

A STUDY OF FLOW AND HEAT TRANSFER OF NANOFLUIDS: BETWEEN  
TWO PARALLEL PLATES, OVER A WEDGE AND PAST A STRETCHING  
SHEET

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This thesis is consecrated to my beloved parents; Late Alhaji Salihu Usman (Madaki)  
and Hajiya Halimatu M. Salihu



PTTA UTHM  
PERPUSTAKAAN TUNKU TUN AMINAH

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## ABSTRACT

This thesis investigates analytically and numerically the flow and heat transfer of nanofluids: between two infinite parallel plates, over a wedge, and past a stretching sheet. Two problems have been considered for the parallel plates. A mathematical model of squeezing unsteady nanofluid flow is studied firstly in the presence of thermal radiation, and secondly, in the presence of both thermal radiation and heat generation/absorption. The solutions are obtained by using homotopy perturbation method (HPM) and fourth-order Runge-Kutta with shooting technique (RK4). The flow of nanofluids over a wedge leads to the derivation of the Falkner-Skan equation and this problem have been solved using the optimal homotopy asymptotic method (OHAM). Finally, three issues have been considered for nanofluids past the stretching sheet. Firstly, we considered a problem of flow and heat transfer of nanofluids over a dynamic stretching sheet with non-linear velocity and variable thickness in the presence of Brownian motion and thermal radiation. Secondly, the effect of a chemical reaction is taken into account. These two problems have been investigated using the OHAM and RK4. Lastly, a mathematical model for the effect of chemical reaction in a natural convective boundary-layer flow of nanofluids has been evolved. The HPM with Pade approximation (HPM-Pade) along with RK4 is used to solve the nonlinear governing equations. It is found that the thermal radiation had recorded a significant influence, in which it has been observed that the growing value of the thermal radiation parameter results to the decrease in the temperature profile in the case of squeezing flow problem. Thereby both the thermal boundary layer thickness and temperature profile have substantially risen in the flow and heat transfer over a stretching sheet cases. From the subsequent cases, we also found that the temperature is high due to the increase in both the Brownian motion and the thermophoresis parameters, while the scenario reverses as the nanoparticle concentration only increases with the strengthen thermophoresis parameter and slow down with an increase in the Brownian motion parameter.

## ABSTRAK

Tesis ini mengkaji secara analitik dan berangka aliran dan pemindahan haba nanobendalir: yang berada di antara dua plat tak terhingga yang selari, terhadap baji dan melintasi helaian yang meregang. Dua masalah telah dipertimbangkan untuk aliran melalui plat selari. Pertamanya, model matematik terhadap aliran tak mantap nanobendalir yang dipicit dengan kehadiran sinaran terma dan keduanya, dengan kehadiran sinaran terma dan penjanaan/penyerapan haba. Penyelesaian diperoleh menggunakan kaedah usikan homotopi (HPM) dan skim Runge-Kutta peringkat empat dengan teknik tembakan (RK4). Aliran nanobendalir terhadap baji pula mendorong kepada pembentukan persamaan Falkner-Skan dan telah diselesaikan menggunakan kaedah homotopi asimptot optimum (OHAM). Akhirnya, tiga isu telah dipertimbangkan untuk nanobendalir yang melintasi helaian yang meregang. Pertamanya, masalah aliran dan pemindahan haba nanobendalir terhadap helaian yang meregang secara dinamik dengan halaju tak linear dan ketebalan yang berbeza diiringi dengan kehadiran pergerakan Brown dan sinaran terma. Keduanya, kesan tindak balas kimia telah diambil kira. Kedua-dua masalah ini telah dikaji menggunakan OHAM dan juga RK4. Akhirnya, model matematik bagi kesan tindak balas kimia pada perolakan tabii di dalam aliran lapisan sempadan nanobendalir telah dikaji. Kaedah HPM dengan anggaran Pade (HPM-Pade) bersama dengan RK4 digunakan untuk menyelesaikan persamaan menakluk tak-linear. Didapati bahawa sinaran terma memberi kesan yang penting, yang mana pada masalah aliran cubitan dengan meningkatkan parameter sinaran terma menghasilkan pengurangan pada profil suhu. Pada waktu yang sama pada masalah helaian merenggang, kedua-dua ketebalan lapisan sempadan terma dan profil suhu meningkat secara ketara. Pada masalah yang terakhir, didapati bahawa suhu meningkat akibat dari pada peningkatan parameter gerakan Brown dan termoforesis. Sementara itu, senario sebaliknya berlaku apabila kepekatan nanozarah hanya meningkat dengan kekuatan parameter termoforesis dan perlahan dengan peningkatan parameter gerakan Brown.



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- 7.15 Effects of  $N$  on  $\theta$  when  $Pr = 7$ ,  $A = M = Bi = 0.5$ ,  $Nb = Nt = 0.1$ ,  $Le = 5$ ,  $Ec = 0.2$ , and  $\theta_C = 1.5$ . 153



**LIST OF ABBREVIATIONS**

HPM	-	Homotopy perturbation method
OHAM	-	Optimal homotopy asymptotic method
MHD	-	Magnetohydrodynamic flow
HAM	-	Homotopy analysis method
BVP	-	Boundary value problem
IVP	-	Initial value problem
ADM	-	Adomian decomposition method
DTM	-	Differential transformation method
LSM	-	Least square method
CM	-	Collocation method
DRA	-	Duan-Rach approach
PHF	-	Prescribed heat flux



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## NOMENCLATURE

### Roman Letters

$a, b$	-	Positive constants along the sheet
$A$	-	Ratio of the rates of free stream velocity to the parameter velocity of the stretching sheet
$A_1, A_2, A_3$	-	dimensionless constants
$Bi$	-	Biot number
$B_0$	-	Magnetic field
$C$	-	nanoparticle volume fraction
$C_f$	-	Skin friction coefficient of the fluid
$C_i (i = 1, 2, \dots, m)$	-	Concentration of each species $i$
$C_j (j = 1, 2, \dots, m), K$	-	positive Convergence-control parameters
$C_w$	-	nanoparticle volume fraction at the sheet surface (wall)
$C_\infty$	-	Ambient nanoparticle volume fraction
$D_B$	-	Brownian diffusion coefficient, $kg/ms$
$D_T$	-	Thermophoretic diffusion coefficient, $kg/msK$
$Ec$	-	Local Eckert number
$f(\eta)$	-	Dimensionless stream function
$\vec{f}$	-	Body force per unit mass
$h$	-	Convective heat transfer coefficient
$h$	-	Step size
$H(p), H(p, \eta)$	-	Auxiliary functions
$\mathbf{J}$	-	Flux for the conserved quantity $\varphi$
$\mathbf{J}_i$	-	Flux
$k_1$	-	Rate of chemical reaction
$k_f$	-	Effective thermal conductivity of base fluid, $W/mK$

$k_{nf}$	-	nanofluid's thermal conductivity
$k_s$	-	thermal conductivity of the solid material
$K$	-	Dissipation function
$l$	-	initial position of the plates
$L$	-	Linear operator
$Le$	-	Lewis number
$M$	-	Magnetic parameter
$n$	-	velocity power index parameter
$N$	-	Radiation parameter
$\mathcal{N}$	-	Nonlinear operator
$Nb$	-	Brownian motion parameter
$Nt$	-	Thermophoresis parameter
$Nu$	-	Nusselt number
$Nur$	-	Reduced Nusselt number
$p$	-	Pressure $N/m^2$
$P$	-	generalized pressure
$P_E(\eta)$	-	Polynomial of degree at most $E$
$Pr$	-	Prandtl number
$q_m$	-	Wall mass flux, $kg/sm^2$
$q_r$	-	Radiative heat flux
$q_w$	-	Wall heat flux, $W/m^2$
$Q$	-	heat generation/absorption coefficient
$Q_G(\eta)$	-	Polynomial of degree at most $G$
$R$	-	Consumption term
$Re_x$	-	Local Reynolds number
$S$	-	squeezing integer
$Sc$	-	Schmidt number
$Sh$	-	Sherwood number
$Shr$	-	Reduced Sherwood number
$T$	-	Local fluid temperature, $K$
$t$	-	dimensionless time
$T_w$	-	Convective surface temperature, $K$

$T_\infty$	-	Ambient temperature, $K$
$u, v$	-	Velocity components along $x$ and $y$ directions, $m/s$
$U_w$	-	Stretching sheet velocity at wall, $m/s$
$U_\infty$	-	Free stream velocity, $m/s$
$\vec{V}$	-	Velocity vector
$x$	-	Coordinate along the sheet
$y$	-	Coordinate normal to the sheet
$z$	-	Independent variable
$'$	-	Differentiation with respect to $\eta$

### Greek Letters

$\alpha$	-	Thermal diffusivity of the base fluid, $m^2/s$
$\beta$	-	Wedge angle parameter
$\eta$	-	Similarity variable
$\gamma$	-	Chemical reaction parameter
$\Gamma$	-	boundary
$\lambda$	-	heat generation parameter
$\mu$	-	Absolute viscosity of the base fluid, $Ns/m^2$
$\nu_f$	-	Kinematic viscosity of the base fluid, $m^2/s$
$\nabla$	-	$\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j}$ (vector operator)
$\omega$	-	vorticity function
$\Omega$	-	domain of the boundary
$\frac{\partial}{\partial n}$	-	Normal derivative pointing outward from $\Omega$
$(\rho C)_f$	-	Effective heat capacity of the base fluid, $kg/m^3K$
$(\rho C)_p$	-	Effective heat capacity of the nanoparticle material, $kg/m^3K$
$\rho_f$	-	Density of the base fluid, $kg/m^3$

$\rho_p$	-	Nanoparticle mass density, $kg/m^3$
$\psi$	-	Stream function
$\sigma_e$	-	Electrical conductivity
$\tau$	-	$(\rho C)_p/(\rho C)_f$
$\bar{\tau}$	-	Viscous stress tensor
$\theta$	-	Dimensional temperature
$\theta_c$	-	Ratio of the temperature of the hot fluid to the ambient temperature
$\varphi$	-	Dimensionless nanoparticle volume fraction

### Superscripts

$'$	-	Differentiation with respect to $\eta$
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### Subscripts

$f$	-	fluid
$nf$	-	nanofluid
$p$	-	solid particle
$w$	-	surface (w)
$\infty$	-	condition at the free stream





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