

ANALYSIS OF TEMPERATURE DISTRIBUTION IN DISC BRAKES

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*To my loving wife, Siti Norfaezah Bt. Mohsangosehek, my daughter, Khayra Zafirah
Bt. Azriszul, my mother, Zamaliah Bt. Suratdi, my family and my supporting friends*

...

"THANK YOU for your support"



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ABSTRACT

Disc brake have been used for many years and still improving in terms of it temperature reach. Many methods have been introduced in order to simulate and predicting the temperature reach of the disc brake (i.e. Lumped analysis, one dimensional analytical method, two dimensional numerical, and three dimensional numerical method). These numerical simulations are range from finite difference to finite elements with their own assumptions. In this report three independent directions of heat flows is determining in order to find it importance in the solution accuracy of temperature reach in brake disc by using Normalizing and order of magnitude analysis which have been introduced by Ludwig Prandtl in his analysis of fluid flow. The results of order of magnitude will be validated with finite difference simulations.

ABSTRAK

Penggunaan brek cakera telah lama digunakan dan masih lagi berkembang dari segi pembaikan suhunya. Pelbagai kaedah telah diperkenalkan bagi menganggarkan suhu yang dicapai oleh brek cakera dari segi simulasi dan eksperimen. Kaedah simulasi yang digunakan termasuk kaedah *Finite Difference* dan *Finite Elements* dengan pelbagai pertimbangan atau anggaran yang di kemukakan. Di dalam laporan ini, kaedah analisa yang digunakan adalah *Normalising* dan *Order of Magnitude* bagi mencari kepentingan setiap aliran haba di dalam brek cakera. Kaedah *Order of magnitude* ini telah diguna pakai oleh Ludwig Prandtl didalam kajian bendalir. Hasil dari analisa diatas akan dibuat perbandingan dengan hasil simulasi.

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LIST OF SYMBOLS & ABBREVIATIONS

θ	Dimensionless temperature difference
k	Thermal Conductivity
C_p	Specific heat Capacity of material
ρ	Density of material
E	Heat Energy (J)
q_z	Heat flow in z-direction (i.e. thickness of disc)
q_r	Heat flow in r- direction (i.e. radius of disc)
q_ψ	Heat flow in ψ -direction (i.e. angular of disc)
A	Area (m^2)
T	Temperature ($^{\circ}C$)
ω	Angular velocity (1/s)
t	Time (s)
α	Thermal diffusivity
L	Thickness of disc
R_d	Radius of disc
t^*/Fo	Fourier number
T_b	Surface temperature ($^{\circ}C$)
T_{∞}/T_o	Ambient temperature ($^{\circ}C$)
h	Convective heat transfer coefficient ($W/m^2\text{ }^{\circ}C$)
Bi	Biot number
q''	Heat flux (W/m^2)
Δx	Incremental thickness of disc (m)
V_{car}	Velocity of the car (m/s)
μ	Coefficient of friction
R_{tire}	Tire radius
\emptyset	Angle of brake pad

ω_{disc}	Angular velocity of disc (1/s)
ω_{tire}	Angular velocity of tire (1/s)
V_1	initial velocity, m/s
V_2	end velocity, m/s
ω_1	initial angular velocity (1/s)
ω_2	end angular velocity (1/s)
M	Mass of the car (kg)
I	Mass moment of inertia (kgm^2)
E_b	Energy of braking (J)
V_{enter}	velocity of a point enter a brake pad (m/s)
V_{exit}	velocity of a point exit from a brake pad (m/s)
t_{pad}	duration of a point inside the brake pad (s)
ρ_R	rotor/disc density (kg/m^3)
ρ_p	brake pad density (kg/m^3)
C_R	rotor/disc specific heat ($\text{Nm/kg}^\circ\text{C}$)
C_p	brake pad specific heat ($\text{Nm/kg}^\circ\text{C}$)
k_R	rotor/disc thermal conductivity ($\text{W/m}^\circ\text{C}$)
k_p	brake pad thermal conductivity ($\text{W/m}^\circ\text{C}$)
q''_R	Heat flux going into rotor (W/m^2)
q''_p	Heat flux going into brake pad (W/m^2)
R_{mean}	Mean radius of brake pad position from center of disc (m)
F_f	Frictional Force (N)
F_n	Normal Force (N)
A_p	Area of brake pad in contact with disc surface (m^2)
$\ddot{\theta}$	Angular acceleration (m/s^2)
m	mass of the disc (kg)
S	displacement of the car (m)
a	acceleration (m/s^2)

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CHAPTER 1

INTRODUCTION

The safe motor vehicles should have continuous adjusting of its speed to changing traffic conditions. The brakes along with tires and steering systems are the most important safety measure, critically in avoiding accident in motor vehicles. Its must perform safely under a variety of operating conditions including slippery, wet, and dry roads; when a vehicle is likely or fully load; when braking straight or in a curve; with new or worn brake lining; with wet or dry brakes; when applied by the novice or experienced driver; when braking on smooth or rough roads; or when pulling a trailer.

These general uses of the brakes can be formulated in terms of three basic functions where braking system must provide:

- Decelerate a vehicle including stopping.
- Maintain vehicle speed during downhill operation
- Hold a vehicle stationary on a grade.

Deceleration of a vehicle involves the change of the kinetic and potential energy (if any) into thermal energy. Important factors that a design engineer must consider when designing a brake includes braking stability, brake force distribution, tire/road friction utilization, braking while turning, pedal force modulation, stopping distance, in-stop fade, and brake wear (brake design and safety).

Maintaining vehicle speed on a downhill is involving the change of potential into thermal energy. Important considerations during this operation are brake temperature, lining fade, brake fluid, vaporization in hydraulic brakes, and brake adjustment of air brakes. There are two types of brake that are usually being used in automobile which are disc brake and drum brakes. Drum brake is a brake in which friction is caused by a set of shoes or pads that press the inner surface of the rotating drum. The disadvantages of the drum brakes over the disc brakes is that when the drum brakes is heated by the frictions between the pads and the rotating drum, the drum will expanded and due to the expansion of the material and the brakes must be further depressed to obtain effective braking action. This increase of pedal motion is known as brake fade and can lead to brake failure in extreme circumstances. For this reason drum brakes have been superseded in most modern automobiles and light trucks with at least front wheel used disc brakes.

Disc brake is a device used in automotive to decelerate a car from high speed to a lower speed or stops. It has been used since 1960's and consists of two blocks of frictional materials which are pressed against each side of rotating annular ring of ferrous material which is the disc either mechanically or hydraulically. The two blocks of frictional materials so called 'pads' is rigidly fixed to the body of cars to reduce the rotational speed of the disc. During this application the pads will give constant frictional forces to the disc and the disc will have friction contact between them and heat will be generated during this applications. Since the pads do not cover all area of the disc, therefore the heat produces will be conducted radials, axially or circumferentially. This heat will be conducted through the disc by conduction and some of it will be dissipated to the surrounding by convection, radiation and some will be conducted to the other components of the car. Simultaneously the disc surface will increase in temperature as a results of this friction contact and because of this, several attempt have been done to analyze this transient heat flows in order to improve the dissipation of the heat from the surface of the disc. As we know, this resulting high temperature on the disc can cause warping which is due to excessive heat build up which soften the metal and can allow it to be disfigured. It can also create abnormal deposits if the brake pad is used above its temperature range which creates 'sticky' spot on one area of the surface of the disc and will grab every

revolution of the disc which creates abnormal movement of the cars. Thermal cracking can also resulted from high temperature gradients in the disc. Thermal cracking often happens in drilled disc where a small cracks of the disc outside edges of the drilled holes near the edge of the disc due to the rotor uneven rate of expansion in severe duty environments. If this mass is removed, the stress will be increased in the rotor due to not enough heat sinks in the rotor. This problems can made the brake malfunction which results in catastrophically accidental cars. This heat variation also is used to analyze the thermal performance of the disc brake where for the effectiveness stop the surface temperature and the associate temperature gradient through rotor material are the critical evaluations. It is shown that an effectiveness stop at high deceleration temperature gradients of sufficient magnitude can develop in rotor material that causes surface rupture. In this report we are concern with analytical method for predicting the temperature variations of the disc and with the time.

1.1. Proposed Project.

This project is about to derive the equation of temperature variation in brake disc where a mathematical equation has to be made in order to predict the thermal behavior of the disc brake. This equation then will be validated with simulations analysis of temperature distribution in disc brake using coding programmed. This result then will be furthered for future analysis such as thermal cracking, warping and thermal stress in disc brake

1.2. Objectives

The objective of these project is to develop the equation of the temperature variations of the brake disc axially, radially and circumferentially with respect to time where it is a three dimensional equations. By using the Order Magnitude of Analysis, this equation will be analyzing whether the three axes are all important to

predict the temperature variations of disc with respect to time. Finalize equations is then being used using appropriate software to get the temperature variations of the disc in graphical method.

1.3. Scope of the Project

This problem is an unsteady thermal distribution where it involves three directions of heat flow (r , θ , z). From the derived equations it is then required to find which independent direction or axis of heat flows is important in determining temperature of brake disc. To find this direction flow, the order of magnitude analysis plays its part to analyzing all the three directions of heat flow. The complexity of this project is related to the boundary conditions where there are convection which is forced convection, natural convection and also radiation. The forced convection is due to the movement of the car and the natural convection is due to the temperature difference between the disc and the air surrounding. Centrifugal force is also much involved in this heat dissipation due to the disc is rotated. To solve this problem the numerical methods must be selected due to its complexity.



CHAPTER 2

LITERATURE REVIEW

The derivation of the equations of temperature variation for the surface temperature rise on brake disc has been long attempted in order to predict the surface temperature of the disc for a given physical properties and inputs. Knowledge of this actual temperature that occurs at the interface between a pad of friction material and the mating metal disc during braking is the vital area of interest to the brake designer.

During the preliminary rotor sizing stage of the design process, the lumped parameter rotor model is an extremely valuable tool. It provides information regarding bulk or spatially averaged temperature of the entire rotor. The results and the agreement obtained with measured temperature variations not only validate the order of magnitude analysis and the numerical procedure but also the brake input power distributions. Lumped parameter models combined with one dimensional model are also formulated and developed to predict the thermal loading of the brake disc rotor. The lumped model can predict transient bulk rotor temperature while one-dimensional model provides peak surface as well as bulk temperature.

One-dimensional transient model governing equations have been used for many years in order to obtain peak surface and bulk temperature. It can also be used to investigate the brake disc temperature distributions in one direction. Newcomb [1] used method of the Laplace transformation in solving the one-dimensional governing

equations in which the disc is assumed to be a semi-infinite slab in analyzing single stop and repetitive braking using these methods. He assumes that:

- the disc is homogenous, that it is heated / cooled only at the friction surfaces
- that the total energy of braking flows only into the disc
- circumferential temperature gradients can be neglected because of the rate of disc rotations is so high
- the effective rate of heat generations at any point is the average for the whole brake.

He also assumed that the deceleration rate during braking is constant, and conduction to and cooling from the hub, flange, brake fluid, pad and pad holder is neglected. The disc material properties and heat transfer coefficient is also assumed to be invariant with temperature increase. Agrawal [2] using the Fractional Derivatives approach to solve one-dimensional governing equations for thermal analysis of the disc brake which differs little from Newcomb's approach. Limpert [3] includes energy absorbed in pads into the consideration in his analysis. He details the analysis by considering the heat flows into the hub, caliper and flange as well as temperature distribution in continued braking and includes the radiative heat transfer in his analysis of disc brake.

Steady state two dimensional analyses of temperature and heat distribution in disc brake are introduced as an improvement to the one-dimensional analysis to predict the temperatures during multi stop driving schedule. Limpert [3] has been detailed analyze the temperature response of a solid-rotor disc. The effect of varying heat transfer coefficient is introduced and fade in the form of decrease in brake effectiveness as well as the effects of burnished and unburnished brakes are taking into consideration. He used finite difference method to compute rotor temperatures where the node points were plotted in radial and axial directions inside the brake. The coefficient of friction was assumed to be a function of brake temperature and the heat transfer coefficient of the solids rotor also was assumed to be the speed dependent in his report. Each part of the brakes such as the hub, flange and bolt of the brake were applied with difference heat transfer coefficient from all boundaries. Sheridan et. al

[4] used same approach but the nodes point were plotted in the axial and circumferential direction inside the brake in Cartesian coordinate. They includes pad's caliper into account for their thermal modeling of the brake. Because of the used of ventilated disc brake, the convective heat losses from the web is also taking into consideration in their report. In their report also the heat generations is assumed to be uniform over the swept area and therefore the inner and outer brake cheek surface temperature can be assumed to be uniform. The energy split between rotor and pad as well as the caliper temperature distribution were calculated using an electrical analog method.

Three-dimensional steady state and transient model is the complete model which was used to obtain much more detail local rotor temperature distributions for any stopping sequences. Since temperature distributions are generated throughout the entire model, they can be used to estimate thermal stresses and thermal distortions in the system. Noyes and Vickers [5] achieve this by using finite difference method of solution. The particular finite difference method used was the implicit one and the resulting set of simultaneous linear algebraic equations was solved by Gauss Seidel iterative method technique. They also introduce the heating rate equations for the front wheel of every disc brake used in the vehicles. The entire rotor of the brake was applied with the appropriate convective and radiative cooling from all boundaries where the vented disc brake rotor was used in this analysis. Sheridan et al. [4] used finite element model where accurate material and fluid property data, convective heat transfer coefficients and proper energy input was applied to the system. A full rotor caliper model also diminishes the energy partitions like the two-dimensional model as previous. Thermal analyses for the brake were conducted using NASTRAN. By using this method, they assumed braking energy was uniformly distributed over the swept area of the rotor. It is also assumed that the temperature gradient in circumferential directions is neglected. Yano and Murata [6] doing an experimental analysis of frictional heat generated between the pads and rotors of disc brakes to determine the paths and amounts of heat flow in tackling the problems of three dimensional analysis of disc brake. The area studied by them consists in six paths where heat conduction to the pad, heat convection to the air from the frictional areas of the inner and outer disc, from the ventilating parts and from the tube section of the

rotor and heat conduction to the rotor flange section. From the experimental results, they then introduce the analytical method to calculating the heat flows to the six paths as mention above. The influence of air temperature inside the wheel was also investigated by them to see whether the heat transfer coefficient is the same with the air outer wheel. Gao and Lin [7] using analytical and transient finite element technique to deal with determinations of temperature distributions. Analytical approach introduced in [7] by using three dimensional non-axisymmetry heat conduction equations in Cartesian coordinate system attached to the fixed part is developed. From the three dimensional equations, the equations is changed by using the standard Galerkin weighted residual formulation to solve with a transient Finite Element technique. The influence of a moving heat source with variable speed effect and operating characteristic of brakes such as variations of velocity, frictional moment, and the convectional coefficient of the disc and duration of braking were also taken into account by them.

Cooling of disc brake rotors is one of the important factors in reducing the rotor from cracking and warping. Limpert [8] stated that cooling capacity of a ventilated rotor is sharply reduced at lower speeds, and most cooling is provided by the increase surface area. In his report, the equations for determining the convective heat transfer coefficients of solid and ventilated disc brakes were also presented. He found that convective heat transfer coefficient only affects brake temperature during repeated and continued braking. It is also stated that from the cooling analysis using experimental result, the cooling analysis can be represented by the product of the dimensionless numbers raised to some power, which are Nusselt, Prandtl and Reynolds numbers. He also introduced the coefficient of heat transfer for solid disc brake for turbulent and laminar flows of air through the disc. The difference in equations is presented in [8]. Ventilated disc brakes have different heat transfer coefficients inside the vanes for turbulent and laminar flow of fluid. Radiative heat transfer coefficient is being approximated in this report when the temperature of the brake is higher. Jancirani et. al. [9] further analyzes the heat transfer coefficient of disc brake through virtual simulation. In their work a convective and radiative heat transfer model being formulated and developed to find heat transfer parameters such

as Reynolds number, Nusselt number, heat transfer coefficients for the rotor pad, and thermal backing plate.

A review of the previous work by the above authors shows that many approaches have been introduced in determining the temperature increase in brake disc. Lumped analysis approaches, one-dimensional approaches, two-dimensional approaches, three-dimensional approaches have all been carried out. Experiments too have been performed in determining the heat distribution in brake disc components. All the methods above have been done and also been validated with the experimental results which shows a reasonably good correlation between analytical simulations and experiments. Sheridan et. al. [4] indicate in their paper that one-dimensional analysis simulation results show quite the same as three dimensional results of simulation where they indicate that the one-dimensional analysis of disc brake actually is already good enough in determining the temperature increase in disc brake.

2.1 PRESENT WORK AND RATIONALE

In the present work a different and more economical approach has been described to determine the temperature increase of the disc and will be introduced in the report. In it, the importance of the three independent directions on solution accuracy in brake disc will be determined. In this analysis, an Eulerian approach is introduced and used in deriving the energy equation in the brake disc. The disc experiences heat conduction in z , r and θ directions and enthalpy flow is also taken into account. The disc experiences frictional heat and convection at the boundary. The conservation of energy is applied here and equations of energy are derived. The derived energy equations are non-dimensionalised and Order of Magnitude Analysis will then be used to determine which direction of temperature analysis in the disc brake will give the most accurate temperature increase in brake disc.

Table 2-1: Summary of Literature Review

<i>Title</i>	<i>Authors</i>	<i>Year Published</i>	<i>Summary</i>
1. Temperatures Reached In Disc Brakes	T. P. Newcomb	1960 Journal Mechanical Engineering Science	Problem of transient temperatures reached is solved using an infinite slab bounded by parallel planes by Laplace transform in solid rotor.
2. Application of Fractional Derivatives in Thermal Analysis of Disc Brakes.	OM Prakash Agrawal	2004 Nonlinear Dynamic Kluwer Academic Publisher	Thermal analysis of disc brake using fractional derivative where it contains fractional semi integral and derivative expressions which is used to compute friction surface temperature and heat flux as a function of time. This formulation also is accounted for convective heat loss from side surface.
3. The Thermal Performance of Automotive Disc Brakes.	Rudolf Limpert.	1975 SAE Technical Paper	New thermal performance measurements introduced are in form of weight and area effectiveness.

4. Approaches to the Thermal Modeling of Disc Brakes	David C. Sheridan et. al.	1988 SAE Technical Paper	<p>Presents four different modeling approaches for the thermal analyses of disc brakes.</p> <ol style="list-style-type: none"> 1. Lumped analysis 2. one dimensional analysis 3. two dimensional steady state analysis 4. three dimensional transient model <p>All the methods above being compared each other and to the experimental results to find which method is excellent.</p>
5. Prediction of Surface Temperature in Passenger Car Disc Brakes	Robert N. Noyes and Paul T. Vickers.	1969 General Motor Corp.	<p>Heat conduction in vented rotor is solved using three-dimensional finite difference method. (Multistop)</p>
6. Heat Flows on Disc Brakes	Masataka Yano and Masahiko Murata	1993 SAE Technical Paper	<p>Experimental and analytical approaches were used to study heat flow in several paths (i.e. heat transfer in various components of braking system).</p>

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