Contact Pressure Prediction Comparison Using Implicit and Explicit Finite Element Methods and the Validation with Hertzian Theory

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Abstract-- This paper presents contact pressure prediction results based on numerical solution using implicit and explicit finite element method (FEM) verified by Hertzian contact theoretical solution. The contact pressure distributions are investigated for a simple cylinder-on-flat contact configuration model subjected to constant normal load. The FEM analysis is performed on two different aeroengine materials; Ti-6Al-4V and Super CMV (Cr-Mo-V) alloys. The predicted characteristics of the contact pressure distributions and the contact half-width lengths are verified with the Hertzian contact theoretical solutions. The results show that both implicit and explicit FEM are comparable with the Hertzian theoretical results. The difference of the calculated maximum contact pressure is less than 1%. The explicit method gives better agreement in terms of maximum contact pressure while the implicit approach has a better agreement in predicting the contact half-width length compared with the Hertzian contact theoretical solutions.

Index Term-- Ti-6Al-4V, Super CMV, Cylinder-on-flat, Contact pressure, Contact area half-width.

INTRODUCTION
Contact mechanics is concerned with the study of deformation of solids when two surfaces are interacting and touching at one or more points. Contact mechanics is considered a crucial subject as the contact occurs in most of the engineering applications, especially in aeroengine transmission components. Two surfaces that fit precisely or closely together without any deformation are considered as having conforming contact while non-conforming contact occurs when one or both surfaces deform with the presence of the contact area between them [1]. A number of studies considering contact mechanics in the elastic regime can be found in the literature [1-4]. Finite element analysis is often used in solving problems involving contact mechanics analysis. Such complex problem is time consuming and costly. Appropriate constitutive model should be employed to represent the contact behaviour of mating surface using numerical simulations [5-8]. In addition, proper contact formulation must be taken into account in order to avoid penetration between two contacting surfaces leading to non-convergent result [9, 10].

Simulating the contact mechanics using Hertzian contact condition offers an interesting, yet challenging subject of research. Hertzian contact theory can be applied with different contact conditions such as the contacting surfaces are continuous and non-conforming, contact with minimum strains, frictionless surfaces and each contacting body considered as an elastic half space [1]. The rapid development of contact mechanics employing Hertzian contact condition leads to various interesting researches on fretting wear, brittle coating and friction drives including experimental and numerical means in order to solve some real life contact problems. Hertz contact solution outcomes is widely used by researchers to validate their research. Application of Hertz contact equations on surface pressures in friction drives is presented by Herak et al. [11] where reduced equations are derived to calculate the surface pressures in friction drives when ball-ball, cylinder-cylinder and cone-cone are in contact. Besides that, survival of the glass in optical lens assembly with the existence of high stress level demonstrated in the study of Cai et al. [12] where the strength of the glass investigated based on Hertzian line contact using finite element method. In addition, a recent experimental framework addressing the elasto-plastic contact of a cylinder-on-flat configuration is investigated by Doca et al. [13] where maximum variation of contact pressure and contact length remaining below 4.6 % as verified with Hertzian contact theoretical solutions. Early studies on Hertzian fretting contacts under normal and tangential loading were conducted by Cattaneo [14] and Mindlin [15], leading to partial slip analytical solution. The development of fretting numerical models, mainly cylinder-on-flat contact configuration were validated with Hertzian contact theory [16-18]. The numerical model used in the study on the interaction between fretting wear and cyclic plasticity for Ti-6Al-4V by Mohd Tobi et al. [16] compared with contact pressure of Hertzian theoretical solution where the finite element model provides excellent agreement with the theoretical solution. A finite element based studies of McColl et al. [17] on fretting wear of aeroengine transmission contact components; the contact pressure accuracy of the unworn model is compared with Hertzian stress distribution. Besides that, Ding et al. [18] presented a fretting contact configuration model for gross sliding and partial slip condition where the numerical model is validated with Hertzian solution and significant effect is noticed on the tangential near-surface.

However, there have been no controlled studies which compare difference between implicit and explicit FEM
approach in quasi static condition to be verified with Hertzian analytical solution. In this paper, contact pressure distribution and contact half-width length for a cylinder-on-flat contact model is predicted using implicit and explicit FEM algorithm. Friction, heat dissipation and wear effects are neglected in this analysis as these conditions are not the major concern. Results are compared with the theoretical Hertzian contact solution. Accuracy of FEM prediction by the two approaches is compared and discussed.

Formulation of Hertzian contact theoretical solution

The Hertzian contact problem that consists of contact between spheres, cylinders, cones and flat surfaces forms the basic foundation of classical contact mechanics studies [13]. Generally, the cylinder-on-flat contact configuration is widely employed in the study of contact mechanics where the cylinder is considered as the indenter while the flat surface as the substrate. A brief summary of Hertz theoretical solutions for the contact pressure and contact area half-width is provided in this section. The Hertz contact pressure distribution, \( p(x) \) [1] is defined as:

\[
p(x) = p_0 \sqrt{1 - \frac{x^2}{a^2}}
\]

(1)

The maximum contact pressure, \( p_0 \) and contact half-width length, \( a \) are given by:

\[
p_0 = \left( \frac{P E^*}{\pi R} \right)^{1/2}
\]

(2)

\[
a = \left( \frac{4 P R}{\pi E^*} \right)^{1/2}
\]

(3)

The term, \( P \) is the normal load acting across the contact surfaces. The relative curvature, \( R \) and the equivalent elastic modulus of the contacting bodies, \( E^* \) is defined by:

\[
R = \left( \frac{1}{R_i^2} + \frac{1}{R_s^2} \right)^{-1}
\]

(4)

\[
E^* = \left( \frac{1-(v^s)^2}{E^s} + \frac{1-(v^i)^2}{E^i} \right)^{-1}
\]

(5)

where \( R_i, R_s \) are the contacting surfaces radii, \( E_i, E_s \) are elastic moduli and \( v_i, v_s \) are the Poisson’s ratios of the indenter and substrate, respectively.

1. **FINITE ELEMENT SIMULATION**

1.1 **Implicit finite element method**

Implicit finite element method is efficient in solving smooth non-linear based problems. Implicit analysis is suitable for solving nonlinear static, low-speed (low frequency response), and nonlinear dynamic, nonlinear heat transfer, coupled temperature-displacement (quasi-static), coupled thermal-electrical, mass diffusion problems and structural-acoustics. Implicit analysis does Newton-Raphson iteration after each increment in order to enforce the internal structure forced equilibrium with the load that is externally applied.

Figure 1 shows the schematic view of the cylinder-on-flat contact configuration where the model is based on the experimental configuration employed in earlier study [16-18]. The corresponding 2D Finite element model illustrated in Figure 2 is used throughout the simulation. The model consists of a half circle with the radius of 6 mm and a rectangle with the dimension of 12 x 6 mm. The implicit finite element simulation is performed using ABAQUS Standard software (version 6.13) in quasi-static condition. The FE model considered as an elastic half space.

![Fig. 1. Schematic view of the cylinder on flat contact configuration](image1)

The materials examined in this analysis are light weight alloy Ti-6Al-4V and high strength steel, Super CMV typically employed in aeroengine applications. The Young’s moduli are 115 GPa for Ti-6Al-4V and 200 GPa for Super CMV while, Poisson’s ratio of 0.3 is taken for both materials [16].

![Fig. 2. 2D Finite element geometry illustrated portioned areas for element mesh optimization](image2)

Linear quadrilateral plane strain elements are used in this simulation for contact problems involving friction [19].
The mesh size refined at the contact region about 5 μm and the transition of the mesh from coarse to fine is achieved via edge seedings. The purpose of refining the mesh node at the contact region as shown in Figure 3 is to get a better contact pressure and contact half-width length prediction with reasonable analysis computational time. Surface to surface contact is employed where two types of the contact pair surfaces introduced such as master and slave; the indenter (cylinder) acts as the master surface while the substrate (the flat surface) as the slave surface. To ensure the exact sticking condition where shear stress is lower than the critical shear stress based on Coulomb friction, the Lagrange multiplier contact algorithm is used in the contact surface interaction properties with the coefficient friction, μ of 0.3. Static general step is assigned with the time period of 1 s and maximum of 100 increments. The bottom surface of the substrate is constrained from any motion in the x and y directions and a concentrated normal load, P of 500 N/mm is applied as the point load to the indenter. The contact pressure distribution and contact area half-width as calculated by the implicit numerical model is compared and verified with the Hertzian contact theoretical solution for validation purpose.

1.2 Explicit finite element method

Explicit finite element method is suitable for solving wave propagation analysis. Explicit analysis is suitable for solving high-speed (short duration) dynamics, drop tests and crash analyses of structural members, large, nonlinear, quasi-static analyses, deep drawing, blow molding, and assembly simulations, highly discontinuous post buckling and collapse simulations, structural acoustics and coupled temperature-2. displacement (dynamic). Based on the explicit central-difference time integration rule, a large number of small time increments are performed efficiently. In addition, the equation of dynamic equilibrium at the beginning of the increment is satisfied by the explicit central-difference operator. The calculated accelerations at time, \( t \) are used in order to advance the displacement solution to time \( t + \Delta t / 2 \) which causes inexpensive increment.

The explicit finite element simulation is performed in ABAQUS-explicit software in quasi-static condition with elastic half space assumption. The contact configuration model, material properties and the finite element meshing are similar with those used in the implicit analysis. Since explicit algorithm requires density in its calculation procedure, the density of the Ti-6Al-4V and Super CMV used in explicit analysis are 4430 kg/m³ and 7940 kg/m³ respectively. The density of the material is taken into account as the explicit analysis requires nodal mass or inertia in order to exist at all activated degrees of freedom.

Unlike implicit element type, the explicit linear quadrilateral and plane strain element is used in this analysis. The transition of the mesh size from fine to coarse and finer mesh at the contact region is employed alike as the implicit approach as illustrated in Figure 3 in order to achieve reasonable computational time without degrading the accuracy. Surface to surface contact approach is used similar as the implicit analysis but different contact constraint is applied in contact surface interaction properties. The Penalty contact algorithm with friction coefficient, \( \mu = 0.3 \) is used as of Lagrange multiplier not applicable in explicit analysis and the Penalty method introduces behaviour of additional stiffness into the model which can affect the stable time increment [20].

Dynamic explicit step is assigned with the period of 0.13 s. In addition, semi-automatic mass scaling throughout the step is practiced with the target time increment of 1 x10⁻⁶ s for every one increment of frequency scale in order to analyze the model in its natural time period [20]. The bottom surface of the substrate (the flat object) is restricted from motion in x and y directions and a body force, P of 500 N/mm is ramped up gradually from zero with the smooth step amplitude loading as shown in Table I. The smooth step amplitude loading rate is practiced instead of instantaneous loading as the instantaneous loading might induce stress wave propagation through the model which will lead to undesired results [20].

<table>
<thead>
<tr>
<th>Time, s</th>
<th>Load amplification factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.13</td>
<td>1</td>
</tr>
</tbody>
</table>

RESULTS AND DISCUSSION

The comparison of contact pressure distribution and contact half-width length of Hertzian contact theoretical solution with the predicted numerical results are presented for two cases: implicit finite element method and explicit finite element method. The theoretical results are calculated for two types of materials, namely Ti-6Al-4V and Super CMV. The Hertzian theory calculates two parameters: contact pressure distribution and the contact half-width length. The numerical investigation results, of implicit and explicit finite element approaches are
compared with the Hertzian contact theoretical solutions for validation. The difference is examined on its maximum contact pressure.

2.1 Evolution of contact variables for Ti-6Al-4V

Figure 4 and Figure 5 show the evolution of FEM predicted contact pressure distribution for Ti-6Al-4V contact case using implicit and explicit analysis comparison with Hertzian theoretical contact pressure distribution. In addition, both numerical simulations for both approaches show a similar evolution as the trend of the contact pressure for both implicit and explicit FE analysis occurred towards the centre of contact area. Based on the Hertzian contact theoretical solution, 1294.636 MPa of maximum contact pressure $p_0$ is calculated for Ti-6Al-4V material. Meanwhile, the maximum contact pressure, $p_0$ recorded by using implicit and explicit analyses are 1304.310 MPa and 1296.840 MPa respectively at the peak of the graph as shown in Figure 4 and Figure 5. The predicted maximum contact pressure, $p_0$ of implicit and explicit analyses show close agreement with the Hertzian contact theoretical solution. But, explicit approach gives better agreement than the implicit as the percentage error of explicit and Hertzian theoretical solution is lower than the implicit method as shown in Table II.

![Fig. 4. Contact pressure and contact half-width length comparison between implicit FEM and Hertzian contact theoretical solution for Ti-6Al-4V.](image)

Next, the contact half-width length, $a$ of Ti-6Al-4V contact is analysed. The theoretical half-width of the contact length, obtained is 0.2459 mm. Based on the implicit analysis, the contact half-width length of 0.2500 mm is predicted as shown in Figure 4. Meanwhile, the contact half-width length predicted using explicit analysis is 0.2500 mm as illustrated in Figure 5. Although, the numerical result for both approach is consistent with the theoretical solution, the contact half-width length, $a$ displayed by implicit approach records lower
percentage error compared with explicit method as shown in the Table II.

Table II

<table>
<thead>
<tr>
<th>Contact variables of Ti-6Al-4V</th>
<th>Theoretical</th>
<th>Implicit</th>
<th>% error</th>
<th>Explicit</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum contact pressure, MPa</td>
<td>1294.6360</td>
<td>1304.3100</td>
<td>0.7472</td>
<td>1296.8400</td>
<td>0.1702</td>
</tr>
<tr>
<td>Contact half-width length, mm</td>
<td>0.2459</td>
<td>0.2500</td>
<td>1.6673</td>
<td>0.2550</td>
<td>3.7007</td>
</tr>
</tbody>
</table>

2.2 The evolution of contact variables for Super CMV

Figure 6 and Figure 7 illustrate the evolution of FE predicted contact pressure distribution for Super CMV contact case using implicit and explicit FE analysis in comparison with Hertzian theoretical contact pressure distribution. The predicted contact pressure distribution of both finite element approach display similar trend of the bell shape graph and the contact pressure occurs towards the centre of the contact. According to Hertzian contact theoretical solutions, of maximum contact pressure, \( p_0 \) recorded by using implicit and explicit analyses are 1718.1300 MPa and 1704.4900 MPa respectively at the peak of the graph as shown in Figure 6 and Figure 7. Close agreement with the Hertzian contact theoretical solution were shown by the predicted maximum contact pressure, \( p_0 \) from implicit and explicit analyses. But, explicit approach gives better agreement in term of maximum contact pressure than the implicit as the percentage error of explicit and Hertzian theoretical solution is lower than the implicit method as shown in Table III.

![Figure 6](image_url)

Fig. 6. Contact pressure and contact half-width length comparison between implicit FEM and Hertzian contact theoretical solution for Super CMV.
In addition, the contact half-width length, \( a \) for Super CMV contact is analysed. The theoretical contact half-width length, \( a \) obtained is 0.1864 mm. By using the implicit analysis, 0.1900 mm of the contact half-width length is predicted as shown in Figure 6. Meanwhile, 0.1950 mm of contact half-width length predicted by using explicit analysis as illustrated in Figure 7. Similar to the Ti-6Al-4V contact analysis, the finite element predicted result for implicit and explicit approach is consistent with the theoretical solution, but the implicit approach records lower percentage error in terms of contact half-width length compared with explicit method as shown in the Table III.

<table>
<thead>
<tr>
<th>Contact variables of Super CMV</th>
<th>Theoretical</th>
<th>Implicit</th>
<th>% error</th>
<th>Explicit</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum contact pressure, MPa</td>
<td>1707.3153</td>
<td>1718.1300</td>
<td>0.6334</td>
<td>1704.4900</td>
<td>0.1655</td>
</tr>
<tr>
<td>Contact length half-width, mm</td>
<td>0.1864</td>
<td>0.1900</td>
<td>1.9313</td>
<td>0.1950</td>
<td>4.6137</td>
</tr>
</tbody>
</table>

2.3 Discussion

Results in this present study are the predicted contact pressure and half-width of the contact length. Hertzian contact theoretical solution are compared with numerical prediction using two different approach of finite element methods, i.e. implicit analysis and explicit analysis. The explicit finite element method gives good agreement for maximum contact pressure while, implicit finite element method shows closer predicted result for contact length half-width as shown in Figure 4 thru Figure 7 using two different ductile materials. Hertzian contact theoretical solution is used as the reference for both analyses.

For the FE predicted maximum contact pressure, explicit analysis gives closer results with the theoretical solution with lower percentage error. This is due to the ability of the explicit approach to solve the high speed dynamic and highly discontinues problems effectively. In addition, the explicit method gives good agreement in predicting the maximum contact pressure as this approach explicitly advancing the state of kinematic from previous increment [21]. So, explicit analysis is said to exhibit the real characteristics of the material by capturing the contact stress field efficiently. As the contact stress is the initial factor for the degradation and failure of the material, the explicit method is recommended as it is an effective approach in analysing
material failure. In addition, the contact case for Super CMV material shows higher contact pressure compared with the Ti-6Al-4V material for both numerical and theoretical solution as Super CMV is high strength steel which can withstand high load or force than the Ti-6Al-4V.

Besides, the implicit analysis gives better agreement compared with theoretical solution for half-width of the contact length by displaying lower percentage error. This is mainly due to the enforcement of Lagrange multiplier contact algorithm. A number of studies have reported the Lagrange multiplier contact algorithm ensures an exact sticking condition where shear stress is lower than the critical shear stress based on Coulomb friction [17-19]. In addition, the Lagrange multiplier contact algorithm provides the exact constraint fulfillment reported by Litewka [22]. Unlike Lagrange multiplier contact algorithm, the penalty method only allows for solving of more general types of contact. This was clearly shown when the implicit approach gives closer result for the contact length for both type of materials with the Hertzian theoretical solution.

As there have been no controlled studies which compare difference between implicit and explicit FEM approach in quasi static condition to be verified with Hertzian analytical solution, this research able to give an insight to practice appropriate approach when practicing finite element method, especially contact based analysis.

3. CONCLUSION
The finite element analysis is carried out only for the elastic regimes counting contact pressure and contact length half-width. The predicted results obtained from implicit and explicit FE method can reproduce a realistic contact behaviour for Ti-6Al-4V and Super CMV materials as close agreement is achieved with the theoretical solution. The difference of the maximum contact pressure and contact length half-width between FE analysis and Hertzian contact theoretical solutions is less than 1% and 5%, respectively. In addition, the explicit FEM is able to solve the high speed dynamic and highly discontinuous problem as it gives better prediction compared with implicit analysis in term of the maximum contact pressure. Meanwhile, the contact constraints applied for both implicit and explicit finite element method such as Lagrange multiplier and Penalty method are perfectly satisfied as only the slight difference between the Hertzian theoretical solutions. But, Lagrange multiplier contact algorithm is able to give better predictions in term of contact length half-width. The application of implicit and explicit finite element method is reliable in solving the contact based problem but both approaches have its own strength where explicit FEM causes small average percentage error about 0.2% with the theoretical maximum contact pressure while implicit FEM causes small percentage error about 0.9% with the theoretical contact length half-width.

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REFERENCES

**List of symbols**

- $p_0$: Maximum contact pressure [MPa]
- $p(x)$: Contact pressure as a function of $x$-position
- $a$: Contact length half-width [mm]
- $P$: Normal load [N/mm]
- $R$: Radius [mm]
- $E$: Elastic modulus [MPa]
- $\nu$: Poisson ratio [No unit]
- $\mu$: Friction coefficient [No unit]