

CHARACTERISTICS OF PLATES EMBEDDED WITH SHAPE MEMORY
ALLOY WIRES

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To my loving father, mother, brother, sister and Sek Eng

My love to you all will always remain...



PTTA UTHM
PERPUSTAKAAN TUNKU TUN AMINAH

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ABSTRACT

Shape memory alloy (SMA) has the potential to be used in engineering applications. It is widely used as sensors, actuators, dampers and when embedded into composite, it demonstrates some unique mechanical and vibration properties such as the strength and damping of the composite material. In this research, the thermo-mechanical properties of SMA wire, the mechanical properties and vibration characteristics of composites embedded SMA wire were determined. There are two groups of SMA arrangement in the composite to be fabricated; they are without SMA wire and with unidirectional fine SMA wires (Flexinol[®] wire) oriented at 0°, 15°, 30° and 45° with the volume fraction of 0.08. The characteristics of SMA wire were determined using the Universal Testing Machine. The Young's modulus, stress – strain curve analysis, fundamental frequency at first mode and damping behavior of the SMA embedded composite were also done. Simulation was carried out using MATLAB based on Brinson model and Reddy approach. The results showed that Flexinol[®] wire has the phase transformation temperatures at 32.84°C, 44.78°C, 52.54°C and 60.90°C and it agree well with the manufacturer technical data. The Flexinol[®] wire has Young's modulus during martensite and austenite of 33.16 GPa and 69.59 GPa and the coefficient of stress influence during martensite and austenite of 10.76 MPa/°C and 9.08 MPa/°C. The Young's modulus of the composite is linearly proportional to temperature increment. The maximum fundamental frequency shifted occur at 45° for the boundary condition of C-F-F-F and C-F-C-F while for the boundary condition of C-C-C-C were at 15°, 75°, 105° and 165° if compared with the fundamental frequency at 0°. The overall highest damping occurred when 0.50 A with DC applied at the boundary condition of C-F-F-F.

ABSTRAK

Aloi Memori Bentuk (SMAs) berpotensi untuk digunakan dalam aplikasi kejuruteraan. Ia digunakan secara meluas sebagai penderia, penggerak, peredam dan apabila ditanamkan dalam rencam, ia menunjukkan sifat mekanikal dan sifat getaran yang unik seperti kekuatan dan sifat peredam bagi rencam. Dalam kajian ini, sifat therma-mekanikal bagi dawai SMA, sifat mekanikal dan sifat getaran bagi rencam yang ditanam dawai SMA ditentukan. Terdapat dua jenis kumpulan rencam dihasilkan iaitu rencam tanpa SMA dan rencam yang ditanam dengan SMA pada sudut 0° , 15° , 30° and 45° dengan nisbah isipadu 0.08. Sifat dawai SMA dilakukan dengan mesin ujikaji semesta. Modulus Young dan graf tegasan-terikan, frekuensi asasi pada mode pertama dan sifat redaman bagi rencam yang ditanam dengan SMA turut dilakukan. Simulasi juga dilakukan dengan menggunakan perisian MATLAB berdasarkan model Brinson dan pendekatan Reddy. Keputusan menunjukkan dawai Flexinol[®] mempunyai suhu penukaran fasa pada 32.84°C , 44.78°C , 52.54°C dan 60.90°C dan ia selari dengan data teknikal daripada pengilang. Dawai Flexinol[®] mempunyai modulus Young semasa martensite dan austenite pada 33.160 GPa dan 69.592 GPa dan pemalar influsi tegasan semasa martensite dan austenite pada $10.761 \text{ MPa}/^{\circ}\text{C}$ dan $9.082 \text{ MPa}/^{\circ}\text{C}$. Modulus Young bagi rencam adalah berkadar terus terhadap peningkatan suhu. Frekuensi asasi terpesong paling tinggi pada 45° pada keadaan sempadan C-F-F-F dan C-F-C-F manakala untuk keadaan sempadan C-C-C-C adalah pada 15° , 75° , 105° dan 165° jika dibandingkan dengan frekuensi tabii pada sudut 0° . Manakala bagi pemalar peredam yang tertinggi secara keseluruhannya berlaku pada arus terus 0.50 A pada keadaan sempadan C-F-F-F.

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LIST OF SYMBOLS

$\varepsilon, \varepsilon_x, \varepsilon_y, \varepsilon_z$	-	Strain, normal strain in X, Y, Z direction
ε_0	-	Initial strain
ε_l	-	Maximum residual strain
$\varepsilon_f, \varepsilon_m$	-	Strain (fiber, matrix)
$\gamma, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}$	-	Shear strain, shear strain in X, Y, Z plane
F, p	-	Load
F_f, F_m	-	Load (fiber, matrix)
$\sum M_x, \sum M_y$	-	Total moment (in axis-x, in axis-y)
m_x, m_y	-	Bending moment per unit length in X, Y, Z Cartesian coordinate system
m_{xy}, m_{yx}	-	Twisting moment per unit length in X, Y, Z Cartesian coordinate system
q_x, q_y	-	Transverse shearing forces in X, Y, Z Cartesian coordinate system
u, v, w	-	Displacement components in X, Y, Z direction
$\sum P_z$	-	Total load in axis-z
$\sigma, \sigma_x, \sigma_y, \sigma_z$	-	Stress, normal stress in X, Y, Z direction
σ_f	-	Flexural stress
$\dot{\sigma}$	-	Stress
σ_0	-	Initial stress
σ_f, σ_m	-	Stress (fiber, matrix)
σ_F, σ_T	-	Stress due to force, force due to temperature

$\sigma_f^{AS}, \sigma_f^{SA}$	-	Stress of SMA fiber (end of austenite to martensite transformation, end of martensite to austenite transformation)
$\tau, \tau_{xy}, \tau_{yz}, \tau_{xz}$	-	Shear stress, shear stress in X, Y, Z plane
A, A_f, A_m	-	Area, Cross section area (fiber, matrix)
E	-	Young's modulus
E_M, E_A	-	Young's modulus (martensite, austenite)
E_f, E_m	-	Young's modulus (fiber, matrix)
$G, G_{xy}, G_{yz}, G_{xz}$	-	Shear moduli, Shear moduli in X, Y, Z plane
V_f, V_m	-	Volume ratio (fiber, matrix)
A_s	-	Austenite start temperature
A_f	-	Austenite finish temperature
M_s	-	Martensite start temperature
M_f	-	Martensite finish temperature
ζ	-	Martensite faction
ζ_0	-	Initial martensite faction
ξ_s	-	Stress induced martensite volume fractions
$\dot{\xi}_s$	-	Stress induced martensite volume fractions
ξ_T	-	Temperature-induced martensite volume fractions
ξ_s	-	Indicative of martensite volume fraction at time, t_{n+1} and $\xi_{s,n}$
Ω	-	Phase transformation coefficient
Θ	-	Thermoelastic coefficient
Θ_M, Θ_A	-	Thermoelastic coefficient (martensite, austenite)
T	-	Temperature
T_0	-	Initial temperature
ΔT	-	Changes of temperature applied
d	-	SMA wire diameter (mm)
λ_s	-	Increments of martensite fraction
$\nu, \nu_x, \nu_y, \nu_{xy}$	-	Poisson ratio, poisson ratio in X, Y, Z plane
$\nabla^4(a)$	-	$\frac{\partial^4 a}{\partial x^4} + 2 \frac{\partial^4 a}{\partial x^2 \partial y^2} + \frac{\partial^4 a}{\partial y^4}$

$m \ddot{x}$	-	inertia force
$c \dot{x}$	-	damping force
kx	-	restoring force
$f(t)$	-	excitation force
$[M]$	-	System mass matrix
$[K_s]$	-	System stiffness matrix
$[K_r]$	-	Geometric stiffness matrix due to recovery stress
$[K_T]$	-	Geometric stiffness matrix due to thermal stress
ω	-	Natural frequency
$\{q^0\}$	-	Mode shape of vibration
C_M, C_A	-	Coefficient of martensite, Coefficient of austenite
φ_x, φ_y	-	Rotations due to the transverse displacement w
θ_x, θ_y	-	The independent correction rotations for the rotations φ_x and φ_y , and due to shearing effects
L	-	Length of the element
B	-	Width of the element
H	-	Thickness of the element
ξ, η	-	Natural coordinate
ψ_j^e	-	Lagrange interpolation functions
$\alpha_T, \alpha, \alpha_a, \alpha_m$	-	Thermal expansion coefficient

LIST OF SHORT FORM

APT	-	Active Property Tuning
ASET	-	Active Strain Energy Tuning
CLPT	-	Classical laminate plate theory
CMC	-	Ceramic matrix composite
DSC	-	Differential Scanning Calorimeter
FSDT	-	First-order shear deformation theory
MEKP	-	Methyl Ethyl Ketone Peroxide
MMC	-	Metal matrix composite
NASA	-	National Aeronautics and Space Administration
PE	-	Polyester
PMC	-	Polymer matrix composite
PVDF	-	Polyvinylidene fluoride
SMA(s)	-	Shape memory alloy(s)
TSDT	-	Third-order shear deformation theory
UTM	-	Universal Testing Machine

CHAPTER I

INTRODUCTION

Shape memory alloys (SMAs) nowadays have the potentials to be used in engineering applications and this has drawn deep interests to many researchers to study and develop new technology based on SMAs. SMAs are usually fabricated with other material to form hybrid composite such as epoxy and Kevlar fibers for wing tail (Simpson and Boller, 2002), ferroelectric ceramic for sensing and actuating to reduce structural vibration (Yang, 2000) and glass-reinforced plastic for vibration control (Friend and Matthey, 1999). The selection of the materials depends on the design application. The polymer matrix composite (PMC) that is embedded with the SMAs is primarily fabricated for the purpose of controlling the static and dynamic properties of composite material. In this research, the PMC had been chosen. Different types of fabrications of the SMA based composites are presented in investigating the basic mechanical properties and vibration characteristics, while another composite plate without SMA is fabricated as a reference for analysis. By measuring the mechanical properties and the vibration mode of a clamped cantilever, the influence of all SMAs arrangement and the temperature on the mechanical properties and vibration characteristic can be clearly seen. The unidirectional fine SMA wires are fabricated at 0° , 15° , 30° and 45° for the angle of SMAs to the composite plate. The mechanical properties involved in this study are Young's modulus and stress – strain curve analysis whereas for the vibration characteristic, investigations are carried out on the natural frequency, first mode frequency and damping behavior of the composite plates.

1.1 Background of the project

Shape memory alloys (SMAs) are widely used as sensors, actuators and dampers. The hybrid composites that are embedded with SMAs demonstrate some unique properties or functions such as self strengthening, active modal modification, high damping, damage resistance, control and self healing. As such they can provide tremendous potential in many engineering applications.

The external skin of the high speed aircrafts, rockets and launch vehicles are subjected to intense thermal load due to the aerodynamic heating. The temperature increase of the skin can induce the thermal buckling and dynamic instability. Previous researcher like Park *et al.* (2004) studied on the vibration of thermally post-buckled composite plates embedded with shape memory alloy fibers. They found out that the stiffness of the plate was affected by controlling the volume fraction and the initial strain of the SMA fibers. Further investigation also found that the thermal large deflection decreased by the SMA fibers. Based on the controllability upon natural frequency, critical temperature and thermal deflection, the SMA fibers can be used for the structure application. Park *et al.* (2004) suggested that further study on the SMA fiber angle, volume fraction and initial strain of the SMA should be carried out in order to optimise the design of the composite plate.

Although the shape memory alloys are manufactured and used in the modern smart structure, the major internal problems of shape memory alloy still occur. This is due to the characteristic of shape memory effect which is only limited to motion along the fibers' longitudinal axes (Kelly and Zweben, 2000). The inter-relationship between the angle of SMAs embedded into the composite and the effect of different angle of SMAs embedded into the composite to the vibration characteristic has not been studied and determined by previous researchers. Besides, as reported by NASA, one of the most challenging and comprehensive projects was applying the advanced textile composites to a more complex structures, such as wings and fuselages (Dow and Dexter, 1997). In this research, different types of orientation of the SMA based composites were presented in investigating the basic mechanical properties and vibration characteristics.

In addition, all the parameters suggested by previous researchers (Gilat and Aboudi, 2004 and Patel *et. al.*, 2005) are studied. This includes the study on the effect of the angle orientation of the SMAs to the vibration characteristics of the composite plate.

1.2 Statement of the problem

The characteristics of the SMA wires are not fully provided by the manufacturer which are very important for the design and simulation (Tan *et. al.*, 2007). Differential Scanning Calorimeter (DSC) is one of the famous test to provide the phase transformation temperatures. However due to specimen preparation, DSC may not suitable for the case of pretension (Tan *et. al.*, 2007 and Zak *et. al.*, 2003c). Zak *et. al.* (2003c) had design a jig to replace DSC and with this jig not only the phase transformation temperatures but other parameters such as the coefficient of stress influence and Young's modulus can also be determine. The jig needs to be precisely fabricated and it has the difficulty to control the temperature by using direct current.

In the report made by Kelly and Zweben (2000), the strain of SMA wires is only limited to the longitudinal axes and this caused the orientation of the SMA fabrication to the composite plate becoming an important factor to be considered for optimum design. The effect of the orientations of the SMA wires fabrication to the composite plate was never studied experimentally by any researchers (Huang, 2005, Lu and Weng, 2000, Gao and Yi, 2003, Tan *et. al.*, 2006 and Tan *et. al.*, 2007).

Most of the simulation works that were done do not consider the effect of the heat transfer from the SMA wires to the composite. When heat increases, the standard properties of materials is no longer valid to be used especially for the polymer. Modifications need to be done on the material properties to show the changes due to the temperature increment.

1.3 Statement of hypothesis

The hypotheses of this research are as follows:-

- a. Universal Testing Machine can used to replace Differential Scanning Calorimeter and the jig designed by Zak, *et.al.* (2003c) to determine the phase transformation temperatures and other characteristics of the shape memory alloy.
- b. The standard material properties are not valid for higher temperature application and with some modification, the results gained from simulation will tally with the experimental results.

1.4 Purpose of study

The objectives of this research are as follows:-

- a. To determine the thermo-mechanical properties of shape memory alloy wires.
- b. To study the mechanical properties and vibration characteristics of composite plate embedded with shape memory alloy wires.
- c. To study the effects on angles of shape memory alloys embedded into the composite to the studied parameters (stress, strain, fundamental frequency, damping characteristic and temperature).

1.5 Importance of the study

A new method has been proposed to determine the characteristics of the shape memory alloy wire. This method can determine other parameters that cannot be determined by DSC such as the Young's modulus and the coefficient of stress

influence as well as the phase transformation temperatures. This method can be used by researchers without having the DSC facility to determine the characteristics of SMA wire. The researchers also do not require a precise rig like Zak, *et. al.* (2003c), but can only use the Universal Testing Machine (UTM) to determine the characteristics of SMA wire.

As suggested by the previous researchers, the orientations of the SMA wires to the hybrid composite need to be studied in optimizing the application of the SMA wire to the hybrid composite product (Huang, 2005, Lu and Weng, 2000, Gao and Yi, 2003, Tan *et. al.*, 2006 and Tan *et. al.*, 2007). However, this important factor was never studied by any previous researchers (Tan *et. al.*, 2006). In this research, the orientation of the SMA wires in the composite plate is studied on the fundamental frequency and the damping.

In the simulation, the effect of heat transfer from the SMA wire to the composite structure is taken into account. Some modification on the standard material properties need to be done. These modified material properties have shifted the fundamental frequency with applied temperature.

1.6 Scope of the study

The scopes of the study are as stated below:-

- a. The bonding of the shape memory alloy wires is assumed to be perfectly bonded to the polyester matrix. The *rule of mixture* is applied for this case.
- b. Brinson's model will be used to predict the amount of recovery stresses.
- c. The constitutive and evolutionary equations in the Brinson's model will be used to solve some cases of the shape memory wires.

- d. The study is limited to the natural vibration analysis in the simulation with MATLAB. The damping of the structure is not taken into account.
- e. The first-order shear deformation plate theory is used.
- f. Heat transfer between shape memory alloy and polyester matrix is assumed to have thermal equilibrium. In addition the heat from direct current is also assumed to be transmitted to the composite structure.



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CHAPTER II

LITERATURE REVIEW

Mechanical properties and vibration characteristic are the main keywords for this research. Some theories and formulas related to the above keywords are discussed in this topic.

2.1 Background study

Composite is one of the major engineering materials which are formed from the combination of two or more types of other engineering materials. The major advantages of these composite materials are ease of fabrication, high fracture energy, potential low cost and etc. Typically, composite is formed of matrix and reinforcement materials. In this research, the matrix used is an unsaturated polyester resin and the reinforcement materials are tissue mat glass fiber and Flexinol[®] wire.

There are three major types of composite that commonly been used for engineering applications based on the classification by matrix material. They are Metal Matrix Composite (MMC), Polymer Matrix Composite (PMC) and Ceramic Matrix Composite (CMC). The PMC is chosen for this research due to its advantages of easy to process, least expensive, good mechanical properties, provides good adhesion and normally wet reinforcements well. The matrix material used for this research is Polyester (PE). The unsaturated polyester resin is mixed with the

catalyst of Methyl Ethyl Ketone Peroxide (MEKP) Butanox M50 which is used for general purpose.

Shape memory alloys (SMAs) are defined as a group of metallic materials that able to return to its original shape when subjected to the appropriate thermal procedure. Shape memory alloys can be plastically deformed to some relatively low temperature and upon exposure to some higher temperature, capable to return to original shape (Licker, 2004). In this research, Flexinol[®] wire is chosen for the shape memory alloy and it is manufactured by Dynalloy (www.dynalloy.com). This Flexinol[®] will form like muscles when electrically driven. When electricity flow through Flexinol[®], it will increase the temperature and automatically activated the Flexinol[®]. The detail technical sheet for the Flexinol[®] is given in the **Appendix A1**.

The major advantages of SMA over other smart materials are their high power to volume ratio and large maximum recovery strain (Lee *et al.*, 2000). Recently, much work has been carried out on hybrid structures which incorporated shape memory alloys with other structural or functional materials. Shimamoto *et al* (2004), studied on the enhancement of mechanical strength by shape memory effect in TiNi fiber-reinforced composites. He also studied on the fatigue crack propagation and local crack-tip strain behavior in TiNi shape memory fiber reinforced composite. Furthermore, Marfia (2005), studied on the micro–macro analysis of shape memory alloy composites, while Park *et al.* (2004), analyzed on the vibration of thermally post-buckled composite plates embedded with shape memory alloy fibers. Hybrid composites show some unique properties or functions such as self strengthening, active modal modification, high damping, damage resistance, control and self healing. The hybrid composites provide tremendous potential in many engineering applications.

Since vibration control is one of the most important and common applications of the shape memory alloys, the study on vibration characteristic of SMA becomes extremely important. For the vibration characteristics, the fundamental frequency and the damping characteristic are the main parameters to be considered. The fundamental frequency is the basic of the vibration study. Every system or

component has its own fundamental frequency and the values of the fundamental frequency are different from each other. The damping characteristic is the energy absorption study, which can be harnessed for use in various damping applications. Lammering and Schmidt (2001), addressed that the SMA is an ideal material in improving the damping properties of structures, which are required to load dynamically within the transformation range from austenite to martensite.

2.2 Rule of mixtures

The long parallel fibers which the strain in directional to fibers must be the same in both the matrix and the fiber, $\varepsilon_f = \varepsilon_m = \varepsilon$. For loading in directional to the fibers, the total load, F is the sum of the forces on the fibers, F_f and the matrix, F_m . In term of the stresses, $F_f = \sigma_f A_f$ and $F_m = \sigma_m A_m$, where σ_m and σ_f are the stresses in the matrix and fibers and where A_m and A_f are the cross-sectional areas of the matrix and fibers. The total force, F is the sum of $F_f + F_m$. For the elastic loading, $\sigma = E\varepsilon$, $\sigma_f = E_f \varepsilon_f$, and $\sigma_m = E_m \varepsilon_m$, which $\varepsilon_f = \varepsilon_m = \varepsilon$ and definition of volume ratio of fiber, $V_f = A_f/A$ and volume ratio of matrix, $V_m = A_m/A$, the rule of mixtures can be summarized as,

$E = E_f V_f + E_m V_m$ or can be written as

$$\frac{1}{E} = \frac{V_f}{E_f} + \frac{V_m}{E_m} \quad (2.1)$$

Equation 2.1 (Ye, 2003) is used in the simulation with MATLAB to determine the Young's modulus of the composite plate.

2.3 Models of analysis

There are several models of analysis available for the shape memory alloys, the most common model are the Tanaka model, Brinson model, and Liang and Rogers model. In these models assumed that the total change in stress of SMA consists of three components. The first is the elastic component which is proportional to the Young's modulus E and the change in strain ε . The second is the transformational component which is proportional to the phase transformation coefficient Ω and certain changes in the martensite volume fraction ξ . The third is the thermal component which is proportional to the thermoelastic coefficient Θ and changes in temperature T .

For the Tanaka and the Liang and Rogers models (Liang and Rogers,1990), the transformational component is assumed to be proportional to the total martensite volume fraction ξ . The constitutive equation for these two models can be expressed by equation 2.2 (Tanaka,1986):

$$(\sigma - \sigma_0) = E(\xi)(\varepsilon - \varepsilon_0) + \Omega(\xi)(\xi - \xi_0) + \Theta(\xi)(T - T_0). \quad (2.2)$$

where ε_0 is the initial strain, ξ_0 is the initial martensite volume fraction and T_0 is initial temperature for the shape memory alloys. However, the Brinson model (Brinson, 1993), introduces the stress induced, ξ_S and the temperature-induced, ξ_T martensite volume fractions. The overall martensite volume fraction, ξ during the transformations is defined in this model as the sum of two fractions. The constitutive equation in the Brinson model can be expressed by equation 2.3 and 2.4:

$$(\sigma - \sigma_0) = E(\xi)(\varepsilon - \varepsilon_0) + \Omega(\xi)(\xi_S - \xi_{0S}) + \Theta(\xi)(T - T_0) \quad (2.3)$$

$$\xi = \xi_S + \xi_T. \quad (2.4)$$

Generally, for each model, the Young's modulus, E and the phase transformation coefficient, Ω , as well as the thermoelastic coefficient, Θ , can be

assumed as linear functions of the martensite volume fraction, ξ as shown in equation 2.5:

$$\begin{aligned} E(\xi) &= E_A + \xi(E_M - E_A), & \Omega(\xi) &= -\varepsilon_L E(\xi) \\ \Theta(\xi) &= \Theta_A + \xi(\Theta_M - \Theta_A), & \Theta(\xi) &= \alpha(\xi)E(\xi) \end{aligned} \quad (2.5)$$

where ε_L is the maximum residual strain and α is the thermal expansion coefficient of shape memory alloy. The functions to describe the martensite volume fractions ξ , ξ_S and ξ_T differ for each model. However, the stresses, σ and temperature, T are commonly chosen as functions.

Recently there is another model known as the Auricchio model (Auricchio and Sacco, 2001). Auricchio model mentions that for small deformation regimes to occur, they are based on selection of a set of linear curves explaining stress and strain relationship. Modeling consideration of ξ_s , indicating martensite volume fraction, and applying forward transformation processes (austenite to martensite conversion) and reverse transformation (conversion of martensite into austenite) as shown in equation 2.6 and 2.7.:

(A) Conversion of austenite into martensite (A-S)

$$\dot{\xi}_s = (1 - \xi_s) \frac{|\dot{\sigma}|}{|\sigma| - \sigma_f^{AS}} \quad (2.6)$$

(B) Conversion of martensite into austenite (S-A)

$$\dot{\xi}_s = (\xi_s) \frac{|\dot{\sigma}|}{|\sigma| - \sigma_f^{SA}} \quad (2.7)$$

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