E	2.3
Kudat	6° 55' N	116° 50'E	3.5
Tawau	4° 16' N	117° 53'E	19.8
Sandakan	5° 54' N	118° 04'E	10.3

The study will also be focusing on using the L-moments method for parameter estimations which is highly recommended by researchers. For the goodness-of-fit test, the probability plot correlation coefficient (PPCC) will be the main test considered as well as the other three tests for comparison such as the relative root mean square error (RRMSE), root mean square error (RMSE) and the maximum absolute error (MAE).

For the probability distribution, a set of eight probability distributions commonly used in distribution fitting studies are chosen and they are the two-parameter Exponential (EXP), Gamma (GAM) and Gumbel (GUM) distributions, the three-parameter Generalized Extreme Value (GEV), Generalized Logistic (GLO), Generalized Pareto (GPA) and Pearson Type III (PE3) distributions, and the five-parameter Wakeby (WAK) distribution.

Malaysian Meteorological Service Station Network

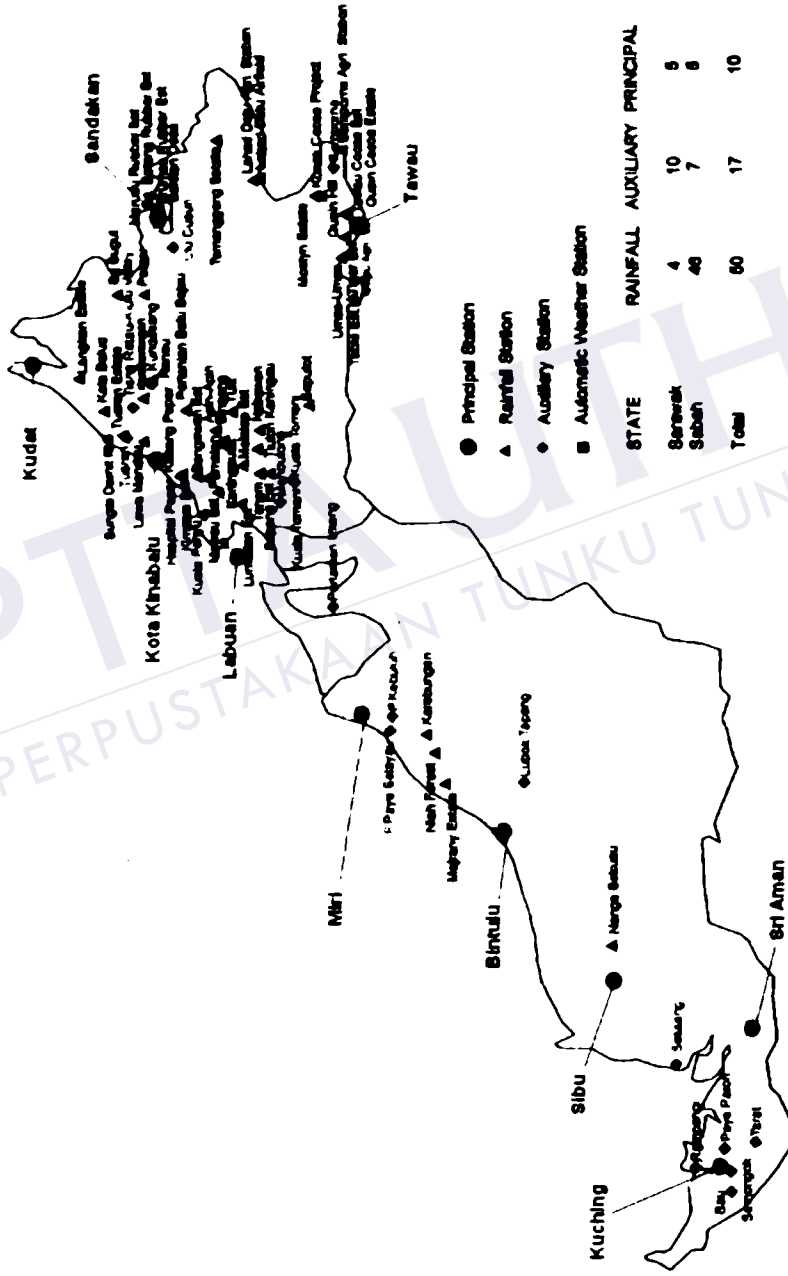


Figure 1.1 : Locations of Stations

1.5 Importance and Contribution of the Study

The study is meant to identify the best fitting probability distribution that best describe the extreme rainfall in East Malaysia and hence can be of great importance for future practitioners to design for water related structures in both the states. It is made more important to various authorities in those two states or federal government because so far there is no formal study on probability distribution of extreme rainfall series has ever been conducted to cover the whole of East Malaysia.

At the moment, Gumbel has been accepted to be used as a probability distribution for the whole of Malaysia. This is done so without any research done on this matter. So with this perception, it is hopefully beneficial to all that the outcome of this study will contribute to a better knowledge of the rainfall behavior in East Malaysia

1.6 Layout of Report

This report is presented in six chapters and three appendices. The objectives and scope of the study are covered in Chapter I, that also including the problem statement and significant of study. Chapter II will basically be dealing with literature review of all the procedures and methods that are in existence. The methodology, explaining all the methods and procedures that are to be used in this study are described in details in Chapter III. The case study and analysis of the probability distribution for this study is discussed in Chapter IV. The results and discussions based on this analysis are presented in Chapter V. The overall conclusion of the study and further suggestions for future research are discussed in Chapter VI. Appendices A will be the summary of all the results from the analysis while Appendices B will tabulate the data collected from MMS. And lastly Appendix C is the sample of Fortran routine for computing L-moment parameters for GEV distribution.

CHAPTER II



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