

NANOPARTICLE SHAPES BOUNDARY LAYER FLOW IN NANOFUID AND
THEIR EFFECTS ON HEAT TRANSFER

JAAFAR ABDUL ABBAS ABBOOD AL-NASRAWI

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DEDICATED TO

In the name of Allah the Merciful

To my dear father, may God have mercy on him, and my beloved mother, may God
prolong her life

To my brothers and sisters and their families

To my heart and my wife

To my children, the light of my eyes, Mohammed, Ali, Hasan, Hussein, and
everyone who helped us.



PTT AUTHM
PERPUSTAKAAN TUNKU TUN AMINAH

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ABSTRACT

This thesis investigated two-dimensional, viscous and incompressible boundary layer flow models subject to different stream conditions. The nanoparticles such as copper or a single-walled carbon nanotube are suspended in a base fluid-water, in order to investigate heat transfer characteristics. The thermophysical properties such as empirical nanoparticle shapes and nanoparticle volume fraction are utilized to examine the nanofluid according to the Tiwari-Das approach in four problems. The governing partial differential equations were transferred to the ODEs using similarity transformations, solved by the Runge-Kutta-Fehlberg method and the shooting technique programmed in Maple 18. Firstly thermal radiation and viscous dissipation are modeled on the mixed convection heat transfer over a nonlinear moving sheet. The stagnation point flow is considered in the presence of a magnetic field with a permeable surface. It is found that the temperature for nanoparticle sphere shapes to be the lowest compared to a cylinder and laminar shapes due to differences in internal energy or kinetic energy and nanoparticle movement on the surface. Secondly is the unsteady Carreau nanofluid model with the squeezed flow between two parallel flat plates, including the sensor surfaces. It is found that the temperature profiles for non-permeable surface is lower than permeable surface. Further, the temperature for nanoparticle lamina shape is the lowest compared to sphere and cylinder shapes. Thirdly is the Carreau nanofluid model on the nonlinear moving surface with variable wall thickness. The electrical field impact with the magnetohydrodynamic flow is scrutinized. It is found that an increase in volume fraction led to an increased heat transfer rate in shear-thinning and shear-thickening. An increase in nanoparticle volume fraction led to a slight increment of mass transfer. Finally, the Cattaneo-Christov heat flux model on Maxwell nanofluid with suction/injection over a stretching sheet was examined. An increase in nanoparticle volume fraction leads to increased heat transfer rate and a slight decrease in mass transfer rate. Various

dynamical parameters and physical properties were presented in graphical of velocity, temperature, and concentration with heat and mass transfer analysis.



ABSTRAK

Tesis ini mengkaji model aliran lapisan sempadan dua-dimensi, likat dan tak termampatkan tertakluk kepada pelbagai syarat strim. Nanozarah seperti tembaga atau nanotiuub karbon dinding tunggal diampaikan di dalam bendalir asas-air, untuk mengkaji pencirian pemindahan haba. Sifat termofizikal seperti bentuk empirikal nanozarah dan pecahan isipadu nanozarah digunakan untuk mengkaji empat masalah nanobendalir menggunakan pendekatan Tiwari-Das. Persamaan terbitan separa menakluk ditukarkan kepada persamaan terbitan biasa menggunakan penjelmaan keserupaan, kemudian diselesaikan menggunakan kaedah Runge-Kutta-Fehlberg dan teknik tembakau yang diprogram didalam Maple 18. Pertamanya radiasi terma dan pelesapan likat dimodelkan keatas pemindahan haba olakan campuran terhadap kepingan bergerak taklinear. Aliran titik genangan dipertimbangkan dalam kehadiran medan magnet dengan permukaan telap. Didapati suhu nanozarah bentuk sfera terendah berbanding bentuk silinder dan lamina disebabkan perbezaan pada tenaga dalaman atau tenaga kinetik dan pergerakkan nanozarah di atas permukaan. Keduanya adalah model tak mantap nanobendalir Carreau dengan aliran terhimpit antara dua plat rata selari termasuk sensor permukaan. Didapati profil suhu bagi permukaan tak telap lebih rendah daripada permukaan telap. Selanjutnya, suhu nanozarah bentuk lamina adalah terendah berbanding bentuk sfera dan silinder. Ketiga adalah model nanobendalir Carreau terhadap permukaan bergerak taklinear dengan ketebalan dinding pembolehubah. Kesan medan elektrik dengan aliran magnetohidrodinamik diteliti. Didapati peningkatan pecahan isipadu meningkatkan kadar pemindahan haba dalam ricihan penipisan dan ricihan penebalan. Peningkatan pada pecahan isipadu nanozarah memacu kepada sedikit peningkatan pemindahan jisim. Akhirnya, model fluks haba Cattaneo-Christov ke atas nanobendalir Maxwell dengan sedutan/semburan terhadap regangan helaian dikaji. Peningkatan pecahan isipadu nanozarah meningkatkan kadar pemindahan haba dan sedikit pengurangan pada kadar

pemindahan jisim. Pelbagai parameter dinamik dan sifat fizikal telah dipersembahkan secara graf halaju, suhu, dan kepekatan dengan analisis pemindahan haba dan jisim.



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LIST OF SYMBOLS AND ABBREVIATIONS

a	-	Constant
b	-	Squeezed flow parameter
B_o	-	Magnetic field strength
c	-	Constant
c_p	-	Specific heat capacity at constant pressure
C	-	Nanoparticle concentration
C_i	-	Concentration
C_w	-	Nanoparticle concentration at the wall
C_∞	-	Ambient nanoparticle concentration
C_f	-	Skin friction coefficient
C_u	-	Copper
d	-	Constant
D_i	-	Diffusion coefficient
E_l	-	Electric field
E_0	-	Electric field factor
Ec	-	Eckert number
f	-	Dimensionless stream function
F	-	External force
F_g	-	Body force term
g	-	Acceleration due to gravity
G	-	Heat generation parameter
h	-	Step size in the Runge-Kutta Fehlberg
$h(t)$	-	Highest squeezed channel
h_o	-	Constant
I	-	Identity tensor

J	-	Electric current
j_x, j_y	-	Electric coefficient densities
k	-	Thermal conductivity
k^*	-	Diffusion coefficient of Rossland spectrum
k_r	-	Reaction coefficient
l	-	Characteristic shape of nanoparticle
L	-	Characteristic length scale
m	-	Nonlinearity of the stretching/shrinking sheet
M	-	Magnetic field parameter
n	-	Power law index parameter
N_i	-	Molar flux
Nu	-	Nusselt number
Nu_x	-	Local Nusselt number
P	-	Pressure
Pr	-	Prandtl number
q	-	Heat flux
q_w	-	Heat flux of the wall
q_r	-	Radiative heat flux
q^{**}	-	Rate of energy input per unit volume by conduction
q^{***}	-	Rate of heat generation per unit volume
Q	-	Heat generation
R	-	Radiation parameter
Ri	-	Richardson number
Re	-	Reynolds number
Re_x	-	Local Reynold number
s	-	Permeable (suction/injection) velocity parameter
s^*	-	Approximate root of the equation
S_i^*	-	Slop of solution ($i= 1, 2...,5$)
Sc	-	Schmidt number parameter
t	-	Time

T	-	Temperature of the fluid
\mathbf{T}	-	Cauchy stress tensor
T_w	-	Temperature at the wall
T_∞	-	Temperature at the stream flow
$T(y)$	-	Temperature gradient in y -directions
u, v	-	Velocity components in x, y directions, respectively
U_∞	-	Velocity of the free stream flow
U_w	-	Velocity at the wall surface
$u(y)$	-	Velocity gradient in y -directions
v_o	-	The reference velocity of permeable surface
V	-	Characteristic velocity of the flow
We	-	Weissenberg number
ω	-	Dissipation function

Greek symbols

α	-	Thermal diffusivity
α^*	-	The wall thickness parameter
β	-	Deborah number
Π	-	Rate of the second invariant strain tensor
Λ	-	Non-dimensional thermal relaxation time
$\dot{\gamma}$	-	The rate of shear stress
γ	-	Chemical reaction
δ	-	Boundary layer thickness
δ_t	-	Thermal boundary layer thickness
ε	-	Stretching/shrinking
η	-	Similarity variable
θ	-	Dimensionless temperature
$-\theta'(\theta)$	-	The rate of heat transfer
μ	-	Dynamic viscosity

μ_∞	-	Viscosity of infinite shear rate
μ_o	-	Low shear rate viscosity
ν	-	Kinematic viscosity
ρ	-	Density of fluid
ρc_p	-	Heat capacity
σ^*	-	Stefan-Boltzmann
σ	-	Electrical conductivity
ϵ	-	Small quantity
$\tau_{xy}, \tau_{yx}, \tau_{xx}, \tau_{yy}$	-	momentum components shear stresses
τ	-	Extra stress tensor
τ_w	-	Shear stress at the wall
φ	-	Dimensionless concentration
ϕ	-	Nanoparticle volume fraction
ψ	-	Stream function
Ω	-	Thermal expansion
Γ	-	Material time constant
Γ^*	-	Thermal relaxation time
γ	-	Nanoparticle volume fraction parameter

Subscript

bf	-	Fluid
nf	-	Nanofluid
p	-	Particle
w	-	Surface
∞	-	Infinity (free stream flow)

Superscripts

T	-	Transpose of matrix
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Abbreviations

bvp4c	-	Boundary value problem with fourth-order accuracy
DTM	-	Differential transform method
IVP	-	Initial value problem
MHD	-	Magnetohydrodynamic
<i>MWCNTs</i>	-	Multi-walled carbon nanotubes
NS	-	Navier-Stokes
RKF	-	Runge-Kutta-Fehlberg
OHAM	-	Homotopy asymptotic method
ODEs	-	Ordinary differential equations
PDEs	-	Partial differential equations
<i>SWCNTs</i>	-	Single-walled carbon nanotubes

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