Feedback Control Schemes for Gantry Crane System incorporating Payload

M.A. Zawawi, W.M.S. Wan Zamani, M.A. Ahmad, M.S. Saealal, R.E. Samin
Faculty of Electrical & Electronics Engineering
Universiti Malaysia Pahang
Pekan, 26600, Pahang, Malaysia

Abstract — **This paper presents theoretical investigations into the dynamic characterisation of a two dimensional gantry crane system. A dynamic model of the system is developed using Euler-Langrange formulation. Simulation exercises are performed in Matlab with three different control strategies; LQR, DFS and PD controllers and then the results are compared with uncontrolled system. To study the effects of payload weight on the response of the gantry crane system, the results are evaluated with different payload weight in the algorithm. Results achieved from simulation work are shown in time and frequency domains. Performance of the feedback controllers in minimizing the sway angle is examined in terms of time response specifications and magnitude of sway. Finally, a comparative assessment of different payload weight to the system performance is assessed and discussed.**

Keywords-gantry crane system, LQR controller, DFS controller, PD controller

I. INTRODUCTION

The main purpose of controlling a gantry crane is transporting the load as fast as possible without causing any excessive swing at the final position. However, most of the common gantry crane results in a swing motion when payload is suddenly stopped after a fast motion [1]. The swing motion can be reduced but will be time consuming. Moreover, the gantry crane needs a skilful operator to control manually based on his or her experiences to stop the swing immediately at the right position. The failure of controlling crane also might cause accident and may harm people and the surrounding.

Various attempts in controlling gantry cranes system based on open loop system were proposed. For example, open loop time optimal strategies were applied to the crane by many researchers such as discussed in [2,3]. They came out with poor results because open loop strategy is sensitive to the system parameters (e.g. rope length) and could not compensate for wind disturbances. Another open loop control strategy is input shaping [4,5,6]. Input shaping is implemented in real time by convolving the command signal with an impulse sequence. The process has the effect of placing zeros at the locations of the flexible poles of the original system. An IIR filtering technique related to input shaping has been proposed for controlling suspended payloads [7]. Input shaping has been shown to be effective for controlling oscillation of gantry cranes especially for load without hoisting in the process [8,9]. Experimental results also indicate that shaped commands can be of benefit when the load is elevated during the motion [10].

As compared to input shaping method and as reported by Omar [11], feedback controllers family which are less susceptible to external disturbances and parameter changes have also been implemented for controlling the gantry crane system. In his work, Omar has discussed the performances of proportional derivative (PD) controller to steer the cart position and to minimize the swing angle of the load/rope. Similarly, Wahyudi and Jalani [12] have proposed a fuzzy-based controller for a gantry crane system for both position and sway movement. Even both conventional and intelligent controllers give a promising results for this system, nevertheless, feedback controllers require complicated sensors at the cart (position) and load (sway movement) and these are becoming harder for real application due to hoisting mechanism that surrounding gantry crane application.

This paper presents an investigation into the development of techniques for anti-swaying of a gantry crane system for varying payload weight at rope tip. Three feedback control strategies; Linear Quadratic Regulator (LQR), Delayed Feedback Signal (DFS) and Proportional-Derivative (PD) controllers are investigated. A simulation environment is developed within Simulink and Matlab for evaluation of performance of the control schemes. Simulation results of the response of the gantry crane with the controllers are presented in time and frequency domains. The performances of the control schemes are examined in terms of swing angle of the rope for multiple payload weight and its respective power spectral density as a comparison between LQR-DFS-PD controllers. Finally, a comparative assessment of the control techniques is presented and discussed.

II. GANTRY CRANE SYSTEM

The two-dimensional gantry crane system with its payload considered in this work is shown in Fig. 1, where x is the horizontal position of the cart, l is the length of the rope, θ is the sway angle of the rope, M and m is the mass of the cart and payload respectively. In this simulation, the cart and payload can be considered as point masses and are assumed to move in two-dimensional, x-y plane. The tension force that may cause the hoisting rope elongate is also ignored. In this study, which consists of results presentation on simulation work via Matlab and simulink software, the length of the cart, l = 1.0 m, M = 2.49 kg, m = (1.0 kg, 2.0 kg and 3.0 kg) and g = 9.81 m/s^2 is considered.

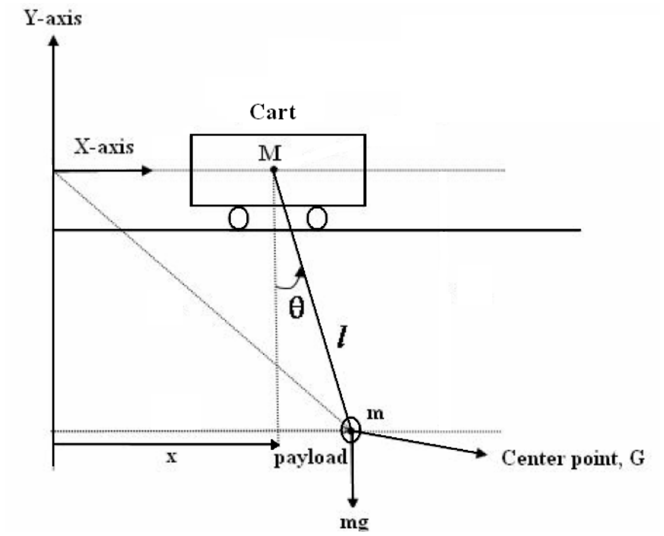

Figure 1. Sketch of the 2D-Gantry Crane System

III. DYNAMIC MODELING OF THE GANTRY CRANE SYSTEM

As a beginning to controlling the sway movement of the gantry crane system in both simulation or/and real environments, it is compulsory to get its dynamic model. After the mathematical equation has been developed, then the feedback control system can be implemented before further assessment can take place. One common method to formulate the dynamic equation of a mechanical system such as gantry crane is by using the Euler-Lagrange formulation. Considering the motion of the gantry crane system on a two dimensional plane, the kinetic energy of the system can thus be formulated as

$$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m(\dot{x}^2 + \dot{l}^2 + l^2\dot{\theta}^2 + 2\dot{x}\dot{l}\sin\theta + 2\dot{x}l\dot{\theta}\cos\theta) \quad (1)$$

The potential energy of the beam can be formulated as

$$U = -mgl\cos\theta \quad (2)$$

To obtain a closed-form dynamic model of the gantry crane, the energy expressions in (1) and (2) are used to formulate the Lagrangian $L = T - U$. Let the generalized forces corresponding to the generalized displacements $\overline{q} = \{x, \theta\}$ be $\overline{F} = \{F_x, 0\}$. Using Lagrangian's equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = F_j \qquad j = 1,2 \quad (3)$$

the equation of motion is obtained as below,

$$F_x = (M+m)\ddot{x} + ml(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) + 2m\dot{l}\dot{\theta}\cos\theta + m\ddot{l}\sin\theta \quad (4)$$

$$l\ddot{\theta} + 2\dot{l}\dot{\theta} + \ddot{x}\cos\theta + g\sin\theta = 0 \quad (5)$$

For a single rope 2D-gantry crane system with payload, the dynamic model can be shown in state space from as below

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (6)$$

where $\boldsymbol{x} = \begin{bmatrix} x & \dot{x} & \theta & \dot{\theta} \end{bmatrix}^T$ and the system matrix, A and input matrix, B are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mg}{M} & 0 & 0 \\ 0 & -\frac{(M+m)g}{Ml} & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ -\frac{1}{Ml} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{D} = \begin{bmatrix} 0 \end{bmatrix} \quad (7)$$

IV. CONTROLLER DESIGN

In this section, three feedback control strategies (LQR, DFS and PD controllers) are proposed and described as necessary. The application of the above stated feedback controllers is only concern on the reduction of the sway angle movement while the cart position is disregarded in this comparison study. All controllers are implemented and analyzed separately before their overall performances are discussed and compared.

A. Linear Quadratic Regulator(LQR) Controller

The first feedback controller used in this paper to minimize the sway angle movement of the rope is LQR controller. As a regulator, LQR controller is used to eliminate the disturbance effects and to change the set-point from time to time in order to achieve the desired output value of the rope movement, which is zero degree (vertically down at natural position). From design perspective, the equation of motion is linearized to get a linear model in state-space form. For an LTI system,

$$\dot{x} = Ax + Bu$$

the technique involves choosing a control law $u = \psi(x)$ which stabilizes the origin (i.e., regulates x to zero) while minimizing the quadratic cost function

$$J = \int_0^\infty x(t)^T Q x(t) + u(t)^T R u(t) dt \quad (8)$$

where $Q = Q^T \geq 0$ and $R = R^T > 0$. The term "linear-quadratic" refers to the linear system dynamics and the quadratic cost function.

In (8), the matrices Q and R are called the state and control penalty matrices, respectively. If the components of matrix Q are chosen relatively bigger than those of R, then deviations of x from zero will be penalized heavily relative to deviations of u from zero. Reversely, if the components of R are larger relative to those of matrix Q, then control effort is more costly and the state will converge to zero at a very slow rate.

Due to Kalman theorem, the control law which minimizes J always takes the form $u = \psi(x) = -Kx$, and the optimal regulator for a LTI system with respect to the quadratic cost function above is always a linear control law. With this observation in mind, the closed-loop system takes the form

$$\dot{x} = (A - BK)x \tag{9}$$

and the cost function, J takes the form

$$J = \int_0^\infty x(t)^T Qx(t) + (-Kx(t))^T R(-Kx(t))dt \tag{10}$$

$$J = \int_0^\infty x(t)^T (Q + K^T RK)x(t)dt$$

Assuming that the closed-loop system is internally stable, which is a fundamental requirement for any feedback controller, the commonly used Kalman's theorem allows the computation value of the cost function for a given control gain matrix K by using Matlab software.

B. Delayed Feedback Signal (DFS) Controller

The next feedback controller type applied to control sway angle of gantry crane system is DFS controller which is calculated based on the delayed position feedback approach described in (11) and displayed by the block diagram shown in Fig. 2.

$$u(t) = k(y(t) - y(t-\tau)) \tag{11}$$

Substituting (11) into (6) and taking the Laplace transform gives

$$sIx(s) = Ax(s) - kBC(1 - e^{-s\tau})x(s) \tag{12}$$

The stability of the system given in (12) depends on the roots of the characteristic equation

$$\Delta(s,\tau) = | sI - A + kBC(1 - e^{-s\tau}) | = 0 \tag{13}$$

Equation (13) is transcendental and results in an infinite number of characteristic roots [13]. Several approaches dealing with solving retarded differential equations have been widely explored. In this study, the approach described in [14] will be used on determining the critical values of the time delay τ that result in characteristic roots of crossing the imaginary axes. This approach suggests that (13) can be written in the form

$$\Delta(s,\tau) = P(s) + Q(s)e^{-s\tau} \tag{14}$$

$P(s)$ and $Q(s)$ are polynomials in s with real coefficients and deg($P(s)$)=n>deg($Q(s)$) where n is the order of the system. To find the critical time delay, τ that leads to marginal stability, the characteristic equation is assessed at $s=j\omega$. By isolating the polynomials $P(s)$ and $Q(s)$ into real and imaginary parts and replacing $e^{-j\omega\tau}$ by $\cos(\omega\tau)-j\sin(\omega\tau)$, equation (14) can be written as

$$\begin{aligned}\Delta(j\omega,\tau) &= P_R(\omega) + jP_I(\omega) \\ &+ (Q_R(\omega) + jQ_I(\omega))(\cos(\omega\tau) - j\sin(\omega\tau))\end{aligned} \tag{15}$$

The characteristic equation $\Delta(s,\tau) = 0$ has roots on the imaginary axis for some values of $\tau \geq 0$ if (15) has positive real roots. A solution of $\Delta(j\omega,\tau) = 0$ exists if the magnitude $|\Delta(j\omega,\tau)| = 0$. Taking the square of the magnitude of $\Delta(j\omega,\tau)$ and setting it to zero lead to the following equation

$$P_R^2 + P_I^2 - (Q_R^2 + Q_I^2) = 0 \tag{16}$$

By setting the real and imaginary parts of (16) to zero, the equation is rearranged as below

$$\begin{bmatrix} Q_R & Q_I \\ Q_I & -Q_R \end{bmatrix} \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix} = \begin{bmatrix} -P_R \\ -P_I \end{bmatrix}, \tag{17}$$

where $\beta=\omega\tau$. Solving for $\sin\beta$ and $\cos\beta$ gives

$$\sin(\beta) = \frac{(-P_R Q_I + P_I Q_R)}{(Q_R^2 + Q_I^2)} \text{ and } \cos(\beta) = \frac{(-P_R Q_R - P_I Q_I)}{(Q_R^2 + Q_I^2)}$$

The critical values of time delay can be determined as follows:

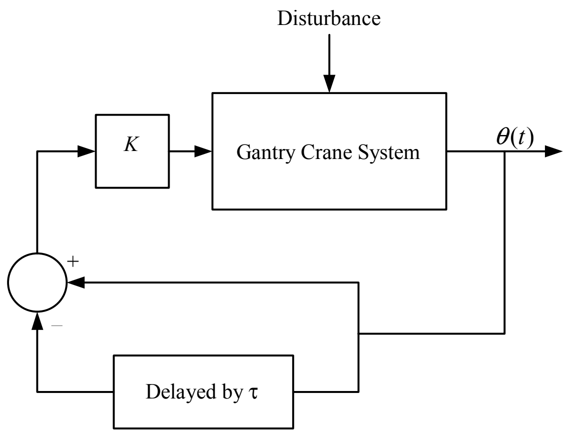

Figure 2. Block diagram of DFS controller

if a positive root of (16) exists, the corresponding time delay τ can be found by

$$\tau_k = \frac{\beta}{\omega} + \frac{2k\pi}{\omega} \tag{18}$$

where $\beta \in [0\ 2\pi]$.

C. *Proportional-Derivative (PD) Controller*

The last feedback controller to be implemented in this study for sway angle reduction is PD controller. A PD controller consists of proportional and derivative components which require critical gains selection to avoid unstable (due to very large proportional gain), less sensitive and poor transient responses. A block diagram of the PD controller is shown in Fig. 3, where K_p and K_d are the proportional and derivative gains, respectively. Meanwhile x , $\dot{x}$, θ and $\dot{\theta}$ represent cart position, cart velocity, sway angle and sway velocity of the hoisting rope, respectively. The control signal *u(t)* in Fig. 3 can be written as,

$$u(t) = K_p \theta(t) + K_d \frac{d}{dt}\theta(t) \tag{19}$$

In this study, the Ziegler-Nichols approach is employed to design the PD controller. The value of K_p and K_d were heuristically but crucially selected to achieve a satisfactory time and frequency responses. For this study, the best values were recorded as, K_p = 150, 140 and 130 and K_d = 60, 90, 150; for respective load weight (*m* = 1 kg, 2 kg, and 3 kg). An appropriate gain values for K_p and K_d will result in stable system and good transient response.

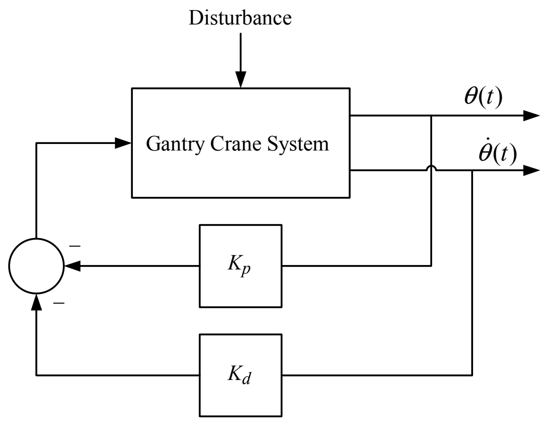

Figure 3. Block diagram of PD controller

V. Implementation and Results

The main concerns in the simulation work presented in this paper is to study LQR, DFS and PD feedback control schemes in improving the transient responses of the sway angle with the present of different load weights. The authors understand that the implementation of all stated controllers is not new in this field, in simulation and hardware experimentation environments. However, in real industrial applications, the effect of the payload being placed at the end tip of the rope cannot simply be neglected. Thus, it is necessary to put the case into further study so that sufficient information can be gathered to improve present knowledge on feedback controller performances for gantry crane system.

In this part, the individual results obtained from each feedback controller are merged together in similar graphs (according to its respective load) and then they are compared with each other and the uncontrolled responses. The rope sway angle for three different payload (m = 1.0 kg, 2.0 kg and 3.0 kg) with DFS, LQR and PD controllers are shown in Fig. 4, Fig. 5 and Fig. 6, respectively. The sway angle responses with controllers are compared with the uncontrolled system to show the controllers performances.

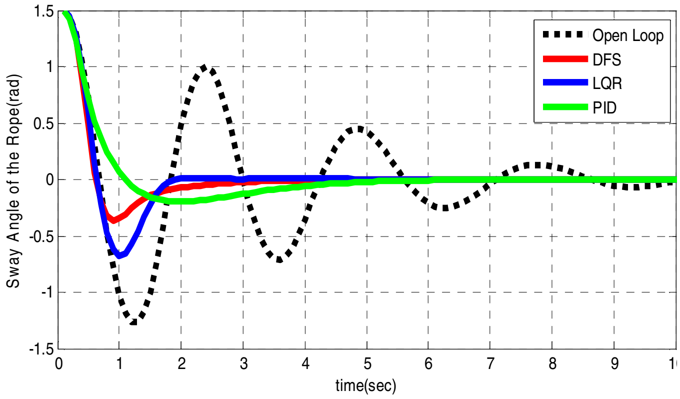

Figure 4. Rope sway angle with payload, m = 1.0 kg

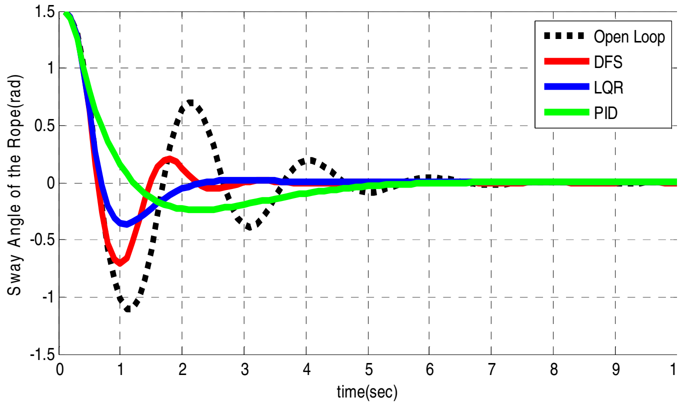

Figure 5. Rope sway angle with payload, m = 2.0 kg

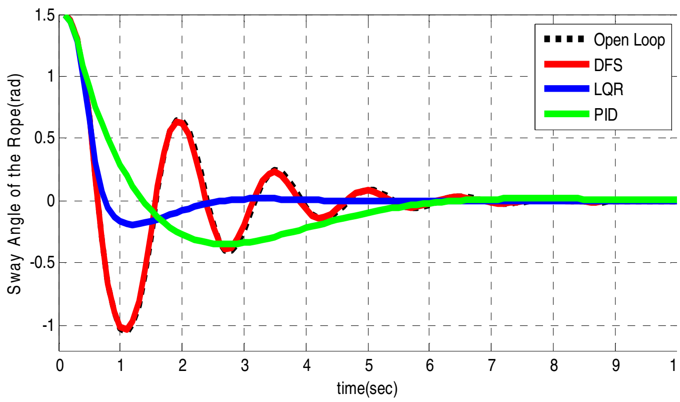

Figure 6. Rope sway angle with payload, m = 3.0 kg

Besides that, Fig. 7, Fig. 8 and Fig. 9 present the PSD of the rope for the respective payload weight. For more details comparative assessment, the sway angle responses in term of settling time, rise time and overshoot, considering three different payload values, under three different controllers are also shown in Table 1 for controller performance comparison.

Fig. 4 shows the sway angle of the rope with a 1 kg payload being attached to the tip of the rope. The sway angle of the rope for the system under three different controllers are compared and studied. For the system without controller, the rope remains swaying at significant amplitude up to 8 seconds after the first movement. But with the implementation of feedback controllers, the oscillation settles down after 4 seconds. Among the three controllers under study, PD controller becomes the best option for a minimum overshoot if compared to DFS and LQR controllers. However, this advantage is back drawn by a slower settling time. On the other hand, LQR shows the fastest settling time with slightly larger overshoot during transient response. With a 0.6 rad. overshoot, this is twice as large as the one produced by PD controller. But if compared to uncontrolled system performance, the overshoot is at acceptable magnitude.

For the next comparison of sway angle performance, the payload is extended to 2 kg load at the tip of the rope. Three similar controllers as in previous discussion are again applied to the system to study the performance of the controller with a changing in payload weight. As shown in Fig. 5, the sway angle under controlled by DFS controller is having an expansion in overshoot magnitude (0.7 rad.) twice as large as for 1 kg load (0.4 rad.). However, a slight improvement for sway angle with LQR controller, where the overshoot is now reduced from 0.7 rad. to 0.4 rad. with a relatively similar settling time. The system under PD does not show any significant changes in term of overshoot and settling time.

When the payload is further increased to 3 kg in weight, as shown in Fig. 6, DFS controller fails to improve the system performance, where the sway angle just shows the same performance as the one without any controller. As for LQR controller, the sway angle performance remains the same as for lighter payload, where the overshoot is at minimal magnitude (0.2 rad.) and settling time at 2.5 seconds. On the other hand, PD controller gives a slower settling time which at 6 seconds, nevertheless, the overshoot remains at less than 0.4 rad.

As an overall comparative assessment, with DFS controller, the sway angle of the rope is getting worse along with the increment of payload weight, in terms of overshoot and settling time. This is proven better by LQR controller which gives a smaller overshoot with the addition of payload but with slightly slower settling time. Finally for PD controller, the sway angle performance shows no significant changes in time domain with the additional payload being attached to the rope tips.

Next on Fig. 7 until Fig. 9, the paper discuss on the system performance with three different controllers in frequency domain. When a payload of 1 kg weight is placed at the end tip of the rope, the magnitude of sway is -80 dB for the uncontrolled system. By applying a feedback control to the system, the sway magnitude is reduced to -118 db for all three controllers. A suppression of 38 dB of sway can be achieved by applying a feedback controller onto the system.

The study is further extended by increasing the payload weight from 1 kg to 2 kg (refer Fig. 8) and later to 3 kg (refer Fig. 9). LQR gives the largest sway suppression if compared to PD and DFS controllers for both weights. The reduction in magnitude of sway with implementation of feedback controllers has highlighted the importance of controllers in ensuring the safe handling of a gantry crane system for various payload weights. For better overview on overall feedback controller performances for gantry crane system discussed previously, the details on the system performance in time and frequency domains for different payload weights are shown in Table I.

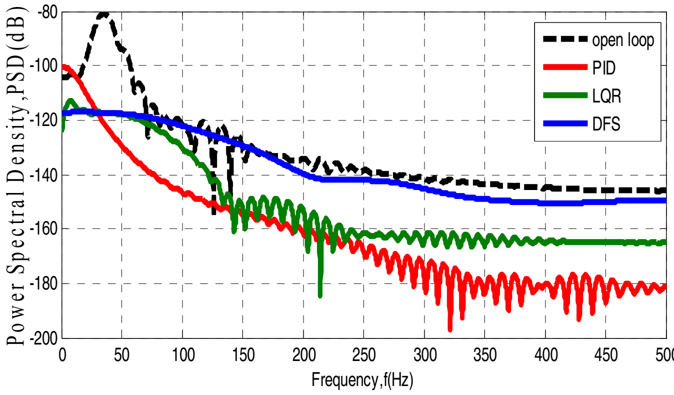

Figure 7. Power Spectral Density with payload, m = 1.0 kg

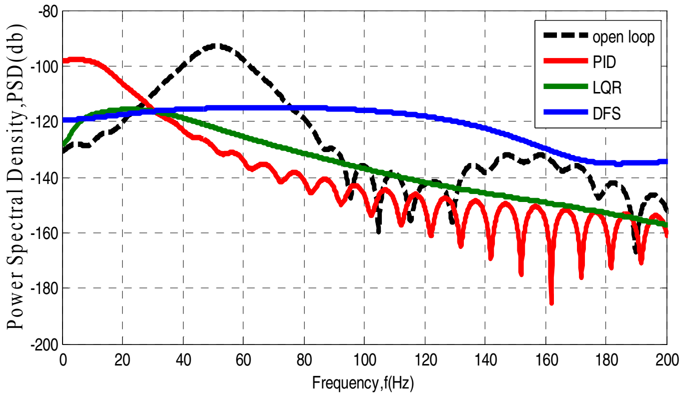

Figure 8. Power Spectral Density with payload, m = 2.0 kg

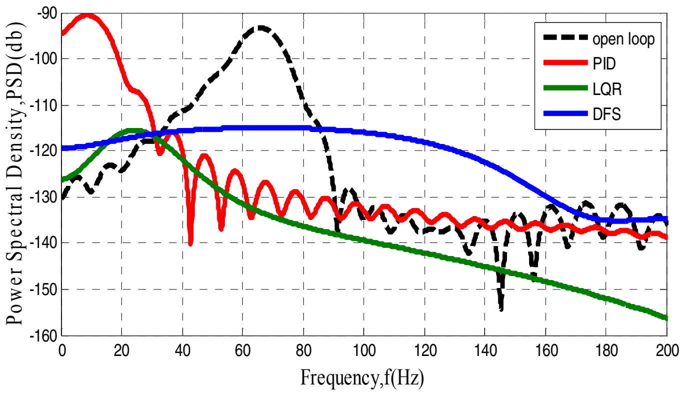

Figure 9. Power Spectral Density with payload, m = 3.0 kg

TABLE I : SWAY ANGLE RESPONSE AND MAGNITUDE OF SWAY

Payload weight (kg)	***Controller Type***	**Sway Angle Response**			**Magnitude of Sway (dB)**
		Settling Time (s)	***Rise Time (s)***	***Overshoot (%)***	
1.0	DFS	3.0	0.6	0.004	-121
	LQR	1.8	0.6	0.007	-118
	PD	4.5	1.0	0.002	-118
2.0	DFS	2.7	0.6	0.007	-118
	LQR	2.2	0.6	0.004	-120
	PD	5.0	1.2	0.003	-130
3.0	DFS	7.5	0.6	0.01	-115
	LQR	2.4	0.7	0.002	-132
	PD	5.5	1.4	0.004	-128

VI. CONCLUSION

The development of three feedback control strategies has been presented to control the sway angle of a gantry crane system. The controlled system performances in time and frequency domains have been shown and compared with the uncontrolled system. The sway angle of the rope and the power spectral density of the sway for the system with several payload weights have been presented and discussed. From simulation perspective, it is noted that LQR gives a better performance in minimizing the overshoot and settling time even when the payload weight increased. On the other hand, PD controller is proven to have the slowest system response for most payload weights. In overall, a heavier payload weight has resulted in a slower settling time and smaller sway magnitude with the overshoot and rise times are minimally affected.

ACKNOWLEDGMENT

This work was supported by Faculty of Electrical & Electronics Engineering, Universiti Malaysia Pahang, under Fundamental Research Grant Scheme (FRGS) RDU100112 which was sponsored by Ministry of Higher Education Malaysia.

REFERENCES

[1] P. Lueg, “Process of silencing sound oscillations,” US Patent 2 043 416, 1936.

[2] G.A. Manson, “Time-optimal control of and overhead crane model”, *Optimal Control Applications & Methods*, Vol. 3, No. 2, 1992, pp. 115-120.

[3] J. Auernig, and H. Troger, “Time optimal control of overhead cranes with hoisting of the load”, *Automatica*, Vol. 23, No. 4, 1987, pp. 437-447.

[4] B.H. Karnopp, F.E. Fisher, and B.O. Yoon, “A strategy for moving mass from one point to another”, *Journal of the Franklin Institute*, Vol. 329, 1992, pp. 881-892.

[5] C.L. Teo, C.J. Ong, and M. Xu, “Pulse input sequences for residual vibration reduction”, *Journal of Sound and Vibration*, Vol. 211, No. 2, 1998, pp. 157-177.

[6] W.E. Singhose, L.J. Porter, and W. Seering, “Input shaped of a planar gantry crane with hoisting”, *Proc. of the American Control Conference*, 1997, pp. 97-100.

[7] J.T. Feddema, “Digital Filter Control of Remotely Operated Flexible Robotic Structures,” *American Control Conference*, San Francisco, CA, Vol. 3, pp. 2710-2715, 1993.

[8] M.W. Noakes, and J.F. Jansen, “Generalized Inputs for Damped-Vibration Control of Suspended Payloads,” *Robotics and Autonomous Systems*, 10(2): p. 199-205, 1992.

[9] N. Singer, W. Singhose, and E. Kriikku, “An Input Shaping Controller Enabling Cranes to Move Without Sway,” *ANS 7th Topical Meeting on Robotics and Remote Systems,* Augusta, GA, 1997.

[10] N.M. Belanger, Control Engineering: A Modern Approach, Saunders College Publishing, 1995.

[11] H.M. Omar, “Control of gantry and tower cranes”, *Ph.D. Thesis*, M.S. Virginia Tech, 2003.

[12] Wahyudi and J. Jalani., “Design and implementation of fuzzy logic controller for an intelligent gantry crane system”, *Proceedings of the 2nd International Conference on Mechatronics*, 2005, pp. 345- 351.

[13] M. Ramesh, S. Narayanan, “Controlling chaotic motions in a two-dimensional airfoil using time-delayed feedback,” *J. Sound Vibration* 239 (5) (2001) 1037-1049.

[14] Z.H. Wang, H.Y. Hu, “Stability switches of time-delayed dynamic systems with unknown parameters,” *J. Sound Vibration* 233 (2) (2000) 215-233.