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10
LINEAR ARX MODELLING OF PNEUMATIC ACTUATOR SYSTEM

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10.1 INTRODUCTION

Pneumatic actuator system refers to a series of pneumatic sourced interconnected components (includes pneumatic actuator) that results in mechanical movement when operated. Pneumatic actuator system is commonly found in manufacturing and automation industries to execute high frequency and repetitive mechanical motion (linear or rotary). Comparing to electrical and hydraulic actuator system, the pneumatic actuator system possesses some absolute advantage, such as providing greener, cleaner and fail-safe environment (oil/spark free), and lower cost in operation and maintenance. Due to these advantages, the pneumatic actuator system gradually become the standard solution in industry by replacing electrical actuator system.

The modelling of pneumatic actuator system can be done by either analytical method [1] [2] [3] or empirical method [4] [5] [6]. Both of the method has their own advantages. However, the pneumatic actuator system is not a singular body system, it consists of several interconnected components such as proportional control valve, pneumatic cylinder, and sensors. Due to this reason, the empirical approach could be the solution in modelling the system, as analytical method is complicated and exhausting, although nonlinearity is ignored as an assumption in modelling.

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10.2 PROBLEM FORMULATION

The typical analytical model of a pneumatic actuator (PA) system usually consists of 3 dynamics models: control valve model, pressure dynamics model, and motion equation of cylinder's piston. In reality, all three models possess high nonlinear properties, such as the dead-zone effect in the control valve and friction effect in the cylinder. Due to the fact that most controller designs require a linearized model, the nonlinear term in each dynamic model must be simplified and linearized as well. Therefore, assumptions must be made to reduce the system's operating condition towards a linearizable environment. According to [7], these assumptions include maintaining the air pressure supply and temperature at a constant level, ignoring heat transfer dynamics between pressurized air and surroundings, assuming the working gas is ideal/isothermal, and taking speed proportional friction.

10.2.1 Proportional Valve Model

Equation (10.1) is a nonlinear pressurize air mass flow rate model of a control valve, where the compressibility dynamics of valve flow are considered. Depending on the downstream and upstream pressure ratio, the flow phase can shift between subsonic (under choked) flow or sonic (choked) flow.

$$\dot{m} = P_u \cdot C_v \cdot \rho_a \cdot \sqrt{\frac{T_a}{T_u}} \cdot \varphi(P_u, P_d, b_{cr}) \tag{10.1}$$

$$\varphi(P_u, P_d, b_{cr}) = \begin{cases} \sqrt{1 - \left(\dfrac{\frac{P_d}{P_u} - b_{cr}}{1 - b_{cr}}\right)^2} & for\ \dfrac{P_d}{P_u} > b_{cr}\ (Subsonic) \\ 1 & for\ \dfrac{P_d}{P_u} \leq b_{cr}\ (Sonic) \end{cases}$$

$\dot{m}$ Mass flow rate (kg/s)
P_u Upstream air pressure (Pa)
C_v Sonic conductance $(m^3/s \cdot Pa)$

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ρ_a Ambient air density (kg/m^3)
T_a Ambient air temperature (K)
T_u Upstream air temperature (K)
P_d Downstream air pressure (Pa)
b_{cr} Critical pressure ratio

To linearize equation (10.1), first, we could establish an equation using constant coefficient, K_2 instead of a nonlinear term like $\varphi(P_u, P_d, b_{cr})$. The assumption in modelling is neglecting valve dynamics and assume both upstream and downstream pressure is almost constant. Introducing the valve input voltage term u_{valve}, we obtain equation (10.2)

$$\dot{m}_n = \pm K_2 \cdot P_n \cdot u_{valve} \quad where\ n \in \{A, B\} \tag{10.2}$$

10.2.2 Air Pressure Dynamics

Figure 10.1 shows the relationship of control valve input voltage, air mass flow rate and cylinder's chamber A and B. The linear model of air pressure dynamics that cause the movement of cylinder piston can be represented using differential pressure equation. The full derivation of the equation is demonstrated in [7].

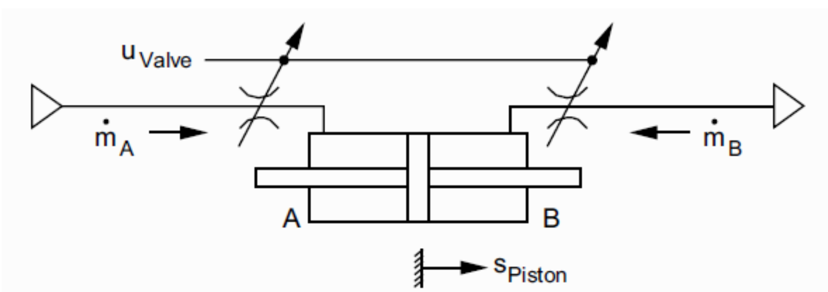

Figure 10.1: Schematic diagram of air pressure dynamics in pneumatic actuator system

If the initial piston displacement s_{piston} is middle of the symmetric cylinder, the volume and pressure change equation of both cylinder chamber A and B can be described as below

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$$V_A = \frac{V_{total}}{2} + A_{piston} \cdot s_{piston}$$
$$V_B = \frac{V_{total}}{2} - A_{piston} \cdot s_{piston} \tag{10.3}$$

$$\dot{P}_A = \frac{P_a \cdot K_2 \cdot P_S \cdot u_{valve}}{2\rho_a \cdot \left(\frac{V_{total}}{2}\right)} - \frac{P_S \cdot A_{piston} \cdot \dot{s}_{piston}}{V_{total}}$$
$$\dot{P}_B = \frac{-P_a \cdot K_2 \cdot P_S \cdot u_{valve}}{2\rho_a \cdot \left(\frac{V_{total}}{2}\right)} + \frac{P_S \cdot A_{piston} \cdot \dot{s}_{piston}}{V_{total}} \tag{10.4}$$

P_S	Air pressure supply (Pa)
$V_{A,B}$	The volume of air in chambers A, B (m^3)
V_{total}	The total air volume of cylinder (m^3)
A_{piston}	Effective piston cross-section area (m^2)
s_{piston}	Piston displacement (m)

By defining the differential pressure as $P_{AB} = P_A - P_B$, we could obtain a differential equation for pressure change.

$$\dot{P}_{AB} = \left[\frac{2P_a \cdot K_2 \cdot P_S}{\rho_a \cdot V_{total}}\right] u_{valve} - \left[\frac{2P_S \cdot A_{piston}}{V_{total}}\right] \dot{s}_{piston} \tag{10.5}$$

10.2.3 Motion Equation of Piston

The piston acceleration can be obtained by derivation using Newton's second law

$$M_{Piston}\ddot{s}_{piston} = A_{piston} \cdot (P_A - P_B) - F_{External} - F_{Friction} \tag{10.6}$$

$\ddot{s}_{piston}$	Piston acceleration (m/s^2)
$F_{External}$	External load force (N)
$F_{Friction}$	Friction force (N)
M_{Piston}	Piston mass (kg)

Both $F_{External}$ and $F_{Friction}$ could be a nonlinear term as well. The external load force is a constant if is a weight, or a sinusoidal term if it from spring force. However, the friction force is clearly nonlinear and hardly predictable, it could be modelled as LuGre model or Stribeck effect friction. Therefore, the linearization process of pneumatic actuator system does not consider $F_{External}$in following derivation and $F_{Friction}$ is modelled into speed proportional friction. Noted that both load and friction force contribute most towards the system dynamic behavior, therefore if one is modelling for analysis, both forces should be described well without simplification.

However, for controller design purposes, we assume for speed proportional friction $F_{Friction} = f_v \cdot \dot{s}_{piston}$ and zero external load forces $F_{External} = 0$. Then equation (10.7) is simplified from (10.6).

$$\ddot{s}_{piston} = \frac{A_{piston} \cdot P_{AB}}{M_{Piston}} - f_v \cdot \dot{s}_{piston} \tag{10.7}$$

By differentiating (2.7) and substitute in $\dot{P}_{AB}$ from (10.5),

$$\begin{aligned}\dddot{s}_{piston} = &\left[\frac{-2P_S \cdot A^2{}_{piston}}{M_{Piston} \cdot V_{total}}\right]\dot{s}_{piston} + [-f_v]\ddot{s}_{piston} \\ &+ \left[\frac{2P_a \cdot K_2 \cdot P_S \cdot A_{piston}}{\rho_a \cdot M_{Piston} \cdot V_{total}}\right]u_{valve}\end{aligned} \tag{10.8}$$

10.2.4 Linearized State-space Equation

From equation (10.8), we could construct a Position-Velocity-Acceleration (PVA) state model, by defining the state and input vector as:

$$\underline{x} = \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} Position \\ Velocity \\ Acceleration \end{bmatrix} \quad u = [u_{valve}]$$

The state-space equation for position tracking system of pneumatic actuator system is as follow:

$$\begin{aligned}\underline{\dot{x}} &= A\underline{x} + Bu \\ \underline{y} &= C\underline{x}\end{aligned} \tag{10.9}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & \dfrac{-2P_S \cdot A^2{}_{piston}}{M_{Piston} \cdot V_{total}} & -f_v \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{2P_a \cdot K_2 \cdot P_S \cdot A_{piston}}{\rho_a \cdot M_{Piston} \cdot V_{total}} \end{bmatrix}$$

$$C = [1 \quad 0 \quad 0]$$

10.2.5 Problem with Analytical Modelling

Generally, the linear analytical model provides insight into the core dynamics working on a pneumatic actuator system. After all, for the case of designing controller using a model-based approach, some problem arises First, noticed that the parameter f_v and K_2 in equation (10.9), these parameters are hardly obtained, even though the parameter range is controlled under a certain boundary by restricting the operating condition. Yet, the parameter is inaccurate and does not represent the system dynamic appropriately, mainly because the parameter is inserted by initial guessing from the range. Secondly, even if both the parameter f_v and K_2 is guessed correctly, the parameters are only suitable for that operating condition. When we are going to operate in a different condition, say increase the supply pressure from 700 kPa to 800 kPa, the parameter would change eventually. In that case, we have to tune the parameter again. Even though the model is assumed to be linear, two problems mentioned earlier make the controller design process more complex.

Contrasting to the analytical model, the empirical modelling approach might solve the stated issues previously. The empirical method, also known as the System Identification technique, build the pneumatic actuator model directly from the input and output data, with a specified operating condition. The modelling process is relatively simpler than the analytical method: defining the input signal, running the system, collecting the data via data acquisition system, and performing model identification via scientific software: MATLAB. Thanks to the development of computers, nowadays, performing iterative computation is no longer a time-consuming task

10.3 METHODOLOGY

The initiation of applying system identification comes when one does not have a reasonable physics law to describe the phenomenon during a process. Still, the necessary accuracy of the model is required. Moreover, system identification deals with the black-box problem, where the analytic characterization is difficult to achieve and make sense if the system is cheap and safe to be experimented on. As we know, the pneumatic actuator system is safe to operate and affordable to experiment with. Therefore, the system identification technique is suitable for a pneumatic actuator system. The following subsection will describe the methodology [8] of performing empirical modelling for the pneumatic actuator system, they include process rig setup, input signal designation, data acquisition, model structure selection, parameter estimation and model validation.

10.3.1 System and Process Rig Setup

The secondary data is obtained from a process rig consist of several components. These components include an air compressor, a pneumatic cylinder, two pressure sensors, an electromagnetic displacement sensor and a proportional control valve. Besides, the process rig is computer-controlled via a data acquisition system with a sampling time of 0.01 *s*. Figure 10.2 shows the rig setup of the pneumatic actuator system, and Table 10.1 is the specification list of all components.

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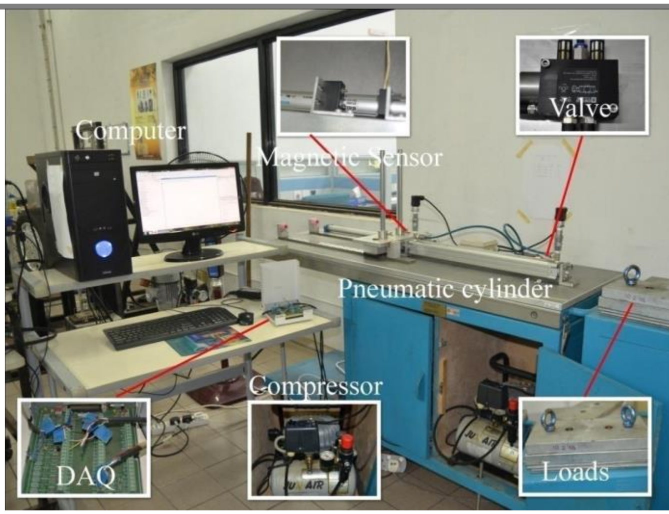
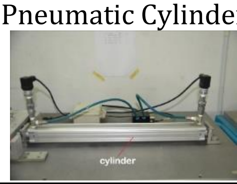


Figure 10.2: Pneumatic actuator system rig setup

Table 10.1: Specification of components

Components	Specification	Description
Air Compressor 	Product	Air Compressor
	Brand	JUN-AIR
	Model No.	3 - 4
	Function	Pneumatic energy source and storage for PA system
Control Valve 	Product	High Speed 5/3 Proportional Directional Valve
	Brand	Enfield Technologies
	Model No.	LS-V15S
	Function	Control compressed air flow between pneumatic cylinder air chambers via voltage manipulation
Pneumatic Cylinder 	Product	Double-acting ISO cylinder
	Brand	FESTO
	Model No.	DNC-40-500-PPV-A
	Function	Generate linear motion via manipulation of pressure

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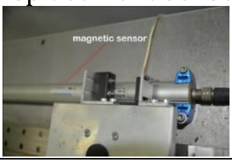
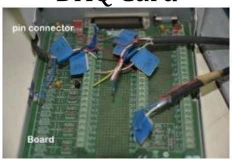
<table>
<tr><td></td><td></td><td>difference in both air chamber</td></tr>
<tr><td rowspan="4">Electromagnetic Displacement Sensor

</td><td>Product</td><td>Magneto-restrictive Sensor</td></tr>
<tr><td>Brand</td><td>Balluff</td></tr>
<tr><td>Model No.</td><td>Micropulse AT style</td></tr>
<tr><td>Function</td><td>Provide cylinder position information as output voltage signal</td></tr>
<tr><td rowspan="4">DAQ Card

</td><td>Product</td><td>Data Acquisition Card</td></tr>
<tr><td>Brand</td><td>National Instrument</td></tr>
<tr><td>Model No.</td><td>SCB-68</td></tr>
<tr><td>Function</td><td>Data acquisition from PA system (analogue) to host PC (digital)</td></tr>
</table>

10.3.2 Input Signal Designation and Data Collection

Model identification requires an experimental run on the process rig to obtain the data. However, the selection of the input signal must be carefully designated. It depends on 3 main factors: the control valve's compatibility with the signal bandwidth, control valve's operating voltage limit and type of signal for system identification. The control valve used in the rig has a maximum bandwidth frequency of 100 Hz. It means that it could perform 100 cycles of switching between 3 valve positions in 1 second, which is extremely robust and fast enough. Besides, the operating voltage limit of the control valve is nominally $-5\ V\ to\ 5\ V$.

Regarding the type of signal used for the system identification, it is advised that step input should be avoided. Landau and Zito [9] states that the convergence towards zero of the prediction error during regression does not always means that the estimated model parameters will converge towards the 'true' parameters of the plant model. The constant (step) input is not a good input signal candidate for system identification. It does not allow satisfactory model parameter estimation, mainly because the steady-state gain of both the estimated model and plant model might not be the same when the frequency gets higher. This can be observed from the frequency characteristic plot of both models. Therefore, we could consider using non-zero frequency sinusoidal signal. In other words, to identify a 'correct' model, it is necessary to apply a frequency rich input.

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Based on the above consideration, three types of multi-frequency input signal $u(t)$ is designed as shown in equation (10.10). The experiments for each input signal will be repeated three times and will result in 9 outputs $y(t)$ in total. Due to some uncontrollable non-linearity and external disturbances, all three experiments from the same input signal are expected to be slightly different. To gain a clearer understanding of the input signal, the statistical properties of all input signal (collected from first 100 *seconds* of experiment) is listed in Table 10.2.

$$
\begin{aligned}
u_1(t) &= 0.5\cos(2\pi \times 0.05t) + 1.5\cos(2\pi \times 0.2t) \\
&\quad + 2.5\cos(2\pi \times 0.8t) \\
u_2(t) &= 0.8\cos(2\pi \times 0.05t) + 0.5\cos(2\pi \times 0.2t) \\
&\quad + 3\cos(2\pi \times 0.8t) \\
u_3(t) &= 0.75\cos(2\pi \times 0.05t) + 0.75\cos(2\pi \times 0.2t) \\
&\quad + 2.5\cos(2\pi \times 0.8t)
\end{aligned}
\tag{10.10}
$$

Table 10.2: Statistical properties of input signals

Statistical Properties	Voltage input signal $u(t)$		
	$u_1(t)$	$u_2(t)$	$u_3(t)$
$Mean, \bar{u}$	4.5×10^{-4}	4.5×10^{-4}	3.5×10^{-4}
$Maximum(u)$	4.5	4.5	3.5
$Minimum(u)$	−3.9878	−3.9308	−3.1562
$Variance, {\sigma_u}^2$	4.3770	3.5645	2.5637

Figure 10.3 shows the data gathering process from the pneumatic actuator system. The experiment run time is fixed at $100\,s$ with a sampling time T_s of $0.01\ s$. Each discrete data would be saved into the host PC iteratively via Simulink Desktop Real-Time software. A total of 10001 (including initial time $0\,s$) data will be used to perform model identification afterwards.

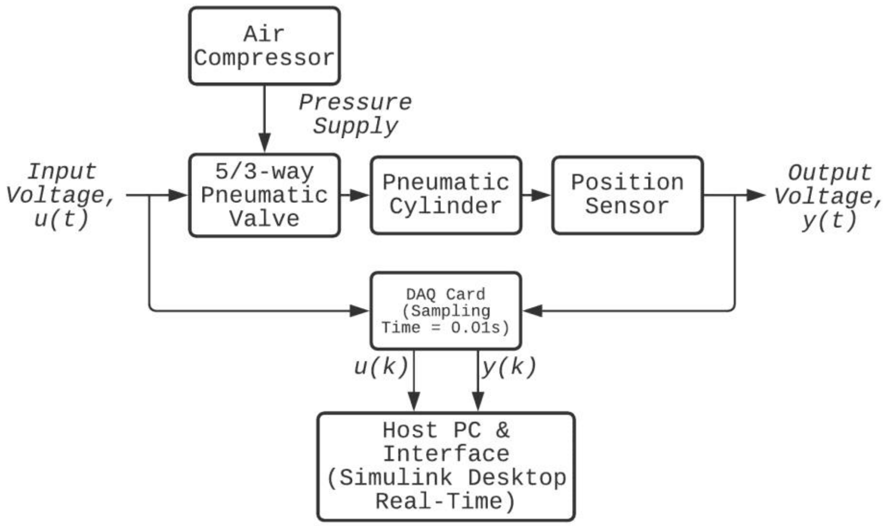

Figure 10.3: Data collection from pneumatic actuator system

Figure 10.4, 10.5 and 10.6 shows the input and response plot after experimentation on pneumatic actuator system using input $u_1(t)$, $u_2(t)$ and $u_3(t)$. All 9 types of data set will be used in parameter estimation process later on.

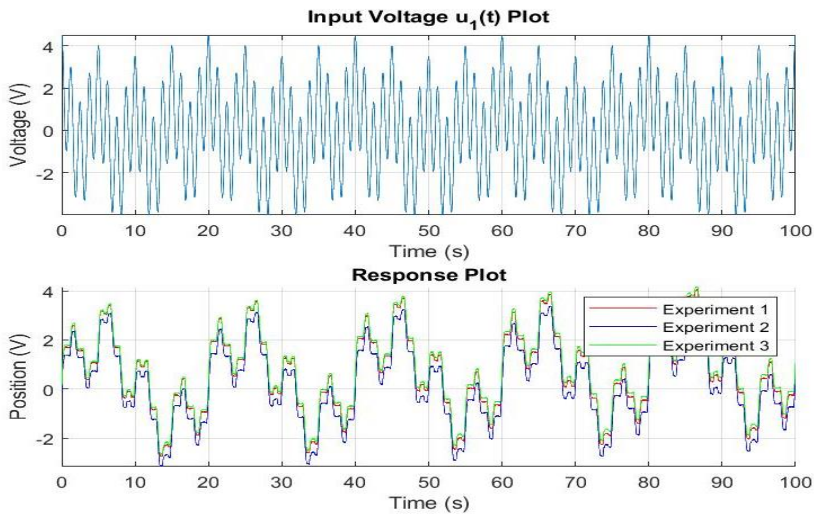

Figure 10.4: Input and response plot of $u_1(t)$

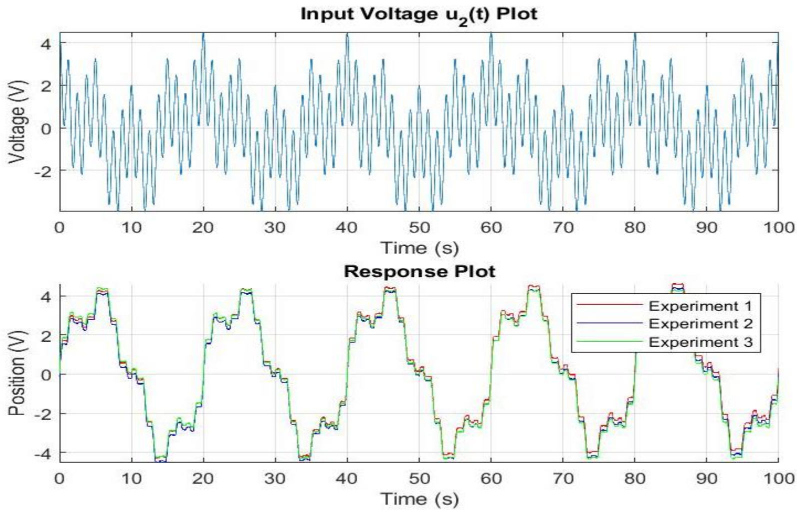

Figure 10.5: Input and response plot of $u_2(t)$

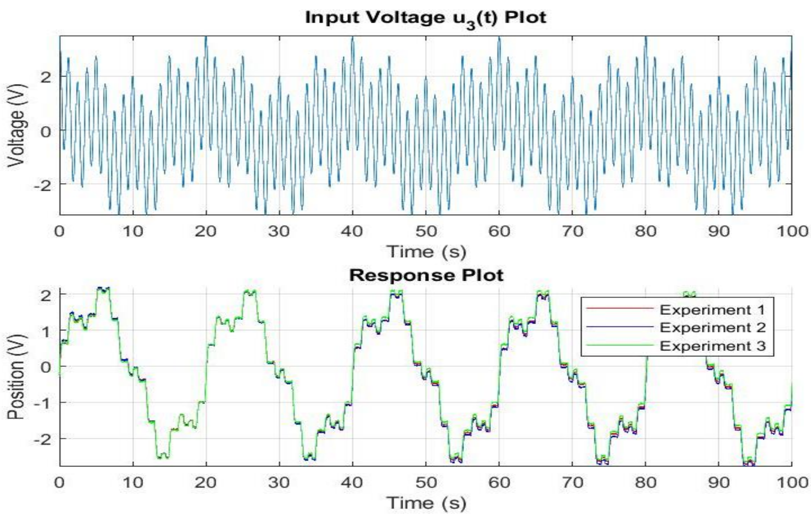

Figure 10.6: Input and response plot of $u_3(t)$

10.3.3 Model Complexity Selection

For the ease of controller design, we assume that the noise from the system is rather minimal, approximately near to zero. Besides, the nonlinear factors are ignored during modelling, these nonlinear factors may be hysteresis, pressure change and frictional effect. Therefore, the auto-regressive exogenous (ARX) polynomial model is chosen for parameter estimation. Figure 10.7 and Equation (10.11) below is the model structure and mathematical representation of ARX model.

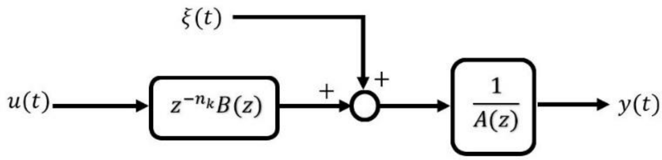

Figure 10.7: ARX model structure

$$y(t) = z^{-n_k}\frac{B(z)}{A(z)}u(t) + \frac{1}{A(z)}\xi(t) \tag{10.11}$$

Where,

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{n_a} z^{-n_a}$$
$$B(z) = b_1 + b_2 z^{-1} + a_3 z^{-2} + \cdots + a_{n_b} z^{-(n_b-1)}$$

n_a	Number of poles
n_b	Number of zeros
n_k	Dead time / Delay term
$\xi(t)$	White noise signal with zero mean and variance, $N(0, \sigma^2)$

10.3.4 Parameter Estimation

The parameter estimation algorithm used is least-squares method. However, we could easily perform parameter estimation without programming the least-squares regression algorithm from scratch, by using either MATLAB System Identification Toolbox graphical user interface (GUI) or MATLAB function. Both methods allow one-click parameter estimation by setting some configuration based on some requirements.

The purpose of system identification for pneumatic actuator system is to estimate a linear model for simulation and controller design purposes. Therefore, simulation focus is chosen in the parameter estimation workflow. Compare to prediction focus, the main difference is that simulation focus model would allow the model to perform better under a wide range of condition, say, using other types of input during simulation. In the other hand, the residual analysis is not suitable in validating simulation focus model as the predictive ability of the model is not prioritized.

Since we are using polynomial ARX model as the model structure, it is important to decide the range of poles, zeros and delay term. For simplicity, we started the parameter estimation from a lower polynomial, such as $2, 3\ and\ 4$ for both n_a and n_b, and set the delay term n_k to 0. This parameter estimation process will be repeated for another 2 delay terms, which is $n_k = 1, 2$.

10.3.5 Model Validation and Selection

Since the selected model will be used as simulation model, the model validation and selection criterion will be based on 3 criteria: Best Fit %, Loss Function value and Mean Squared Error.

1. Best Fit % - A percentage expression on normalized root mean square error (NMRSE), it indicates that how well fitted is the real data towards the model data

$$BF\% = 100\left(1 - \frac{\|y - \hat{y}\|}{\|y - \bar{y}\|}\right)$$

where,

y	Experiment output data
$\hat{y}$	Simulated data
$\bar{y}$	Mean of experiment output data
$\|\cdot\|$	Euclidean norm, or 2-norm of a vector

2. Loss Function – A positive function of squared simulation error that minimized towards θ.

$$J(\theta) = \frac{1}{N}\sum_{k=1}^{N} e^T(k,\theta) \cdot W(\theta) \cdot e(k,\theta)$$

where,

N	Data size
$e(k,\theta)$	Error vector term
$W(\theta)$	Positive semidefinite weighting matrix

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3. MSE – Mean Squared Error

$$MSE = \frac{1}{N}\sum_{k=1}^{N} e^{T}(k) \cdot e(k)$$

10.4 RESULT AND DISCUSSION

Table 10.3, 10.4 and 10.5 the results of parameter estimation for 3 different delay term, which is $n_k = 0,1,2$. The similarities from all 3 tables show that model with 2 zeros / 2 poles using input $u_2(t)$ has the highest best fit percent. Although the loss function value and mean squared error is not the lowest among all estimation, but the value is convincing enough, when compare to results in 4 zeros /4 poles.

Table 10.3: System identification result for $n_k = 0$

	Exp	(2,2,0) Best Fit %	(2,2,0) LF	(2,2,0) MSE	(3,3,0) Best Fit %	(3,3,0) LF	(3,3,0) MSE	(4,4,0) Best Fit %	(4,4,0) LF	(4,4,0) MSE
u_1	1	65.93	5.02E-04	0.2917	67.86	5.50E-04	0.2595	78.73	1.91E-03	0.1137
	2	78.98	4.15E-04	0.1107	78.68	5.18E-04	0.1140	78.71	1.63E-03	0.1137
	3	61.10	4.98E-04	0.3744	63.23	5.23E-04	0.3346	75.02	1.72E-03	0.1544
u_2	1	90.24	3.91E-04	0.0697	87.68	3.98E-04	0.1109	88.93	1.86E-03	0.0896
	2	91.83	4.20E-04	0.0490	89.98	3.95E-04	0.0737	91.91	1.78E-03	0.0481
	3	93.04	3.82E-04	0.0363	90.71	3.83E-04	0.0645	92.84	2.30E-03	0.0384
u_3	1	87.88	4.47E-04	0.0311	68.02	2.37E-04	0.2164	72.44	1.34E-03	0.1607
	2	85.04	4.34E-04	0.0482	65.70	2.50E-04	0.2533	69.07	1.29E-03	0.2060
	3	89.30	4.27E-04	0.0243	71.97	2.51E-04	0.1667	80.83	1.22E-03	0.0780

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Table 10.4: System identification result for $n_k = 1$

	Exp	$(n_a, n_b, 1)$ (2,2,1) Best Fit %	LF	MSE	(3,3,1) Best Fit %	LF	MSE	(4,4,1) Best Fit %	LF	MSE
u_1	1	65.96	4.68E-04	0.2911	69.89	5.31E-04	0.2277	76.114	1.91E-03	0.1433
	2	79.01	3.99E-04	0.1105	78.71	5.21E-04	0.1136	78.712	1.63E-03	0.1136
	3	61.18	4.71E-04	0.3730	67.82	4.92E-04	0.2563	77.781	1.72E-03	0.1222
u_2	1	90.32	3.88E-04	0.0685	87.93	3.96E-04	0.1066	89.058	1.86E-03	0.0875
	2	91.94	4.18E-04	0.0477	90.35	4.00E-04	0.0683	91.940	1.78E-03	0.0477
	3	93.10	3.81E-04	0.0356	90.65	3.80E-04	0.0654	92.883	2.29E-03	0.0379
u_3	1	87.78	4.47E-04	0.0316	68.14	2.37E-04	0.2148	72.787	1.34E-03	0.1567
	2	84.95	4.33E-04	0.0488	65.59	2.52E-04	0.2549	69.856	1.30E-03	0.1956
	3	89.10	4.27E-04	0.0252	72.50	2.55E-04	0.1604	81.363	1.22E-03	0.0737

Table 10.5: System identification result for $n_k = 2$

	Exp	(2,2,2) Best Fit %	LF	MSE	(3,3,2) Best Fit %	LF	MSE	(4,4,2) Best Fit %	LF	MSE
u_1	1	66.49	4.17E-04	0.2822	70.37	5.46E-04	0.2205	76.787	1.92E-03	0.1354
	2	79.03	3.88E-04	0.1102	78.15	5.32E-04	0.1197	78.670	1.63E-03	0.1141
	3	61.89	4.28E-04	0.3594	66.93	5.11E-04	0.2707	76.546	1.72E-03	0.1361
u_2	1	90.44	3.85E-04	0.0669	88.23	4.00E-04	0.1013	89.388	1.86E-03	0.0823
	2	92.02	4.18E-04	0.0468	90.44	4.02E-04	0.0671	91.857	1.79E-03	0.0487
	3	93.13	3.80E-04	0.0353	90.75	3.84E-04	0.0641	92.730	2.30E-03	0.0396
u_3	1	88.44	4.46E-04	0.0283	66.70	2.43E-04	0.2346	69.635	1.34E-03	0.1951
	2	85.81	4.31E-04	0.0434	66.15	2.59E-04	0.2468	69.166	1.29E-03	0.2047
	3	89.41	4.26E-04	0.0238	73.16	2.60E-04	0.1528	78.662	1.22E-03	0.0966

Based on observation, the 3rd experiment using input $u_2(t)$ produce the highest best fit percent, which is 93.04%, 93.10% *and* 93.13%. Therefore, the ARX discrete polynomial model will adopt the estimated parameter of the configuration (*arx220, arx221 and arx222*). However, arx220 model will be selected for simulation purpose due to its simplicity. Equation 10.12 shows the selected model parameter.

$$y(t) = z^{-n_k}\frac{B(z)}{A(z)}u(t) + \frac{1}{A(z)}\xi(t) \tag{10.12}$$

where,

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2}$$
$$B(z) = b_1 + b_2 z^{-1}$$

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For $n_k = 0$

$$a_1 = -1.988 \qquad a_2 = 0.9876$$
$$b_1 = 0.007788 \qquad b_2 = -0.007671$$

For $n_k = 1$

$$a_1 = -1.988 \qquad a_2 = 0.9885$$
$$b_1 = 0.007966 \qquad b_2 = -0.007856$$

For $n_k = 2$

$$a_1 = -1.988 \qquad a_2 = 0.9893$$
$$b_1 = 0.008141 \qquad b_2 = -0.00804$$

Figure 10.8 shows the best fit % plot with the comparison of all simulated model from *arx220, arx221, arx222* and measure data from 3rd experiment using input $u_2(t)$. It can be observed that the nonlinearity and chattering behaviour from disturbance signal is not demonstrated by the simulation model response. The enlarged plot is shown in Figure 10.9.

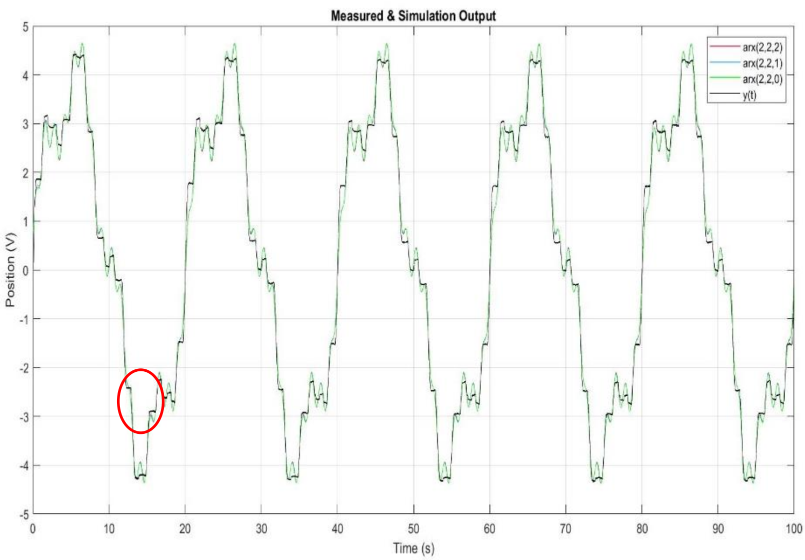

Figure 10.8: Best fit % plot

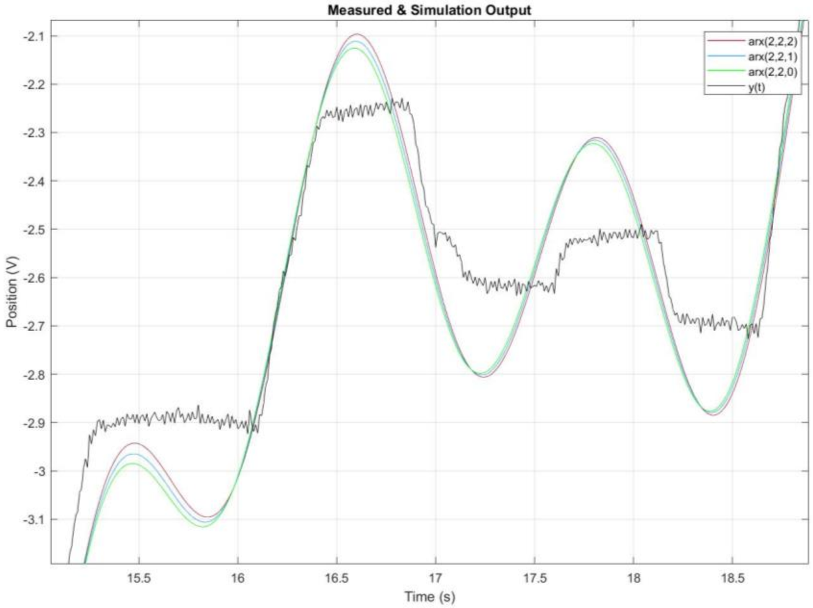

Figure 10.9: Best fit % plot (enlarged)