SUCCESSIVE OVER RELAXATION TECHNIQUE FOR STEADY STATE AND DYNAMIC CHARACTERISTICS OF A CYLINDRICAL BORE BEARING

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DEDICATIONS

TO MY FAMILY,

THANK YOU FOR BEING THERE FOR ME.

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First of all I would like to express my gratitude to the most Gracious and Most Merciful ALLAH S.W.T. Praiseworthy to Almighty Allah for giving me the will and strength to complete this thesis.

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ABSTRACT

The knowledge of static and dynamic characteristics of journal bearings is crucial for the accurate determination of the critical speed of a shaft and also for studying the stability of the rotating shaft against self-excited vibrations. These characteristics are determined from the solutions of Reynolds equation numerically using finite difference methods with successive over relaxation technique (SOR). In order to implement SOR effectively, the optimum value for over relaxation factor Ω had to be found first. In this thesis, the exact value of Ω was calculated by using a formula proposed by G.D. Smith. Khonsari and Booser (K&B) found the value of Ω, by trial and error which is not exact and time consuming. While Orcutt and Arwas (O&A) used Gauss-Seidel technique which has a much slower convergence rate compared to SOR, also they used two convergence limits which had to be satisfied before terminations of the iteration procedure. This thesis is intended to improve both works by calculating the exact value for Ω and employed the SOR technique using only one convergence limit. The dynamic coefficients were then used as an input data for studying the stability characteristics of the rotor-bearing system and the threshold of instability were also plot. The computer program was written using FORTRAN 95 programming language and run in the Microsoft Developer Studio environment. Method in this thesis shows that the time taken for a complete solution for the steady state and dynamic characteristics of a cylindrical bore bearing were greatly shortened in terms of number of iterations (about 90%) and the automatic calculation of Ω. The accuracy of the results were good with less than 10% in difference when compared to results from both K&B and O&A. It is then concluded that the finite difference method and successive over relaxation technique used in this thesis can predict accurately and effectively the static and dynamic characteristics of a cylindrical bore bearing.

ABSTRAK

Pengetahuan mengenai ciri-ciri statik dan dinamik galas jurnal adalah penting untuk penentuan secara tepat halaju kritikal bagi suatu galas dan juga untuk mengkaji kestabilan galas yang berpusing terhadap getaran. Ciri-ciri ini ditentukan melalui penyelesaian berangka persamaan Reynolds menggunakan kaedah beza terhingga serta teknik santaian secara berturutan (SOR). Untuk menggunakan SOR secara efektif, nilai optimum faktor santaian Ω harus dicari terlebih dahulu. Dalam tesis ini nilai Ω dicari menggunakan formula yang diperkenalkan oleh G.D. Smith. Khonsari dan Booser (K&B) mencari nilai Ω dengan kaedah cuba jaya yang memakan masa dan tidak tepat. Orcutt dan Arwas pula menggunakan teknik Gauss-Seidel yang mempunyai kadar penumpuan yang jauh lebih perlahan berbanding SOR, juga dua had penumpuan terpaksa dipenuhi sebelum prosedur lelaran ditamatkan. Tesis ini bertujuan untuk memperbaiki kedua-dua kerja tersebut dengan mengira nilai Ω yang tepat dan menggunakan teknik SOR dengan hanya satu had penumpuan diperlukan. Pemalar-pemalar dinamik yang diperolehi digunakan sebagai data input untuk mengkaji kestabilan sistem rotor-galas dan juga memplotkan kemasukan ketidakstabilan. Program komputer ditulis menggunakan bahasa pengaturcaraan FORTRAN 95 dan dilarikan di dalam persekitaran Microsoft Developer Studio. Kaedah di dalam tesis ini menunjukkan masa yang diambil untuk penyelesaian penuh ciri-ciri statik dan dinamik suatu galas bergerek silinder dapat dikurangkan dengan ketara dari segi bilangan lelaran (kira-kira 90%) dan pengiraan Ω secara automatik. Ketepatan keputusan adalah baik dengan kurang dari 10% perbezaan apabila dibandingkan dengan kedua-dua keputusan K&B dan O&A. Maka dapat disimpulkan bahawa kaedah unsur terhingga dan teknik santaian secara berturutan yang digunakan di dalam tesis ini dapat mengagak dengan tepat dan berkesan ciri-ciri statik dan dinamik suatu galas bergerek silinder.

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NOMENCLATURE

$A_i, B_i, C_i, D_i, G1, G2$	-	Finite difference parameters
$B_{xx}, B_{xy}, B_{yx}, B_{yy}$	-	Damping coefficients
$\hat{B}_{xx}, \hat{B}_{xy}, \hat{B}_{yx}, \hat{B}_{yy}$	-	Dimensionless damping coefficients
C	-	Radial clearance
D	-	Journal diameter (= $2R$)
e	-	Eccentricity
e_x	-	Component of eccentricity ratio in x direction
e_y	-	Component of eccentricity ratio in y direction
ES	-	Convergence factor
ER	-	Eccentricity ratio used in flowchart
ER_x, ER_y	-	Component of eccentricity ratio in x and y direction used in flowchart
F	-	Oil film force
$\hat{F}$	-	Dimensionless oil film force
Fr	-	Journal frictional force
$\hat{F}_x, \hat{F}_y$	-	Dimensionless oil film forces in x and y direction
f	-	Friction coefficients
h	-	Oil Film thickness
$\hat{H}$	-	Dimensionless oil film thickness
$K_{xx}, K_{xy} K_{yx}, K_{yy}$	-	Stiffness coefficients
$\hat{K}_{xx}, \hat{K}_{xy} \hat{K}_{yx}, \hat{K}_{yy}$	-	Dimensionless stiffness coefficients
L	-	Bearing axial length

M, N	-	Number of finite difference mesh in circumferential and axial direction
$(\hat{M}_a)_{cr}$	-	Dimensionless critical mass of the rotor-bearing system
O_B	-	Centre of the bearing
O_J	-	Centre of the journal
η	-	Rotational speed (rpm)
P	-	Oil Film pressure
P_m	-	Maximum oil film pressure
P_a	-	Atmospheric pressure
$\hat{P}$	-	Dimensionless oil film pressure
$\hat{P}_m$	-	Maximum of $\hat{P}$
Q_L	-	Side leakage
$\hat{Q}_L$	-	Dimensionless side leakage
R	-	Journal radius
S	-	Sommerfeld number
t	-	Time
T	-	Variable in quadratic equation defined in text
U	-	Surface speed of shaft
$\hat{V}$	-	Dimensionless squeeze film velocity
$\hat{V}_x, \hat{V}_y$	-	Dimensionless velocity of rotor centre in x and y coordinates
W	-	Load carrying capacity
$\hat{W}$	-	Dimensionless load carrying capacity
$\Delta\theta, \Delta z$	-	Fluid film mesh in circumferential and axial direction
$\Delta F_x, \Delta F_y$	-	Difference of oil film forces in x and y direction
Ω	-	Optimum over relaxation factor
μ	-	Lubricant viscosity
λ	-	Length over diameter ratio

$= \dfrac{\partial x}{\partial t}, \dot{y} = \dfrac{\partial y}{\partial t}$	-	Journal center velocities
ρ	-	Film density
ε	-	Eccentricity ratio
$\varepsilon_x, \varepsilon_y$	-	Component of eccentricity ratio in x and y direction
$\hat{Z}$	-	Dimensionless length of bearing
$\Delta x, \Delta y$	-	Deflections measured in x and y direction
ϕ	-	Attitude angle
ω	-	Angular velocity of journal/shaft $= 2\pi\eta$
$\hat{\omega}$	-	Dimensionless angular velocity
Θ	-	Angle measured from vertical
θ	-	Angle between position of maximum film thickness and some point around the bearing circumference

SUBSCRIPTS

r	-	Radial coordinate
t	-	Coordinate tangential to radial coordinate
x,y	-	Cartesian coordinate
i,j	-	Coordinate of mesh point in θ and Z direction
o	-	Equilibrium position
a	-	Atmospheric
m	-	Maximum
cav	-	Cavitation
in	-	Inlet
out	-	Outlet
L	-	Leakage

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TITLE

CHAPTER I

INTRODUCTION

In this chapter, the motivation of the research described in this thesis will be summarized and a brief survey of some literature on journal bearings and rotordynamics will be given. The objectives will be presented and an overview will explain the organization of the rest of the thesis.

1.1 Introduction

If two bodies are in contact and in relative motion to each other, a tangential force, the force of sliding friction, results on the surfaces of contact. These surfaces will wear out rapidly which is the major cause of material wastage, loss of mechanical performance of machine elements and shortened the life of the machines used. Reduction in wear can be achieved by improving friction control and any reduction in wear can result in considerable savings. Lubrication is an effective means of controlling wear and reducing friction, and it has wide applications in the operation of machine element such as bearings.

Bearings are representative mechanical elements used in many classes of rotating machinery. They are classified into rolling element bearings and plain bearings. depending on whether they are in rolling contact or in sliding contact. A sliding bearing typically uses a lubricant to reduced friction between the sliding surfaces. A shaft and bushing bearing are known as a journal bearing. Cylindrical bore bearing is a journal bearing with plain cylindrical sleeve (bushing) wrapped around the journal (shaft). The journal is rotating inside the bore of the sleeve with a thin clearance. In journal bearings, the bearing surfaces are parallel to the axis of rotation.

The journal and bearing surfaces are separated by a film of lubricant that is supplied to the clearance space between the surfaces through a hole or a groove. When there is a continuous fluid film separating the surfaces we speak of fluid film lubrication. When the journal bearing begin rotating there is very little lubricant between the bearing and shaft at some contact point and rubbing occurs. After the bearing has reached sufficient speed, the lubricants begins to wedge into the contact area and the relative motion of the surfaces causes the fluid pressure to support the load without metal to metal contact. This lubrication phenomenon is known as hydrodynamic lubrication.

The understanding of hydrodynamic lubrication began with the classical experiments of Beauchamp Tower in 1883 in which the existence of a film was detected from measurements of pressure within the lubricant, and of Nikilay Petroff in 1883 who reached the same conclusion from friction measurements (Hamrock, 1994). Tower's works was closely followed by Osborne Reynolds celebrated analytical paper in 1886 in which he used a reduced form of the Navier-Stokes equations in association with the continuity equation to generate a second-order differential equation for the pressure in the narrow converging gap between bearing surfaces. He derived and published not only the descriptive differential equation that today bears his name but also certain solutions to this equation that agree well with the experimental measurements of Tower (Gross, et al., 1980). Since then Reynolds equation has become the foundation of hydrodynamic analysis of bearing performance.

Other than from the full Navier-Stokes equation, more often Reynolds equation is derived by simply applying a typical engineering approach and considering the equilibrium of an element of liquid subjected to viscous shear and applying the continuity of flow principle. It is assumed that the lubricant is incompressible and that the viscosity is constant throughout the film. This approach is known as isoviscous model where thermal effects in hydrodynamic film are neglected. Journal bearing design parameters such as load carrying capacity, flow requirement and friction coefficients can be determined by solving the Reynolds equation analytically or numerically.

Reynolds equation in its full form is very difficult to solve analytically. However, nowadays there are two types of analytical method available for journal bearing solutions known as infinitely short bearing and infinitely long bearing solutions. Sommerfeld introduced the infinitely long bearing solution in 1904, while Dubois and Ocvirk presented the infinitely short bearing solution in 1953.

Numerical analysis had allowed models of hydrodynamic lubrication to describe the characteristics of real bearings. Numerical methods that were usually employed to solve the governing Reynolds equation are finite difference method and finite element method. The finite difference method is simpler to use compare to finite element method when the region of interest is rectangular with uniformly spaced nodes. Furthermore, an extensive amount of time is needed to write the computer codes when using finite element method. In addition, the results obtained by finite difference or finite element method are negligible in difference. In finite difference method, the partial derivatives in Reynolds equation were replaced by appropriate finite difference operators, thus sampling the pressure field at discrete points in a rectangular grid representing the unwrapped film of a journal bearing.

Rotordynamics is the study of rotating machines and has a very important part to play throughout the modern industrial world. Rotating machinery is commonly supported in fluid film hydrodynamic bearings such as journal bearing. These bearings generally combine relatively low frictional resistance to turning with viscous damping to reduce lateral vibrations in the machines. Dynamic analysis of hydrodynamic bearings is concerned primarily with the forces developed in the bearing due to motions imposed in the shaft. Fluid film bearings in rotating machinery generally have a set of stiffness and damping characteristics that can be used to calculate bearing stability data.

1.2 Literature Survey

In this section, a brief survey of some literature on journal bearings and rotordynamics with regard to the research described in this thesis is given.

1.2.1 Experimental Studies

The development and wide acceptance of the steam engine in the early nineteenth century brought about a need for both thrust and journal bearings. Some satisfactorily journal bearings were built first before efficient thrust bearings appeared. The first studies of a shaft and bearing running under full hydrodynamic conditions were performed by F.A. von Pauli in 1849 and G.A. Hirn in 1854 (Khonsari and Booser, 2001). Hirn reported some remarkable and perhaps inadequately recognized experiments on journal bearing friction (Dowson, 1987). He tested a wide range of lubricants, including air, water and mineral oils in a bronze half bearing (Dowson, 1987). The bearing formed part of a simple friction balance and Hirn's measurements clearly made him aware of a linear relationship between journal bearing friction and speed at light load (Dowson, 1987). He was also aware of the advantages of copious lubrication and the phenomenon of running-in (Dowson, 1987).

It was general assumption that bearing friction was a function of the bearing materials. Not until 1883 when a respected Russian engineer Nikolay Petrov made the first significant attempt to analyze theoretically the friction effect of film lubrication was it recognized that the film rather than the bearing material could be a prime consideration (Gross, et al., 1980). Petrov explained his theory for film lubrication of journal bearing. From his theory, he derived a friction equation that gives results that agree closely with experiments for lightly loaded bearings. These mean that the physical assumptions made in Petrov theory satisfactorily approximate actual conditions.

In the same year that Petrov published his theory, unexpected results were reported by Beauchamp Tower who had been employed to study the friction in railroad journal bearings and learn the best methods of lubricating them. Tower observed that oil in a journal bearing always leaked out of a hole beneath the load. He tried to block this by pounding cork and wooden stoppers into the hole, but the hydrodynamic pressure forced them out. A pressure gauge connected to the hole indicated a pressure of more than twice the unit-bearing load.

The results obtained by Tower had such regularity that an English scientist Osborne Reynolds in 1886 concluded that there must be a definite equation relating the friction, the pressure and the velocity. The present mathematical theory of lubrication is based upon Reynolds work, which followed the experiment by Tower. The original differential equation, developed by Reynolds was used to explain Tower's results. The solution is a challenging problem, which has interested many investigators ever since then, and it is still the starting point for lubrication studies. At the beginning of the 20th century, Michell and Kingsbury successfully applied the theory of hydrodynamic lubrication to thrust bearings and as a result, the pivoted pad thrust bearing was developed. In Europe, the pivoted pad thrust bearing commonly bears the name Michell, whereas in the United States the name Kingsbury is used. The Kingsbury or

Michell bearing is one in which sector pads with heavily plane surfaces are connected to the housing by pivot or elastic support (Gross, et al., 1980). The operating surfaces of such bearings were fully separated by a lubricating film, so there is no metal-to-metal contact and therefore a very low frictional force was maintained. The near simultaneous invention of the tilting pad bearing by Michell and Kingsbury represents the most remarkable and perhaps the most impressive practical development of modern bearings.

Literature related to rotordynamics covers several general stages. In the beginning, rotordynamics was the study of vibrations related to the rotor's structural dynamics, without the concerns for the bearings. The first recorded fundamental theory of rotordynamics can be found in a paper written by Jeffcott in 1919 (Yamamoto and Ishida, 2001). Stodola (1927), Biezeno and Grammel (1959), examine critical speed calculations for flexible rotors (Yamamoto and Ishida, 2001). Then, beginning in the early 1960s, most of the attention focused on hydrodynamic bearings, which was largely stimulated by Lund and Sterlicht (1962) and Lund (1964). Gunter's work in 1966 related to rotordynamic stability problems, combined with Lund's method in 1974 for calculating damped critical speeds, stimulated a great deal of interest in rotor-bearing stability problems (Yamamoto and Ishida, 2001).

1.2.2 Theoretical Studies

Analytical solutions of Reynolds equation are difficult to obtain except for some special cases like infinitely long and short bearing theory. Numerical analysis has allowed models of hydrodynamic lubrication to include closer approximations to the characteristics of real bearings. Computer solutions now dominate many design analysis for evaluating load capacity, power loss, temperature rise, and material and lubrication factors in machine element behaviour.

Initiated by Oscar Pinkus and by Albert Raimondi and John Boyd, digital computer analysis of journal bearing performance has come into widespread use for obtaining numerical solutions of Reynolds equation. Raymondi and Boyd (1958) produced computerized solution of the full Reynolds equation in the form of dimensionless charts and tables, which are required for journal bearing design.

Numerical methods that are used to solved Reynolds equation for the pressure distribution in a journal bearing is usually the finite difference method or finite element method. These methods have been used in the solution of fluid film lubrication problems for some years. Pinkus (1956) used finite difference method to solved Reynolds equation for steady state analysis of elliptical bearings. The results were also checked experimentally and good agreement was obtained. Orcutt and Arwas (1967) find the steady state and dynamic characteristics of a full circular bearing and a partial arc bearing in the laminar and turbulent flow regimes. They used finite difference method to solve Reynolds equation but with direct iteration or Gauss-Seidel method, which has, much slower convergence rate (about 90%) compared to SOR. In addition, they used two convergence limits, which need an extra calculation compared to this thesis that used only one convergence limit. These matters will be discussed more deeply in the following chapter. Woodcock and Holmes (1969) used finite difference method to solve the Reynolds equation for the oil pressure distribution in a journal bearing. They also calculated the eight oil film dynamic coefficient to be used in predicting the performance of a real rotor. Close agreement between theoretical and experimental results were obtained despite the 30% scatter in experimental damping coefficients because of measuring difficulties.

Booker and Huebner (1972) used finite element method for the solution of hydrodynamic problems. Jones and Martin (1986) presented the dimensionless steady state performance characteristics for turbulent journal bearings using finite difference method. Hayashi and Taylor (1980) used the finite element technique to predict the characteristics of a finite width journal bearing. They also found out the results from finite difference method and the agreement between these results was good. As an

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