AN IMPLEMENTATION OF PEAK TO AVERAGE POWER RATIO REDUCTION FOR MULTICARRIER SYSTEM (ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING)

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Orthogonal frequency division multiplexing (OFDM) has been becoming more popular modulation technique in the high-speed wireless communication system. It is used especially in Large Term Evaluation technique (LTE) which depended from the fourth-generation (4G) of wireless communication system. OFDM proves high efficiency to transmit data rate as high as 100 Mbps, the capability to combat multipath fading channel and utilization the whole bandwidth. Although, OFDM technology has more advantages, the same time has some obstacles also. The highest Peak to average power ratio (PAPR) considers the main restrict which cause non-linearity at receiving end. Coding, clipping and phase rotation among many PAPR reduction techniques are proposed to overcome this problem. In this project, we investigated the PAPR reduction performance with two PAPR reduction techniques selective mapping (SLM) and partial transmit sequence (PTS). These two PAPR reduction methods consider sub-parts of signal scrambling technique that depend on phase rotation technique in its operation. The simulation results show SLM and PTS methods have improved the PAPR reduction performance with different parameters. Moreover, different kinds of SLM and PTS schemes are also plotted. Generally, PTS and SLM techniques are leading the PAPR reduction better performance. The results are verified using MATLAB software.
ABSTRAK

Frekuensi Ortogon Bahagian Pemultipleksan (OFDM) merupakan teknik modulasi yang semakin popular dalam sistem komunikasi pantas tanpa wayar terutamanya teknik Large Term Evaluation (LTE) yang berasaskan kepada sistem komunikasi tanpa wayar 4G. OFDM terbukti mempunyai keberkesanan yang tinggi untuk menghantarkan data dengan kadar sehingga 100 Mbps, keupayaan untuk melalui saluran pudar pelbagai arah dan penggunaan kesemua lebar jalur. Walaupun teknologi OFDM mempunyai banyak kelebihan, tetapi pada masa yang sama ia juga mempunyai banyak kekangan.

Nisbah Kuasa Puncak ke Kuasa Purata (PAPR) merupakan kekangan utama yang menyebabkan keadaan tak linear pada bahagian penerima. Pengkodan, keratan dan putaran fasa diantara kebanyakan teknik pengurangan PAPR diutarakan untuk mengatasi masalah tersebut. Dalam projek ini, prestasi pengurangan PAPR telah dikaji dengan dua teknik PAPR iaitu selective mapping (SLM) dan Partial Transmit Sequence (PTS). Kedua-dua teknik pengurangan PAPR ini boleh dikatakan pecahan dari teknik gegasan isyarat yang berdasarkan kepada teknik putaran fasa dalam operasinya.

Keputusan simulasi menunjukkan kaedah SLM dan PTS telah meningkatkan prestasi pengurangan PAPR pada parameter yang berlainan. Tambahan lagi, jenis – jenis skim SLM dan PTS yang berlainan juga dipaparkan pada graf. Umumnya, teknik SLM dan PTS menunjukkan pencapaian yang lebih baik untuk pengurangan PAPR. Keputusan dibuktikan menggunakan perisian MATLAB.
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1.1 Introduction

In the early nowadays, third-generation (3G) mobile communication systems have become popular all around the world. However, its services cannot provide a very big dynamic range of data rates, nor can it meet the requirements of a variety of business types. Besides, voice transportation in 3G still relies on circuit switching technology, which is the same method as used in second-generation (2G) communications systems, rather than pure Internet Protocol (IP) approach. Thus, based on consideration listed above, many countries have already carried out research on the next completely evolutionary fourth generation (4G) communication systems which provide a comprehensive and secure IP solution where voice, data, and multimedia can be offered to users at "any time, anywhere" with higher data rates than previous generations [1, 2].
Since bandwidth resource in 4G mobile communications is still scarce, in order to improve spectrum efficiency and achieve as high as 100Mbps wireless transmission rate, it requires more advanced techniques to be employed. The limitation of modulation schemes in existing communication systems has become an obstruction in further increasing the data rate. Hence, next generation mobile communication systems need more sophisticated modulation scheme and information transmission structure.

Orthogonal frequency division multiplexing (OFDM) has therefore been adopted due to their superior performance. It promises to become key high-speed wireless communication technologies and can provide wireless industry evolution from 3G to 4G system.

The growth of mobile wireless communications has been producing the demand for high-speed, efficient and reliable communication over the hostile wireless medium. As a modulation scheme for such applications, Orthogonal Frequency Division Multiplexing (OFDM) possesses several desirable attributes, such as immunity to the inter-symbol interference, robustness with respect to multi-path fading, and ability for high data rates. Thus, OFDM has been proposed in various wireless communication standards such as IEEE802.11a standard for wireless Local Area Networks (WLAN), IEEE802.16a standard for Wireless Metropolitan Area Networks (WMAN), digital audio/video broadcasting, Terrestrial Digital Video Broadcasting (DVB-T), the ETS1 HIPERLAN/2 standard and high-speed cellular data [3]. However, one of the major drawbacks of OFDM system has been its high Peak-to-Average Power Ratio (PAPR).

To reduce the PAPR several techniques have been proposed such as partial transmit sequences (PTS) [4, 5], selective mapping (SLM) [4, 6], clipping [7] clipping and filtering [8], coding [9], tone reservation (TR) and tone injection (TI) [10, 11]. Each of these methods has a different cost for the reduced PAPR. Although some techniques of PAPR reduction have been summarized, it is still necessary to give a comprehensive review of PAPR reductions in terms of transmission power, data rate loss, implementation complexity and BER performance, etc.
1.2 Problem Statement

In OFDM system, its output produces a superposition of multiple sub-carriers. In this situation, some instantaneous power outputs may increase greatly and become so far greater than the mean power of the system with the condition the phases of these carriers are same. This is defined large Peak-to-Average Power Ratio (PAPR). High PAPR is one of the biggest problems in OFDM system. To transmit signals with high PAPR, it requires power amplifiers with very high power scope. These kinds of amplifiers are very expensive and have low efficiency-cost. If the peak power is too high, it could be out of the scope of the linear region of a power amplifier. This gives an increase to non-linear region distortion which that affects and changes the superposition of the signal spectrum resulting degeneration in performance. OFDM system has encountered many restrictions in practical applications if there is no arrangement to PAPR reduction.

One traditional solution to combat high PAPR is to adopt amplifiers to have larger trade-off range. However, these types of amplifiers are generally costly and have low efficiency-cost therefore it is difficult to use in the practical application. On the other hand, many certain algorithms were introduced and they have been proved a superior performance to PAPR reduction.
1.3 Objective

Although OFDM has some advantages that make it suitable for fading channels, it presents a high peak-to-average power ratio (PAPR), which is one of the main drawbacks of OFDM systems. The goal of this project is to analysis and investigates the PAPR reduction performance as the following:

I. To investigate the PAPR reduction performance with two different PAPR reduction techniques: partial transmit sequence (PTS) and selective mapping (SLM).

II. To study corresponding modified algorithms with consideration to balance between the performance and the applicability.

III. To analyze the comparison between two methods (SLM and PTS) and indicate the best method leading to PAPR reduction performance and less computational complexity.

1.4 Project Scope

The scopes of this project will use two techniques based on scrambling i.e. Selected Mapping (SLM) and Partial Transmit Sequence (PTS). Many scopes should be bound in order to make this project achieve the objectives:

I. In SLM method, the input data block is subjected to scramble or (rotation in phase) by U different phase sequence. The resulting data sequence generates a new sequence, among which the one with the lowest PAPR is selected for transmission.

II. In another approach that is PTS, the data block is partitioned into non-overlapping sub-blocks and each sub-block is rotated with a statistically independent rotation factor. The rotation factor, which generates the time domain data with the lowest PAPR amplitude, is also transmitted to the receiver as side information.
III. The PAPR of the original OFDM signal is used as a reference for each comparison.

IV. The complementary cumulative distribution function (CCDF) is employed for comparison the PAPR values with different parameters.

V. 1000 samples of the OFDM signal are conducted for each process compared.

VI. Simulate PAPR reduction performance and compare SLM and PTS by applying Computer simulation Technology Software (MATLAB).
2.1 Introduction

In this chapter, the peak-to-average power ratio (PAPR) reduction for an orthogonal frequency-division multiplexing (OFDM) is presented. It is a method of digital modulation in which a signal is split into several narrowband channels at different frequencies. The technology was first conceived in the 1960s and 1970s, during research into minimizing interference between channels near each other in frequency.

2.2 OFDM System

The descents of OFDM development started in the late 1950’s with the introduction of Frequency Division Multiplexing (FDM) for data communications. In 1966 Chang patented the structure of OFDM and published the concept of using orthogonal overlapping multi-tone signals for data communications. In 1971 Weinstein and Ebert [12] introduced the idea of using a Discrete Fourier Transform (DFT) for
implementation of the generation and reception of OFDM signals, eliminating the requirement for banks of analog subcarrier oscillators. This presented an opportunity for an easy implementation of OFDM, especially with the use of Fast Fourier Transforms (FFT), which are an efficient implementation of the DFT. This suggested that the easiest implementation of OFDM is with the use of Digital Signal Processing (DSP), which can implement FFT algorithms [13]. It is only recently that the advances in integrated circuit technology have made the implementation of OFDM cost effective.

The reliance on DSP prevented the wide spread use of OFDM during the early development of OFDM. It wasn’t until the late 1980’s that work began on the development of OFDM for commercial use [14], with the introduction of the Digital Audio Broadcasting (DAB) system. Cyclic prefix (CP) or cyclic extension was also first introduced by Peled and Ruiz in 1980 [15] for OFDM systems. In 1985, Cimini introduced a pilot-based method to reduce the interference emanating from the multipath and co-channels [16]. In the 1990s, OFDM was exploited for wideband data communications over mobile radio FM channels, high-bit-rate digital subscriber lines (HDSL; 1.6 Mbps), asymmetric digital subscriber lines (ADSL; up to 6 Mbps), very-high-speed digital subscriber lines (VDSL; 100 Mbps), digital audio broadcasting (DAB), and high definition television (HDTV) terrestrial broadcasting [17].

2.3 Principle of OFDM

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier transmission technique, which divides the bandwidth into many carriers; each one is modulated by a low rate data stream. In terms of multiple access technique, OFDM is similar to FDMA in that the multiple user access is achieved by subdividing the available bandwidth into multiple channels that are then allocated to users [18]. However, OFDM uses the spectrum much more efficiently by spacing the channels much closer together. This is achieved by making all the carriers orthogonal to one another, preventing interference between the closely spaced carriers. Figure 2.1 shows the difference between the
conventional non-overlapping multicarrier technique and overlapping multicarrier modulation technique. As shown in Figure 2.1, by using the overlapping multicarrier modulation technique, we save almost 50% of the bandwidth. To realize the overlapping multicarrier technique. However, we need to reduce crosstalk between subcarriers, which means that we want orthogonally between the differently modulated carriers.

Figure 2.1: Concept of OFDM signal orthogonal multicarrier technique and versus conventional multicarrier technique [19].
The orthogonally of the carriers means each carrier has an integer number of cycles over a symbol period. Due to this, the spectrum of each carrier has a null at the center frequency of each of the other carriers in the system. This results in no interference between the carriers, allowing them to be spaced as close as theoretically possible. This overcomes the problem of overhead carrier spacing required in FDMA. Each carrier in an OFDM signal has a very narrow bandwidth (i.e. 1 kHz), thus the resulting symbol rate is low. This results in the signal having a high tolerance to multipath delay spread, as the delay spread must be very long to cause significant inter-symbol interference (e.g. > 500 μsec).

2.4 Operation of OFDM

As stated above OFDM is a multi-carrier modulation technology where every sub-carrier is orthogonal to each other. The "orthogonal" part of the OFDM name indicates that there is a precise mathematical relationship between the frequencies of the carriers in the system. It is possible to arrange the carriers in an OFDM signal so that the sidebands of the individual carriers overlap and the signals can still be received without adjacent carrier’s interference. In order to do this the carriers must be mathematically orthogonal. If the integral of the product of two signals is zero over a time period, then these signals are orthogonal to each other, since the carriers are all sine/cosine wave which satisfy this criterion. If a sine wave of frequency \( m \) is multiplied by a sinusoid (sine or cosine) of a frequency \( n \), then the product is given as:

\[
f(t) = \sin nt \sin mt \]

Where both \( m \) and \( n \) are integers by simple trigonometric relationship, this is equal to a sum of two sinusoids of frequencies \((n-m)\) and \((n+m)\), since these two components are each a sinusoid, the integral is equal to zero after one period. The integral under this product is given as below:
\[ f(t) = \frac{2\pi}{2} \int_0^1 \cos(m-n) - \frac{1}{2} \cos(m+n) wt \]  

(2.2)

So when a sinusoid of frequency \( n \) multiplied by a sinusoid of frequency \( m \), the area under the product is zero. In general, for all integers \( n \) and \( m \), \( \sin mx \), \( \cos nx \), \( \sin nx \) are all orthogonal to each other.

These frequencies are called harmonics. As the sub-carriers are orthogonal, the spectrum of each carrier has a null at the center frequency of each of the other carriers in the system. This results in no interference between the carriers, allowing them to be spaced as close as theoretically possible. The orthogonality allows simultaneous transmission on a lot of sub-carriers in a tight frequency space without interference from each other. So on the receiver side easily we can extract the individual sub-carriers. But in traditional FDM systems overlapping of carriers are not possible, rather a guard band is provided between each carrier to avoid inter-carrier interference.

Figure 2.2: Multi-carriers of OFDM signal [20].
Figure 2.3 shown typical OFDM systems. At a sender, first of all, input binary serial data stream is first processed by channel encoder, constellation mapping and serial to parallel (S/P) conversion. A single signal is divided into $N$ parallel routes after $N$-point inverse fast Fourier transform (IFFT). Each orthogonal sub-carrier is modulated by one of the $N$ data routes independently. By the mean, the $N$ parallel points constitute one OFDM symbol. Next, convert parallel data sequence to serial sequence and then applying cyclic prefix (CP) by copy the last samples of one symbol to the front as. Finally, processing, digital to analog (D/A) conversion and radio frequency (RF) modulation then transmit the signal.

Figure 2.3: OFDM system block diagram
At the receiver end, firstly, demodulate received signals then demodulated signals are converted from analog to digital (A/D) converter, sample output and take time estimation to find an initial position of OFDM symbol. The CP added in transmission process is removed and N-Points fast Fourier transforms (FFT) transformation will be conducted on the sample points to recover the data in the frequency domain. Finally, the output of baseband demodulation is passed to the channel decoder to recover the original data.

### 2.4.1 Multicarrier System and Single Carrier

#### 2.4.1.1 Single Carrier System Basic Structure

In the single carrier system, the signals are pulse-formed through transmitter filter \( g(t) \) before being applied for the multi-path channel. The incoming signal at the receiver is passed over receiving match filter \( g^*(-t) \) to make as large the signal-to-noise ratio (SNR). Figure 2.4 shows the single carrier system basic structure diagram.

![Figure 2.4: Single carrier system basic structures.](image-url)
2.4.1.2 Multicarrier System Basic Structure

In a multicarrier system, input signals which are divided by a multiplexer are applied to pulse-formed $g(t)$ filters before being transmitted through multipath environment. Correspondingly, the receiving ends consist of $N$ parallel paths. Each one is passed through a respective match filter $g^*(-t)$ to realize maximum SNR. The basic structure diagram of a single the multi-carrier system is shown in Figure 2.5. In a classical wireless communication model, the transmitted signal arrives at the receiver via various paths. Thus, extracting the original signal at the receiving end becomes extremely difficult. If the signal is transmitted at time intervals $T$, then the parameter concerning the multipath channel is the delay $\tau_{max}$ of the longest path with respect to the earliest path. The received signal can be theoretically influenced by $\tau_{max} / T$ previous signals, which must be considered seriously by receiver [20].

Figure 2.5: Multicarrier system basic structure.

In a single carrier system, it is assumed that transmission rate $R = 1/T$ and maximum channel delay is $\tau_{max}$. In the multi-carrier system, the original data stream of
rate R is multiplexed into N parallel data streams with rate \( Rmc = 1/Tmc = R/N \). Each of the sub streams is modulated with a different sub-carriers frequency and all the data streams are transmitted in the same band. In this case, the ISI of each sub-system reduces to \( \tau_{max}/Tmc = \tau_{max}/N\cdot T \). As the value of N increases, inter-symbol interference (ISI) becoming decreases.

In a single carrier system, fading or interference can make the entire link fail. However, in the multi-carrier system, only a small part of sub-carriers will be affected. Error correction coding methods can be employed to correct the errors which were happening in sub-carriers. OFDM is a special form of multicarrier modulation (MCM), in which a signal is transmitted over a number of lower rate sub-carriers.

### 2.4.2 Cyclic Prefix of OFDM System

In OFDM system, the use of Cyclic Prefix (CP), can ensure signals Orthogonality even when they travel multi-path channels \([21]\). To avoid ISI, the case; \( TG > Tmax \) should be satisfied, where the \( TG \) is the CP length, while the maximum delay spread is \( Tmax \) \([22]\).

Figure 2.6 shown, the copy of the later part is CP for OFDM symbol moved into the front of the symbol. On the assumption that, \( NG \) is an extended OFDM symbol number, consequently the \( T + TG \) is practical OFDM symbol period, where FFT transform cycle is \( T \), and the guard interval length is \( TG \), while inserting toward suppress ISI through multipath distortion. Can be included CP into OFDM symbol showing in below equation:

\[
S' n = S'(t)\big|_{t=nT_G} = \sum_{i=0}^{n-1} d_i \cdot e^{j2\pi n \frac{t}{N}}, n = NG, \cdots, -1, 0, \cdots, N-1
\]  
\hspace{0.5cm} (2-3)
Where $d_i$ is the complex modulation symbols, the process between the channel and signal changes from linear convolution to cyclic convolution when is used the CP in OFDM system. The linear weighing will be used in the frequency domain. These one changes sidestep inter-symbol interference when guarantee orthogonality between the sub-carriers in all time.

![OFDM symbols with added cyclic prefix](image)

**Figure 2.6: OFDM symbols with added cyclic prefix.**

### 2.4.3 OFDM Advantage and Disadvantage

OFDM is vastly used for communication method working within combating multi-path distortion. The OFDM applications have been extended from high radio frequency (HF) of communication to telephone networks, digital audio broadcasting and digital television terrestrial broadcasting. The OFDM method of multi-carrier modulation has several advantages compared with single carrier modulation.

**I. Advantages:**

1. Makes efficient use of the spectrum by allowing overlap.
2. By dividing the channel into the narrowband flat fading sub channels, OFDM is more resistant to frequency selective fading than single carrier systems are.
3. Eliminates ISI through using a cyclic prefix.
4. Using proper channel coding and interleaving one can recover symbols lost due to the frequency selectivity of the channel.

5. Channel equalization becomes simpler than by using adaptive equalization techniques with single carrier systems.

6. It is possible to use maximum likelihood decoding with reasonable complexity.

7. OFDM is computationally efficient by using FFT techniques to implement the modulation and demodulation functions.

8. Is less sensitive to sample timing offsets than single carrier systems are.

9. Provides good protection against co-channel interference and impulsive parasitic noise channel equalization becomes simpler than by using adaptive equalization techniques with single carrier systems.

II. Disadvantages:

1. The OFDM signal has a noise like amplitude with a very large dynamic range. Therefore, it requires RF power amplifiers with a high peak to average power ratio.

2. It is more sensitive to carrier frequency offset and drift than single carrier systems.

2.5 Peaks-to-Average Power Ratio in OFDM System

The instantaneous output of an OFDM system often has large fluctuations compared to traditional single-carrier systems. This requires that system devices, such as power amplifiers, A/D converters and D/A converters, must have large linear dynamic ranges. If this is not satisfied, a series of undesirable interference is encountered when the peak signal goes into the non-linear region of devices at the transmitter, such as high out of band radiation and inter-modulation distortion. PAPR reduction techniques are therefore of great importance for OFDM systems [23].
2.5.1 PAPR Definition

Theoretically, large peaks in OFDM system can be expressed as Peak-to-Average Power Ratio, or referred to as PAPR, in some literatures, also written as PAR. It is usually defined as [24]:

\[
PAPR = \frac{P_{\text{peak}}}{P_{\text{average}}} = 10 \log_{10} \left( \frac{\max |x_n|^2}{E[|x_n|^2]} \right)
\]

(2.4)

Where \( E[\cdot] \) denotes the expected value, \( x_n \) represents the transmitted OFDM signals which are obtained by taking IFFT operation on modulated input symbols \( X_k \). Mathematically, \( x_n \) is expressed as:

\[
x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k w_n^k \quad (2.5)
\]

In an OFDM system when the carriers are divided into \( N \) sub-carriers, the peak power at the received signal equal \( N \) times of the average power with respect to the same phase values. Theoretically the PAPR of received signal will be maximum at PAPR (dB) = 10\( \log N \). For example, if we have 16 sub-carriers in the system, the PAPR will reach a maximum value 12 dB, the value considers the theoretical hypothesis. However, in reality the probability of PAPR to reach this maximum is very low.

The figure 2.7 illustrates the amplitude (dB) of the OFDM system with 16 sub-carriers. It is clearly seen from the figure the maximum amplitude of the OFDM signals is less than the upper value (16) also the PAPR is lower than the maximum theoretical value 12dB.
Figure 2.7: An OFDM signal waveform in the time domain [20].

The special case happens when signal subcarriers are modulated by symbols which have the same initial phase. For example, we have an input binary sequence contains ‘1’ for the whole sequence. After PSK constellation mapping (modulation) and IFFT operation, the instant power reaches its theoretical maximum. Figure 2.8 shows the result when input binary sequence contains 16 ‘1’, denoted by [11111111111111].

In this scenario, the maximum amplitude of the OFDM signal reaches the value of 16 and a PAPR can be calculated from $=10\log N$ so that it is reached to 12dB.
Figure 2.8: OFDM when subcarriers are modulated by the same phases [20].

From Figure 2.8, we can conclude the amplitude (dB) of OFDM signal reaches the peak value when the input sequence has very large of consistency. At the same time, the maximum PAPR value will be reached as well as the same condition.

2.5.2 Criteria for PAPR Reduction

In this section, the criteria are defined for the techniques used for the PAPR, and BER reduction. There are six different techniques and some hybrid techniques (in which two techniques from these six techniques are combined) are used for PAPR and BER reduction. But still no one gave acceptable results. For an
acceptable technique, that technique must reduce the PAPR and BER largely plus the following performance factors must be considered for OFDM based system.

2.5.2.1 Capability of PAPR Reduction

The primary factor of selecting PAPR reduction technique is the capability of PAPR reduction. A technique is considered best if it reduces PAPR largely. Out Of Band radiation (OOB) and In Band (IB) distortion are few considerable factors for selecting a technique.

2.5.2.2 Low Average Power

A technique must reduce PAPR as well as the average power of the signal not increased from an acceptable region. If so it will require a large linear region for operation in High Power Amplifier (HPA), which will increase the BER rate of the OFDM system.

2.5.2.3 Low Complexity

The technique should also not increase the complexity of the overall system. Complexity includes both time and hardware requirements for implementation of the system.
2.5.2.4 Less Bandwidth Expansion

Some techniques such as scrambling techniques need side information, which increase the bandwidth usage. Some coding techniques also expand the bandwidth due to code rate generation. A technique must not increase the bandwidth in value which causes degradation in the throughput.

2.5.2.5 Less BER Performance Degradation

The main goal of the PAPR reduction technique is to gain better performance, including BER as compared to the conventional OFDM system.

2.5.2.6 Less Additional Power Need

The technique required no additional power for PAPR reduction, as it will degrade the BER performance of the system plus power efficiency is the main goal of wireless based systems.

2.5.2.7 Good Spectral Efficiency

If a technique destroys the ICI or immunity to multipath fading or some other advantage related to spectrum should not be considered a good PAPR reduction technique.
2.6 Probability Distribution Function of PAPR

In accordance to the centric limit theorem, for sub-carriers great number in multi-carrier signal, the sample values real and imaginary part in time-domain will obey Gaussian distribution with mean 0 and variance of 0.5 average values. For that, the multi-carrier signals amplitude follows Rayleigh distribution with zero average and \( N \) times difference, and the one complex sinusoid difference \[25\]. Its power value comply the \( \chi^2 \) distribution with zero average and freedom 2 degrees. The Cumulative Distribution Function (CDF) written as follows:

\[
F(z) = 1 - \exp(-z)
\]

\((2-6)\)

On the assumption that the different sub-channels, sampling values are reciprocally freelance, and free for oversampling operation, the PAPR probability distribution function less than the confirmed threshold value, then equation written as follows:

\[
P(\text{PAPR} < z) = F(z)^N = (1 - \exp(-z))^N
\]

\((2-7)\)

PAPR in reality it favored to take into account the probability exceeding and the index measuring the threshold for a representation of the distribution of PAPR. Thus, it can be called as a “Complementary Cumulative Distribution Function” (CCDF), and is written as:

\[
P(\text{PAPR} \geq z) = 1 - P(\text{PAPR} \leq z) = 1 - F(z)^N = 1 - (1 - \exp(-z))^N
\]

\((2-8)\)
2.7 Studying of PAPR Reduction Techniques

There are many different algorithms that have been proposed to solve the high PAPR problem of OFDM system. These reduction solutions can be roughly divided into three categories:

2.7.1 Signal Scrambling Methods

The essential precept of this method is to scramble every OFDM signal with various scrambling sequences while it’s smallest for transmission of PAPR value. Obviously, this method does not ensure that to decrease the value of PAPR lower to a certain threshold, but it can decrease the high PAPR appearance probability to a great range. Thus, this approach type implicates, firstly the Selective Mapping SLM and then the Partial Transmit Sequences PTS. Applying scrambling rotation of SLM technique to all sub-carriers autonomously, but scrambling of PTS technique applied to the part of sub-carriers. The two techniques can be considered to apply each scenario without constraint within sub-carriers number and modulation type. Nevertheless, in order to indicate the successful recovery at the receiver, additional information is needed. That leads to the use of low-bandwidth and high-complexity hardware implementation.

2.7.1.1 SLM Selective Mapping

A set of V dissimilar data blocks is created at the receiver side which consist identical information and a block with minimum PAPR is selected for transmission. This technique is used in SLM which is shown in Figure 2.9, multiplied with the dissimilar phase sequence $V$ of length $N$. ($v=1, 2, \ldots V$) results an altered data block. Let’s an
altered data block for \(v^{th}\) phase, where \(v = 1, 2 \ldots V\) with all data blocks, the data block with minimum PAPR should be selected for transmission. Side information about the selected phase (Vth phase) must be sent to the receiver for decoding the received signal.

![Block diagram of PTS based OFDM system](image)

Figure 2.9: SLM technique for PAPR reduction.

### 2.7.1.2 PTS (Partial Transmit Sequence)

In the literature PTS is another popular PAPR reduction technique. It partitioned the input data blocks into dissimilar data blocks of \(N\) symbols, then weighted these subcarriers dissimilar blocks with phase sequence. The block diagram of the PTS based OFDM system is shown in Figure 2.10.
REFERENCES


