OPTIMAL ROUTH-HURWITZ CONDITIONS AND PICARD'S SUCCESSIVE APPROXIMATION METHOD FOR SYSTEM OF FRACTIONAL DIFFERENTIAL EQUATIONS

NG YONG XIAN

A thesis submitted in
fulfillment of the requirement for the award of the
Doctor of Philosophy in Science

Faculty of Applied Sciences and Technology
Universiti Tun Hussein Onn Malaysia

JULY 2022

To my beloved family, my supervisor, Associate Professor Dr. Phang Chang, and you as a reader.

ACKNOWLEDGEMENT

This thesis is successfully completed with the assistance and co-operation of various authorities. I wish to express my utmost gratitude to those who guided, helped and supported me in completing this thesis.

First of all, I would like to thank Universiti Tun Hussein Onn Malaysia (UTHM) for allowing me to pursue my study and provided financial support through GPPS H049. Throughout this thesis, I gained numerous experience, knowledge, and exposure to situations that can not be found in textbooks. This will certainly benefit me in future.

Secondly, I would like to express my gratefulness and appreciation to my supervisor, Associate Professor Dr. Phang Chang, who led me by sacrificing his time to guide me throughout the study. His patience and advice ensure this thesis proceeds well. I truly appreciate what he had done to assist me.

Finally, I must express my very profound gratitude to my parents and my friends for providing me with unfailing support and continuous encouragement throughout my years of study and through the process of researching and writing this thesis. This accomplishment would not have been possible without them.

Ng Yong Xian

ABSTRACT

Fractional calculus is a branch of mathematical analysis investigating the derivatives and integrals of arbitrary order. Fractional calculus has a wide application since many realistic phenomena are defined in fractional order derivative and integral. Moreover, fractional differential equations provide an excellent framework for discussing the possibility of unlimited memory and hereditary properties, considering more degrees of freedom. In this thesis, the stability criteria of the fractional Shimizu-Morioka system and fractional ocean circulation model in the sense of Caputo derivative are developed analytically using optimal Routh-Hurwitz conditions. Hence, Routh-Hurwitz conditions for cubic and quadratic polynomials are presented. The advantage of Routh-Hurwitz conditions is that they allow one to obtain stability conditions without solving the fractional differential equations. In this case, we find the critical range for adjustable control parameter and fractional order α, which concludes that the equilibria of systems are locally asymptotically stable. Aftermath, the numerical results are presented to support our theoretical conclusions using the Adams-type predictor-corrector method. On the other hand, we derive the analytical solution for the inhomogeneous system of differential equations with incommensurate fractional order $1 < \alpha, \beta < 2$, where the fractional orders α and β are unique and independent of each other. The systems are first written in Volterra integral equations of the second kind. Further, Picard's successive approximation method is performed, which is an explicit analytical method that converges very close to exact solutions, and the solution is derived in multiple series and some special function expressions, such as Gamma function, Mittag-Leffler functions and hypergeometric functions. Some special cases are discussed where all the solutions are verified using substitution.

ABSTRAK

Kalkulus pecahan ialah satu cabang analisis matematik yang mempelajari pembezaan dan pengamiran dengan terbitan arbitrari. Kalkulus pecahan mempunyai aplikasi yang luas kerana banyak fenomena menjadi lebih realistik jika ditakrifkan dalam pembezaan dan pengamiran pecahan. Persamaan pembezaan pecahan memberikan satu kerangka kerja yang sangat baik untuk perbincangan mengenai kemungkinan ingatan tanpa had dan ciri-ciri keturunan, dan pertimbangan untuk darjah kebebasan yang lebih tinggi. Dalam tesis ini, kriteria kestabilan untuk sistem *Shimizu-Morioka* pecahan dan model peredaran lautan pecahan dengan menggunakan definisi terbitan *Caputo* akan dibangunkan secara analitik dengan menggunakan syarat *Routh-Hurwitz* yang optimum. Oleh itu, syarat *Routh-Hurwitz* untuk polinomial kubik dan kuadratik akan ditunjukkan. Kebaikan menggunakan syarat *Routh-Hurwitz* ialah syarat-syarat ini memberikan kriteria kestabilan untuk sistem tanpa menyelesaikan persamaan pembezaan pecahan. Dalam kes ini, kami cari julat kritikal untuk parameter kawalan boleh laras dan tertib pecahan α, seterusnya menghasilkan kesimpulan tentang kestabilan untuk keseimbangan sistem secara asimtotik. Selain itu, penyelesaian berangka akan ditunjukkan untuk menyokong kesimpulan secara teori kami dengan menggunakan kaedah peramal-pembetul jenis *Adams*. Selain itu, kami menerbitkan penyelesaian analitik untuk sistem persamaan pembezaan yang bukan homogen dengan pembezaan pecahan tidak setara $1 < \alpha, \beta < 2$, manakala α dan β merupakan dua pecahan nombor bebas yang tidak mempunyai hubungan antara satu sama lain. Sistem itu pada mulanya ditulis dalam persamaan kamiran *Volterra* jenis kedua dan kemudian kami menjalankan penghampiran berturut-turut *Picard*. Kaedah penghampiran berturut-turut *Picard* adalah satu cara analitik eksplisit yang menumpu penyelesaiannya terhadap penyelesaian sebenar, dalam bentuk siri berganda dan beberapa jenis bentuk fungsi khas, seperti fungsi *Gamma*, fungsi *Mittag-Leffler* dan fungsi hipergeometrik. Beberapa kes khas dibincangkan dan semua penyelesaiannya akan disahkan dengan menggunakan kaedah penggantian.

CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS AND ABBREVIATIONS

B	-	Beta function
C	-	Caputo
F_1, F_2	-	Freshwater flux
k	-	Hydraulic constant
RL	-	Riemann-Liouville
S_c	-	Total salt content
S_{ref}	-	Reference salinity
T	-	Temperature
V	-	Volume
α	-	Fractional order
β	-	Fractional order
Γ	-	Gamma function
γ	-	Lower incomplete gamma function
η	-	Positive value parameter
ρ	-	Density
σ	-	Positive value parameter
ϕ	-	Expansion coefficient for temperature
ψ	-	Expansion coefficient for salinity
LHS	-	Left-hand side
RHS	-	Right-hand side

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Background of research

Fractional calculus is a branch of mathematical analysis that studies the derivatives and integrals of real number, real number fractional or complex number order (Malinowska & Torres, 2012). The concept of a fractional derivative is more than 300 years old, coined by the mathematician Leibniz in 1695 (Petráš, 2011). Although fractional calculus has a long history, its applications are still important and are more realistic for many real-life problems if defined in fractional derivative and integral (Tavares *et al.*, 2016). Nevertheless, the fractional calculus can be applied in physics, specifically in the framework of anomalous diffusion, and is related to features observed in many physical systems (Riascos & Mateos, 2014). In particular, Chang *et al.* (2018) demonstrated that anomalous diffusion in natural media follows a non-Gaussian distribution, which deviates from the Brownian motion for the classical integer-order diffusion model. They presented a comparison of experimental data and numerical simulation to support the assertion. The other applications of fractional calculus also include computational biology, material sciences, physical kinetics, economics and long-range interaction (Odzijewicz & Torres, 2011; Bourdin *et al.*, 2013).

In other aspects, dynamical systems are used to describe various physical processes. For example, when the integer order derivatives are replaced by non-integer, fractional order of dynamical systems are obtained. Nowadays, fractional dynamical systems have become widely researched, especially due to hereditary properties, more degrees of freedom, and other advantages of fractional modeling (Čermák &

Nechvátal, 2017). Meanwhile, chaotic-like behaviour is an important dynamical phenomenon that has recently draw researchers' attention since the pioneering work of Lorenz in 1963 (Matouk & Elsadany, 2016). Since then, many papers on fractional chaotic systems have been investigated, focusing on fractional chaos control, chaos synchronization and other topics related to performing the stability and bifurcation analysis of dynamical systems (Matouk & Elsadany, 2016).

The Routh-Hurwitz criterion is a procedure extensively used to discuss the stability of linear systems, which included continuous-time and discrete-time linear systems (Wang *et al.*, 2019; Pereda *et al.*, 2001). This criterion enables us to determine the zeros of coefficient polynomial location without solving the zeros (Pereda *et al.*, 2001). Generally, Routh-Hurwitz conditions provide necessary and sufficient conditions for all the zeros to have negative real parts (Pena, 2004; Wang *et al.*, 2018). In this work, we study the Routh-Hurwitz conditions to develop the stability analysis for fractional dynamical systems. Stability analysis for fractional Shimizu-Morioka system and fractional ocean circulation 3-box model are developed in this thesis. As a comparison, a numerical approach, Adams-type predictor-corrector method is applied to achieve the stability analysis for fractional dynamical systems.

Shimizu-Morioka system is a three-dimensional model of ordinary differential equations proposed by Shimizu and Morioka in 1980, investigating the dynamics of the well-known Lorenz system for the special case of large Rayleigh number (Huang *et al.*, 2020). Recently, the Shimizu-Morioka system has been studied in order to apply it to real-world scenarios as well as bursting oscillations (Ma & Cao, 2018) and electronic circuits (Kapche Tagne *et al.*, 2021). The effect of the fractional-order derivative on the dynamical behaviour of the classical Shimizu-Morioka system was investigated by Kapche Tagne *et al.* (2021), whereas Ma & Cao (2018) exclusively discussed in the integer-order case.

Besides, the ocean circulation 3-box model is a simple model covering the large-scale thermohaline circulation behaviour in the Atlantic ocean (Titz *et al.*, 2002a). Ocean's salt content is described as the important dynamical behaviour of this ocean circulation model. The stable oscillations in the same ocean circulation

3-box model were observed by Keane *et al.* (2022) with delayed feedback, whereas Titz *et al.* (2002b) and Alkhayuon *et al.* (2019) used distinct higher-order models to investigate the dynamical behaviour of Atlantic ocean circulation. However, these researchers studied the model in the integer-order case or with a time-delay term. The ocean circulation models are still rarely extended to fractional-order cases.

The fractional order differential equation systems can be divided into two categories, which are commensurate and incommensurate order systems. If the fractional orders of a system are different and independent to each other, it is then known as an incommensurate fractional order system; otherwise, it is known as a commensurate fractional order system. Hence, a commensurate order system is categorized as a special case of an incommensurate order system (Daşbaşi, 2020). However, the incommensurate order system does not reduce its difficulty of finding explicit solutions compared to the commensurate order system. The approximation solutions are usually obtained for the fractional differential equation systems in the incommensurate case. Some numerical methods, such as predictor-corrector method, Adomian decomposition algorithm and reduced-order model approximation via genetic algorithm, have been developed by researchers (Diethelm *et al.*, 2002; Diethelm & Ford, 2004; Liao *et al.*, 2018; Soloklo & Bigdeli, 2020). Therefore, it is still challenging for researchers to obtain the analytical solution of fractional differential equation systems in incommensurate order.

1.2 Problem statement

In the past decade, fractional calculus has become more important as many phenomena are more realistic if they defined in fractional derivative and integral instead of integer order derivative and integral. For instance, Hilfer (2000) claimed that the integer order calculus sometimes contradicts the experimental result. In this research direction, the fractional calculus will consider the evolution of the system by taking the global correlation, not only local characteristics (Tavares *et al.*, 2016). Therefore, many problems have been extended to fractional order problems or the problem with arbitrary order.

Due to this fractional order, the solution of these problems may not be easy to obtain, or the exact analytical solution may not exist. Many efficient numerical methods had been derived to obtain the approximate solution of these fractional calculus problems. However, these numerical schemes had some drawbacks, which may include the expensive computational cost, and the accuracy of the solution may not be satisfied. In addition, there are no mathematical software that can directly solve fractional dynamical systems, either stability analysis or explicit solution for incommensurate or commensurate cases. In other words, commercial mathematical software such as Maple is only applicable for solving problem in an integer-order derivative.

Hence, obtaining the exact analytical solution is always the concern of the researchers in this area. In this regard, this thesis investigates the analytical solutions for the stability analysis of fractional dynamical systems by using optimal Routh-Hurwitz conditions. Besides, Picard's successive approximation method is applied to derive the explicit analytical solution for an incommensurate differential equation system with order $1 < \alpha, \beta < 2$, where α and β are non-integer values and are independent of each other.

To obtain the stability solution, fractional Shimizu-Morioka system and fractional ocean circulation 3-box model are considered in this study to represent the analysis for two types of systems, which are 3-dimensional and 2-dimensional fractional-order systems, respectively. Fractional-order Shimizu-Morioka system is defined as follows:

$$\mathrm{D}^{\alpha}x = y,\ \mathrm{D}^{\alpha}y = (1-z)x - \sigma y,\ \mathrm{D}^{\alpha}z = -\eta z + x^2, \tag{1.1}$$

exhibits chaotic dynamic behaviours, where D^{α} denotes the fractional derivative of order α. However, the stability and bifurcation analysis of fractional-order Shimizu-Morioka system is still not developed analytically. While, the ocean circulation 3-box model is a model that drive the Atlantic ocean thermohaline circulation behaviour into a simple 2-dimensional model. The fractional-order ocean circulation 3-box model is defined as follows:

$$\mathrm{D}^{\alpha} S_1 = \frac{1}{V} S_{\text{ref}} F_1 + \frac{1}{V} m(S_2 - S_1), \mathrm{D}^{\alpha} S_2 = -\frac{1}{V} S_{\text{ref}} F_2 + \frac{1}{V} m(S_3 - S_2). \quad (1.2)$$

Bifurcation study of this ocean circulation model is computed by Titz *et al.* (2002a) for integer-order derivative, but still has not been extended into fractional-order. For the incommensurate fractional differential equation systems as follows:

$$\mathrm{D}^{\alpha} x_1(t) = a_{11} x_1(t) + a_{12} x_2(t) + g_1(t), \ \mathrm{D}^{\beta} x_2(t) = a_{21} x_1(t) + a_{22} x_2(t) + g_2(t), \quad (1.3)$$

Huseynov *et al.* (2021) introduced the explicit solution, but is limited for order $0 < \alpha, \beta < 1$. Hence, we extend their works rigorously to fractional order $1 < \alpha, \beta < 2$.

1.3 Objectives of research

This study embarks on the following objectives:

(i) Develop the stability analysis of the fractional Shimizu-Morioka system via optimal Routh-Hurwitz conditions for cubic polynomials.

(ii) Investigate the stability analysis of fractional order ocean circulation box model via optimal Routh-Hurwitz conditions for quadratic polynomials.

(iii) Determine the analytical solution of incommensurate fractional differential equation systems with fractional order $1 < \alpha, \beta < 2$.

1.4 Scope of study

Many real phenomena problems have been extended to fractional, but the solutions of fractional order systems are still few to be derived analytically. Therefore, this study aims to analyze the stability analysis using optimal Routh-Hurwitz conditions and analytical solutions for the systems of fractional differential equations. Our study will cover the fractional differential equation systems in the Caputo sense. The

Caputo fractional differential operator is commonly used and available for zero initial conditions cases. Still, it introduces a limitation since the Caputo fractional derivative is ineffective in explaining the singular kernel of phenomena.

The dynamical systems such as the Shimizu-Morioka system and ocean circulation 3-box model are extended to fractional order models in our study. Stability analysis of the fractional systems are developed analytically by applying the Routh-Hurwitz conditions. In this case, we will obtain the critical point for the adjustable control parameter and fractional order α where the Hopf bifurcation occurs. Application of Routh-Hurwitz conditions limits the findings of stability criterion with the fractional order $\alpha \in (0, 2)$. Perturbate the adjustable control parameter or fractional order α from critical value which lead to dissolve of limit cycle to stable or unstable condition, yielding the conclusions that the equilibria of systems are locally asymptotically stable. To support the theoretical conclusions from optimal Routh-Hurwitz conditions, we will evaluate the numerical results using the Adams-type predictor-corrector method introduced by Diethelm *et al.* (2002).

Afterwards, we perform the explicit analytical solution of the systems for linear fractional differential equations with incommensurate order $1 < \alpha, \beta < 2$. The inhomogeneous case is considered. We introduce the analytical solution by transferring the system into the Volterra integral equation of the second kind and then applying Picard's successive approximations. Initial conditions are considered to derive the analytical solutions. Subsequently, the solution can be simplified by some algebraic manipulation and combinatorial identities. The application of Volterra integral equation and Picard's successive approximation method produce the final explicit solution in the expression of Mittag-Leffler and bivariate Mittag-Leffler functions, multiple summation series of gamma and hypergeometric functions, and is verified by substitution. Moreover, some special cases should be considered, such as the case for zero value determinant and denominator. The scope of the study is limited to the linear case and fractional orders are limited in $\alpha, \beta \in (1, 2)$.

1.5 Research gap

Fractional calculus and its applications are wide research areas that need more attention from mathematicians and scientists. In general, many dynamical systems are extended to fractional order as the fractional order models are claimed better in describing the real phenomena. However, the solutions of fractional dynamical systems are not easy to obtain since the concepts of fractional derivative and integral are different and more complicated if compared with the integer-order. Regarding the complexity of the problems, the existing research tends to focus on numerical methods to solve the fractional dynamical systems. There are very few studies have been done on investigating an exact analytical solution for models in fractional order.

Investigation of analytical solutions is important because the numerical solutions sometimes come with error estimation, where the accuracy of the solution may not be satisfied. This serves to establish the demand for further research in this area. On the other hand, the analytical solution of stability analysis is still rarely explored for many dynamical models that extend to fractional order. Therefore, our proposed framework will provide researchers with a stronger theoretical stability analysis for fractional order dynamical systems. The fractional Shimizu-Morioka system and fractional ocean circulation box model are considered in this thesis. Besides, for the incommensurate fractional order systems, the prior works of Huseynov *et al.* (2021) and Ahmadova *et al.* (2021) are limited to order $0 < \alpha, \beta < 1$.

1.6 Structure of thesis

This thesis consists of seven chapters. Chapter 1 describes the background of research, problem statement, objectives, scope of study and research gap.

Chapter 2 describes a review of the research on fractional calculus, stability and bifurcation analysis for fractional dynamical systems, and fractional differential equation systems. Some fundamental understanding of fractional operators and special functions of fractional calculus is provided. This is followed by some discussion of different approaches used to perform stability and bifurcation analysis. This section focuses on the Routh-Hurwitz conditions and the predictor-corrector method. A review of Picard's successive approximation method is provided. It is the analytic approach used for solving incommensurate fractional systems in this thesis. In addition, the chapter provides a critical review of related works on fractional differential equation systems, including fractional Shimizu-Morioka system, fractional ocean circulation box model, incommensurate fractional differential equation systems, and numerical methods for fractional differential equation systems.

Chapter 3 explains the methodology used in this study. Some basic properties of fractional derivatives and integrals are discussed. Routh-Hurwitz conditions for cubic and quadratic polynomials, and the predictor-corrector method are presented as analytical and numerical methods to develop the stability analysis, respectively. The chapter also presents the definitions of the special functions of fractional calculus, such as gamma function, beta function, hypergeometric function and Mittag-Leffler function. The Volterra integral equation of second kind and Picard's successive approximation method are the methods used to derive the analytical solution for an incommensurate fractional differential equation system.

Chapter 4 is devoted in presenting the computation of stability criterion for the fractional Shimizu-Morioka system. Some basic mathematical concepts for the Shimizu-Morioka system in integer order and fractional order are discussed. The major part of this chapter performs the results of stability analysis of the fractional Shimizu-Morioka system by using optimal Routh-Hurwitz conditions. The chapter also compares the stability results from Routh-Hurwitz conditions with results from the numerical approach, which is the predictor-corrector method.

Chapter 5 elaborates on the computation of stability criterion for the fractional ocean circulation box model. This chapter mainly presents the analysis and observations to achieve the stability criterion of the fractional ocean circulation box model. In this case, the optimal Routh-Hurwitz conditions for quadratic polynomials are applied. The chapter also compares results from the Routh-Hurwitz conditions with results from the predictor-corrector method.

Chapter 6 discusses the explicit analytical solutions of incommensurate fractional differential equation systems with fractional order $1 < \alpha, \beta < 2$. The chapter starts with the derivation of an explicit analytical solution for a general incommensurate fractional order system. This is followed by some special cases such as the $A = 1$, $a_{11} = 0$, $a_{22} = 0$, and $a_{11} = a_{22} = 0$. Examples of explicit analytical solutions of incommensurate fractional differential equation systems are presented using theorems obtained.

Chapter 7 describes the conclusion of the overall findings and discussions, and is followed by recommendations for further work in the future.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter will be structured as follows: Section 2.2 is devoted to present some fundamental understanding of the fractional calculus. In this section, some special functions of fractional calculus, including the gamma function, Mittag-Leffler function, and hypergeometric function, are studied. These special functions are useful in helping us develop the solutions of incommensurate fractional differential equation systems. Some fundamental understanding of fractional derivatives and integrals is provided in Section 2.2.4.

Section 2.3 is devoted to present some discussion of different approaches used to perform stability and bifurcation analysis. This section focuses on the Routh-Hurwitz conditions and the predictor-corrector method, which are discussed in Sections 2.3.1 and 2.3.2, respectively. Section 2.4 presents the Picard's successive approximation method for solving systems of fractional differential equations.

Section 2.5 provides a critical review of the related work on fractional differential equation systems. The fractional Shimizu-Morioka system, the fractional ocean circulation box model, and incommensurate fractional differential equation systems are briefly discussed. At the end of the section, several numerical methods for systems of fractional differential equations are discussed.

2.2 Fractional calculus

Fractional calculus has a long history and is one of the classical branches of mathematics. Fractional calculus is the calculus of non-integer order derivatives and integrals that involves fractional derivatives and integrals (Bourdin *et al.*, 2013). Leibniz carried out the concept of a fractional derivative in 1695 in his letter to L' Hopital (Petráš, 2011). Leibniz raised the question about the possibility of the derivatives being generalized to non-integer orders (Petráš, 2011). Hence, contributions of several mathematicians, such as Liouville, Riemann and Weyl, were made to the theory of fractional calculus (Petráš, 2011).

A fractional differential equation is a generalization of the differential equation through the application of fractional calculus. Fractional calculus fundamentals have been studied in many researches, including those of Petráš (2011) and Baleanu *et al.* (2012). Fractional calculus may also be applied in fractional conservation of mass (Dinh, 2019), groundwater flow problem (Mahantane, 2019), fractional advection-dispersion equation (Singh *et al.*, 2019), time-space fractional diffusion equation model (Jia *et al.*, 2018), structural damping models (Praharaj & Datta, 2020), acoustical wave equations for complex media (Näsholm & Holm, 2013) and fractional heat conduction model (Baleanu *et al.*, 2020).

Besides that, the special functions such as Euler's gamma function, Mittag-Leffler function and hypergeometric function always play essential roles in the research related to fractional calculus. To derive the analytical solutions of incommensurate fractional differential equation systems in Chapter 6, a precise application of these special functions is required and permitted us to simplify the solution. Here, we will briefly present some related works for the special functions.

2.2.1 Gamma function

Gamma function $\Gamma(x)$ is a generalized factorial form with its argument shifted down to complex numbers (Baleanu *et al.*, 2012; Wang, 2016),

$$\Gamma(x) = (x-1)!, \tag{2.1}$$

if x is a positive integer. This gamma function is considered to extend the factorial to different definitions for a non-integer or complex number x, for instance, in the form of Euler's integral, infinite product and Euler limit (Wang, 2016). Researchers such as Daniel Bernoulli, Legendre, Gauss, Liouville, Weierstrass and Hermite studied this gamma function, which was introduced by Euler (Kim & Kim, 2020). The various definitions given by these different researchers are equivalent to each other. In particular, the Euler's integral gamma function is used throughout this thesis, especially in Chapter 6, and the definition is given in Section 3.5.

2.2.2 Mittag-Leffler function

Mittag-Leffler function can be derived in the integral representation (Baleanu *et al.*, 2012). In recent years, this special transcendental function shows its importance in treating problems related to fractional-order differential and integral equations (Mainardi, 2020). However, the Mittag-Leffler function has a long history. In 1903, the Swedish mathematician Mittag-Leffler introduced a function with power series given by

$$E_{\alpha}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)}, \tag{2.2}$$

namely Mittag-Leffler function, where z is a complex variable and $\Gamma(\cdot)$ is a gamma function (Mainardi, 2020). When $\alpha = 1$, the Mittag-Leffler function is reduced to an exponential function (Shukla & Prajapati, 2007). Mittag-Leffler function is important due to its wide applications in applied sciences and mathematics as well as

REFERENCES

Abdelaty, A. M., Roshdy, M., Said, L. A., & Radwan, A. G. (2020). Numerical simulations and FPGA implementations of fractional-order systems based on product integration rules. *IEEE Access*, *8*, pp. 102093–102105.

Abdeljawad, T., Amin, R., Shah, K., Al-Mdallal, Q., & Jarad, F. (2020). Efficient sustainable algorithm for numerical solutions of systems of fractional order differential equations by Haar wavelet collocation method. *Alexandria Engineering Journal*, *59(4)*, pp. 2391–2400.

Abu Arqub, O. (2019). Numerical algorithm for the solutions of fractional order systems of Dirichlet function types with comparative analysis. *Fundamenta Informaticae*, *166(2)*, pp. 111–137.

Aggarwal, S., Sharma, N., & Chauhan, R. (2018). Solution of linear Volterra integral equations of second kind using Mohand transform. *International Journal of Research in Advent Technology*, *6(11)*, pp. 3098–3102.

Ahmadova, A., Huseynov, I. T., Fernandez, A., & Mahmudov, N. I. (2021). Trivariate Mittag-Leffler functions used to solve multi-order systems of fractional differential equations. *Communications in Nonlinear Science and Numerical Simulation*, *97*, pp. 105735.

Ahmed, E., El-Sayed, A., & El-Saka, H. A. (2006). On some Routh–Hurwitz conditions for fractional order differential equations and their applications in Lorenz, Rössler, Chua and Chen systems. *Physics Letters A*, *358(1)*, pp. 1–4.

Akinlar, M. A., Secer, A., & Bayram, M. (2014). Stability, synchronization control and numerical solution of fractional Shimizu-Morioka dynamical system. *Applied Mathematics and Information Sciences*, *8(4)*, pp. 1699–1705.

Al-Dhaifallah, M., Nassef, A. M., Rezk, H., & Nisar, K. S. (2018). Optimal parameter design of fractional order control based INC-MPPT for PV system. *Solar Energy*, *159*, pp. 650–664.

Albadarneh, R. B., Batiha, I. M., Tahat, N., & Alomari, A. (2021). Analytical solutions of linear and non-linear incommensurate fractional-order coupled systems. *Indonesian Journal of Electrical Engineering and Computer Science*, *21(2)*, pp. 776–790.

Alkhayuon, H., Ashwin, P., Jackson, L. C., Quinn, C., & Wood, R. A. (2019). Basin bifurcations, oscillatory instability and rate-induced thresholds for Atlantic meridional overturning circulation in a global oceanic box model. *Proceedings of the Royal Society A*, *475(2225)*, pp. 20190051.

Atangana, A. (2014). On the singular perturbations for fractional differential equation. *The Scientific World Journal, 2014*.

Atangana, A., & Alkahtani, B. S. T. (2015). Extension of the resistance, inductance, capacitance electrical circuit to fractional derivative without singular kernel. *Advances in Mechanical Engineering*, *7(6)*, *doi:10.1177/1687814015591937*.

Bagatinsky, V., & Diansky, N. (2021). Variability of the North Atlantic thermohaline circulation in different phases of the Atlantic multidecadal oscillation from ocean objective analyses and reanalyses. *Izvestiya, Atmospheric and Oceanic Physics*, *57(2)*, pp. 208–219.

Baleanu, D., Diethelm, K., Scalas, E., & Trujillo, J. J. (2012). *Fractional Calculus: Models and Numerical Methods, 3*. Singapore: World Scientific.

Baleanu, D., Jleli, M., Kumar, S., & Samet, B. (2020). A fractional derivative with two singular kernels and application to a heat conduction problem. *Advances in Difference Equations*, *2020(1)*, pp. 1–19.

Bourafa, S., Abdelouahab, M., & Moussaoui, A. (2020). On some extended Routh–Hurwitz conditions for fractional-order autonomous systems of order $\alpha \in$ (0, 2) and their applications to some population dynamic models. *Chaos, Solitons and Fractals*, *133*, pp. 109623.

Bourdin, L., Cresson, J., Greff, I., & Inizan, P. (2013). Variational integrator for fractional Euler–Lagrange equations. *Applied Numerical Mathematics, 71*, pp. 14–23.

Capiński, M. J., Turaev, D., & Zgliczyński, P. (2018). Computer assisted proof of the existence of the Lorenz attractor in the Shimizu–Morioka system. *Nonlinearity*, *31(12)*, pp. 5410.

Caputo, M. (1967). Linear models of dissipation whose Q is almost frequency independent II. *Geophysical Journal International, 13(5)*, pp. 529–539.

Čermák, J., & Kisela, T. (2015). Stability properties of two-term fractional differential equations. *Nonlinear Dynamics*, *80(4)*, pp. 1673–1684.

Čermák, J., & Nechvátal, L. (2017). The Routh–Hurwitz conditions of fractional type in stability analysis of the Lorenz dynamical system. *Nonlinear Dynamics*, *87(2)*, pp. 939–954.

Čermák, J., & Nechvátal, L. (2018). Local bifurcations and chaos in the fractional Rössler system. *International Journal of Bifurcation and Chaos*, *28(08)*, pp. 1850098.

Chang, A., Sun, H., Zheng, C., Lu, B., Lu, C., Ma, R., & Zhang, Y. (2018). A time fractional convection–diffusion equation to model gas transport through heterogeneous soil and gas reservoirs. *Physica A: Statistical Mechanics and its Applications*, *502*, pp. 356–369.

Chatibi, Y., El Kinani, E., & Ouhadan, A. (2019). Variational calculus involving nonlocal fractional derivative with Mittag–Leffler kernel. *Chaos, Solitons and Fractals*, *118*, pp. 117–121.

Clark, R. N. (1992). The Routh-Hurwitz stability criterion, revisited. *IEEE Control Systems Magazine*, *12(3)*, pp. 119–120.

Daşbaşi, B. (2020). Stability analysis of the HIV model through incommensurate fractional-order nonlinear system. *Chaos, Solitons and Fractals*, *137*, pp. 109870.

Demir, A., Bayrak, M. A., & Ozbilge, E. (2019). A new approach for the approximate analytical solution of space-time fractional differential equations by the homotopy analysis method. *Advances in Mathematical Physics*, *2019*.

Desai, R., Salehbhai, I., & Shukla, A. (2020). Integral equations involving generalized

Mittag-Leffler function. *Ukrainian Mathematical Journal*, *72(5)*, pp. 712–721.

Diethelm, K., & Ford, N. J. (2004). Multi-order fractional differential equations and their numerical solution. *Applied Mathematics and Computation*, *154(3)*, pp. 621–640.

Diethelm, K., Ford, N. J., & Freed, A. D. (2002). A predictor-corrector approach for the numerical solution of fractional differential equations. *Nonlinear Dynamics*, *29(1)*, pp. 3–22.

Dinh, V. D. (2019). On instability of standing waves for the mass-supercritical fractional nonlinear Schrödinger equation. *Zeitschrift für angewandte Mathematik und Physik*, *70*, pp. 1–17.

Doungmo Goufo, E. F. (2016). Application of the Caputo-Fabrizio fractional derivative without singular kernel to Korteweg-de Vries-Burgers equation. *Mathematical Modelling and Analysis*, *21(2)*, pp. 188–198.

Duan, J. (2018). A generalization of the Mittag–Leffler function and solution of system of fractional differential equations. *Advances in Difference Equations*, *2018(1)*, pp. 1–12.

El-Dessoky, M., Yassen, M., & Aly, E. (2014). Bifurcation analysis and chaos control in Shimizu–Morioka chaotic system with delayed feedback. *Applied Mathematics and Computation*, *243*, pp. 283–297.

Emmanuel, F. S., James, A. K., Nathaniel, O. S., & Temitayo, O. J. (2020). Review of some numerical methods for solving initial value problems for ordinary differential equations. *International Journal of Applied Mathematics and Theoretical Physics*, *6(1)*, pp. 7.

Fareed, A. F., Semary, M. S., & Hassan, H. N. (2022). An approximate solution of fractional order Riccati equations based on controlled Picard's method with Atangana–Baleanu fractional derivative. *Alexandria Engineering Journal*, *61(5)*, pp. 3673–3678.

Fernandez, A., Kürt, C., & Özarslan, M. A. (2020). A naturally emerging bivariate Mittag-Leffler function and associated fractional-calculus operators.

Computational and Applied Mathematics, *39(3)*, pp. 1–27.

Garashchuk, I., Kudryashov, N., & Sinelshchikov, D. (2017). On the analytical properties and some exact solutions of the Glukhovsky-Dolzhansky system. In *Journal of Physics: Conference Series*, vol. 788. Bristol: Institute of Physics and IOP Publishing Limited. pp. 012013.

Garrappa, R. (2015). Trapezoidal methods for fractional differential equations: Theoretical and computational aspects. *Mathematics and Computers in Simulation*, *110*, pp. 96–112.

Gholamin, P., Sheikhani, A. R., & Ansari, A. (2021). Stabilization of a new commensurate/incommensurate fractional order chaotic system. *Asian Journal of Control*, *23(2)*, pp. 882–893.

Gómez-Aguilar, J. (2017). Space–time fractional diffusion equation using a derivative with nonsingular and regular kernel. *Physica A: Statistical Mechanics and its Applications*, *465*, pp. 562–572.

Hajipour, A., & Tavakoli, H. (2016). Analysis and circuit simulation of a novel nonlinear fractional incommensurate order financial system. *Optik*, *127(22)*, pp. 10643–10652.

Heris, M. S., & Javidi, M. (2018). On fractional backward differential formulas methods for fractional differential equations with delay. *International Journal of Applied and Computational Mathematics*, *4(2)*, pp. 72.

Hilfer, R. (2000). *Applications of Fractional Calculus in Physics*. Singapore: World Scientific.

Huang, K., Shi, S., & Li, W. (2020). Integrability analysis of the Shimizu–Morioka system. *Communications in Nonlinear Science and Numerical Simulation*, *84*, pp. 105101.

Huseynov, I. T., Ahmadova, A., Fernandez, A., & Mahmudov, N. I. (2021). Explicit analytical solutions of incommensurate fractional differential equation systems. *Applied Mathematics and Computation*, *390*, pp. 125590.

Jain, S., & Hote, Y. V. (2019). Reduced order approximation of incommensurate fractional order systems. In *2019 IEEE Conference on Control Technology* *and Applications (CCTA)*. IEEE. pp. 1056–1061.

Jayaram, S., Kapoor, S., & DeVor, R. (2000). Analytical stability analysis of variable spindle speed machining. *Journal of Manufacturing Science and Engineering*, *122(3)*, pp. 391–397.

Jia, J., Peng, J., Gao, J., & Li, Y. (2018). Backward problem for a time-space fractional diffusion equation. *Inverse Problems and Imaging*, *12(3)*, pp. 773–799.

Kalmykov, S., & Karp, D. (2017). Log-concavity and Turán-type inequalities for the generalized hypergeometric function. *Analysis Mathematica*, *43(4)*, pp. 567–580.

Kapche Tagne, F., Honoré Kom, G., Motchongom Tingue, M., Kamdoum Tamba, V., & Kisito Talla, P. (2021). Analysis and electronic circuit implementation of an integer and fractional-order Shimizu-Morioka system. *Revista mexicana de física*, *67(6)*.

Keane, A., Pohl, A., Dijkstra, H. A., & Ridgwell, A. (2022). A simple mechanism for stable oscillations in an intermediate complexity Earth System Model. *arXiv preprint arXiv:2201.07883*.

Kessler, A. (2020). *Atlantic Thermohaline Changes and its Implications on the Carbon Cycle During the Last Interglacial*. Ph.D. thesis, University of Bergen.

Kim, H. J., An, S. I., Kim, S. K., & Park, J. H. (2021). Feedback processes modulating the sensitivity of Atlantic thermohaline circulation to freshwater forcing timescales. *Journal of Climate*, *34(12)*, pp. 5081–5092.

Kim, T., & Kim, D. S. (2020). Note on the degenerate gamma function. *Russian Journal of Mathematical Physics*, *27(3)*, pp. 352–358.

Kiryakova, V. (2021). A guide to special functions in fractional calculus. *Mathematics*, *9(1)*, pp. 106.

Kucche, K. D., Mali, A. D., & Sousa, J. (2019). On the nonlinear Ψ-hilfer fractional differential equations. *Computational and Applied Mathematics*, *38(2)*, pp. 1–25.

Kumar, S., Kumar, A., Nieto, J., & Sharma, B. (2019). Atangana–Baleanu derivative with fractional order applied to the gas dynamics equations. In *Fractional*

Derivatives with Mittag-Leffler Kernel, pp. 235–251. New York: Springer.

Lamba, R., Singla, S. K., & Sondhi, S. (2017). Fractional order PID controller for power control in perturbed pressurized heavy water reactor. *Nuclear Engineering and Design*, *323*, pp. 84–94.

Lekdee, N., Sirisubtawee, S., & Koonprasert, S. (2019). Bifurcations in a delayed fractional model of glucose–insulin interaction with incommensurate orders. *Advances in Difference Equations*, *2019(1)*, pp. 1–22.

Lenka, B. K. (2019). Time-varying Lyapunov functions and Lyapunov stability of nonautonomous fractional order systems. *International Journal of Applied Mathematics*, *32(1)*, pp. 111.

Li, H., Cheng, J., Li, H.-B., & Zhong, S.-M. (2019). Stability analysis of a fractional-order linear system described by the Caputo–Fabrizio derivative. *Mathematics*, *7*(2), pp. 200.

Liao, H., Ding, Y., & Wang, L. (2018). Adomian decomposition algorithm for studying incommensurate fractional-order memristor-based Chua's system. *International Journal of Bifurcation and Chaos*, *28(11)*, pp. 1850134.

Liu, W., Fedorov, A. V., Xie, S.-P., & Hu, S. (2020). Climate impacts of a weakened Atlantic Meridional Overturning Circulation in a warming climate. *Science advances*, *6(26)*, pp. eaaz4876.

Llibre, J., & Pessoa, C. (2015). The Hopf bifurcation in the Shimizu–Morioka system. *Nonlinear Dynamics*, *79(3)*, pp. 2197–2205.

Lund, L. A., Omar, Z., Alharbi, S. O., Khan, I., & Nisar, K. S. (2019). Numerical investigation of multiple solutions for Caputo fractional-order-two dimensional magnetohydrodynamic unsteady flow of generalized viscous fluid over a shrinking sheet using the Adams-type predictor-corrector method. *Coatings*, *9(9)*, pp. 548.

Luo, R., & Su, H. (2018). The stability of impulsive incommensurate fractional order chaotic systems with Caputo derivative. *Chinese Journal of Physics*, *56(4)*, pp. 1599–1608.

Lyons, R., Vatsala, A. S., & Chiquet, R. A. (2017). Picard's iterative method for Caputo fractional differential equations with numerical results. *Mathematics*, *5(4)*, pp. 65.

Ma, X., & Cao, S. (2018). Bursting oscillations in Shimizu-Morioka system with slow-varying periodic excitation. *Shock and Vibration*, *2018*.

Mahantane, M. L. (2019). *A New Method for Modeling Groundwater Flow Problems: Fractional-Stochastic Modeling*. Ph.D. thesis, University of the Free State.

Mainardi, F. (2020). Why the Mittag-Leffler function can be considered the Queen function of the Fractional Calculus? *Entropy*, *22(12)*, pp. 1359.

Malinowska, A. B., & Torres, D. F. (2012). *Introduction to the Fractional Calculus of Variations*. Singapore: World Scientific Publishing Co Inc.

Mansour, M. (1988). A simple proof of the Routh-Hurwitz criterion. In *Internal Report 88-04, Automatic Control Laboratory, Swiss Federal Institute of Technology*.

Marasi, H., Joujehi, A. S., & Aydi, H. (2021). An extension of the Picard theorem to fractional differential equations with a Caputo-Fabrizio derivative. *Journal of Function Spaces*, *2021*.

Matignon, D. (1996). Stability results for fractional differential equations with applications to control processing. In *Computational engineering in systems applications*, vol. 2. Citeseer. pp. 963–968.

Matouk, A., & Elsadany, A. (2016). Dynamical analysis, stabilization and discretization of a chaotic fractional-order GLV model. *Nonlinear Dynamics*, *85(3)*, pp. 1597–1612.

Mirzaei, R., Ghalambaz, M., & Noghrehabadi, A. (2021). Study of the flow and heat transfer of a viscoelastic fluid using hybrid neural network-particle swarm optimization (HNNPSO). *Journal of Thermal Engineering*, *7(4)*, pp. 791–805.

Moze, M., Sabatier, J., & Oustaloup, A. (2005). LMI tools for stability analysis of fractional systems. In *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, vol. 47438. pp. 1611–1619.

Näsholm, S. P., & Holm, S. (2013). On a fractional Zener elastic wave equation. *Fractional Calculus and Applied Analysis, 16(1)*, pp. 26–50.

Nguyen, T. B., & Jang, B. (2017). A high-order predictor-corrector method for solving nonlinear differential equations of fractional order. *Fractional Calculus and Applied Analysis*, *20(2)*, pp. 447–476.

Odzijewicz, T., & Torres, D. F. (2011). Fractional calculus of variations for double integrals. *Balkan Journal of Geometry and Its Applications, 16(2)*, pp. 102–113.

Ortigueira, M., & Machado, J. (2017). Fractional definite integral. *Fractal and Fractional*, *1(1)*, pp. 2.

Ortigueira, M. D., & Machado, J. T. (2015). What is a fractional derivative? *Journal of Computational Physics*, *293*, pp. 4–13.

Patil, A. (2021). Routh-Hurwitz criterion for stability: an overview and its implementation on characteristic equation vectors using MATLAB. *Emerging Technologies in Data Mining and Information Security*, pp. 319–329.

Pena, J. M. (2004). Characterizations and stable tests for the Routh–Hurwitz conditions and for total positivity. *Linear Algebra and its Applications*, *393*, pp. 319–332.

Pereda, A., Vielva, L. A., Vegas, A., & Prieto, A. (2001). Analyzing the stability of the FDTD technique by combining the von Neumann method with the Routh-Hurwitz criterion. *IEEE Transactions on Microwave Theory and Techniques*, *49(2)*, pp. 377–381.

Petráš, I. (2011). Fractional derivatives, fractional integrals, and fractional differential equations in Matlab. *Engineering Education and Research Using MATLAB, 10*, pp. 239–264.

Praharaj, R., & Datta, N. (2020). Dynamic response spectra of fractionally damped viscoelastic beams subjected to moving load. *Mechanics Based Design of Structures and Machines*, pp. 1–15.

Rahmstorf, S. (1996). On the freshwater forcing and transport of the Atlantic thermohaline circulation. *Climate Dynamics*, *12(12)*, pp. 799–811.

Riascos, A. P., & Mateos, J. L. (2014). Fractional dynamics on networks: Emergence of anomalous diffusion and Lévy flights. *Physical Review E*, *90(3)*, pp. 032809.

Samaee, S., Yazdanpanah, O., & Ganji, D. (2015). New approaches to identification of the Lagrange multiplier in the variational iteration method. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, *37(3)*, pp. 937–944.

Sarwar, S., & Iqbal, S. (2018). Stability analysis, dynamical behavior and analytical solutions of nonlinear fractional differential system arising in chemical reaction. *Chinese Journal of Physics*, *56*(1), pp. 374–384.

Shatnawi, M. T., Djenina, N., Ouannas, A., Batiha, I. M., & Grassi, G. (2021). Novel convenient conditions for the stability of nonlinear incommensurate fractional-order difference systems. *Alexandria Engineering Journal, https://doi.org/10.1016/j.aej.2021.06.073*.

Shukla, A., & Prajapati, J. (2007). On a generalization of Mittag-Leffler function and its properties. *Journal of Mathematical Analysis and Applications*, *336(2)*, pp. 797–811.

Singh, H., Pandey, R. K., Singh, J., & Tripathi, M. (2019). A reliable numerical algorithm for fractional advection–dispersion equation arising in contaminant transport through porous media. *Physica A: Statistical Mechanics and its Applications*, *527*, pp. 121077.

Singh, J., Kumar, D., Qurashi, M. A., & Baleanu, D. (2017). A novel numerical approach for a nonlinear fractional dynamical model of interpersonal and romantic relationships. *Entropy*, *19(7)*, pp. 375.

Soloklo, H. N., & Bigdeli, N. (2020). Direct approximation of fractional order systems as a reduced integer/fractional-order model by genetic algorithm. *Sādhanā*, *45(1)*, pp. 1–15.

Sondhi, S., & Hote, Y. V. (2014). Stability testing and IMC based fractional order PID controller design for heating furnace system. In *2014 Annual IEEE India Conference (INDICON)*. IEEE. pp. 1–5.

Sousa, J. V. d. C., & De Oliveira, E. C. (2018). On the ψ-Hilfer fractional derivative.

Communications in Nonlinear Science and Numerical Simulation, *60*, pp. 72–91.

Tavares, D., Almeida, R., & Torres, D. F. (2016). Caputo derivatives of fractional variable order: Numerical approximations. *Communications in Nonlinear Science and Numerical Simulation, 35*, pp. 69–87.

Tavazoei, M., & Asemani, M. H. (2020). On robust stability of incommensurate fractional-order systems. *Communications in Nonlinear Science and Numerical Simulation*, *90*, pp. 105344.

Tavazoei, M. S. (2020). Passively realizable approximations of non-realizable fractional order impedance functions. *Journal of the Franklin Institute*, *357(11)*, pp. 7037–7053.

Teodoro, G. S., Machado, J. T., & De Oliveira, E. C. (2019). A review of definitions of fractional derivatives and other operators. *Journal of Computational Physics*, *388*, pp. 195–208.

Tigan, G., & Turaev, D. (2011). Analytical search for homoclinic bifurcations in the Shimizu-Morioka model. *Physica D: Nonlinear Phenomena*, *240(12)*, pp. 985–989.

Titz, S., Kuhlbrodt, T., & Feudel, U. (2002a). Homoclinic bifurcation in an ocean circulation box model. *International Journal of Bifurcation and Chaos*, *12(04)*, pp. 869–875.

Titz, S., Kuhlbrodt, T., Rahmstorf, S., & Feudel, U. (2002b). On freshwater-dependent bifurcations in box models of the interhemispheric thermohaline circulation. *Tellus A: Dynamic Meteorology and Oceanography*, *54(1)*, pp. 89–98.

Toh, Y. T., Phang, C., & Loh, J. R. (2019). New predictor-corrector scheme for solving nonlinear differential equations with Caputo-Fabrizio operator. *Mathematical Methods in the Applied Sciences*, *42(1)*, pp. 175–185.

Uçar, S., Uçar, E., Özdemir, N., & Hammouch, Z. (2019). Mathematical analysis and numerical simulation for a smoking model with Atangana–Baleanu derivative. *Chaos, Solitons and Fractals*, *118*, pp. 300–306.

Veeresha, P., & Prakasha, D. (2020). Solution for fractional generalized Zakharov equations with Mittag-Leffler function. *Results in Engineering*, *5*, pp. 100085.

Velasco, J. A., Estrada, F., Calderón-Bustamante, O., Swingedouw, D., Ureta, C., Gay, C., & Defrance, D. (2021). Synergistic impacts of global warming and thermohaline circulation collapse on amphibians. *Communications Biology*, *4(1)*, pp. 1–7.

Wang, J. L., Adhikari, G., Kobayashi, H., Tsukiji, N., Hirano, M., Kurihara, K., Nagahama, A., Noda, I., & Yoshii, K. (2019). Analysis and design of operational amplifier stability using Routh-Hurwitz stability criterion. In *Applied Mechanics and Materials*, vol. 888. Trans Tech Publications Ltd. pp. 1–10.

Wang, J. L., Tsukiji, N., & Kobayashi, H. (2018). Design of operational amplifier phase margin using Routh-Hurwitz method. In *Proceedings of International Conference on Mechanical, Electrical and Medical Intelligent System 2018*.

Wang, W. S. (2016). Some properties of k-gamma and k-beta functions. In *ITM Web of Conferences*, vol. 7. EDP Sciences. pp. 07003.

Wei, Z., & Zhang, X. (2021). Chaos in Shimizu-Morioka model with fractional order. *Frontiers in Physics*, *9*, pp. 38.

Wrzesiński, D., Marsz, A. A., Styszyńska, A., & Sobkowiak, L. (2019). Effect of the North Atlantic thermohaline circulation on changes in climatic conditions and river flow in Poland. *Water*, *11(8)*, pp. 1622.

Yakubu, H., & Aboiyar, T. (2018). A chaos-based image encryption algorithm using Shimizu-Morioka system. *International Journal of Communication and Computer Technologies*, *6(1)*, pp. 7–11.

Yang, X.-J., & Zhang, Y. (2012). A new successive approximation to non-homogeneous local fractional Volterra equation. *Advances in Information Technology and Management*, *1(3)*, pp. 138–141.

Yang, Y., Liao, X., & Dong, T. (2017). Anti-control of Hopf bifurcation in the Shimizu–Morioka system using an explicit criterion. *Nonlinear Dynamics*, *89(2)*, pp. 1453–1461.

Yu, S., Tang, W. K., Lu, J., & Chen, G. (2008). Generation of $n \times m$-wing Lorenz-like attractors from a modified Shimizu–Morioka model. *IEEE Transactions on Circuits and Systems II: Express Briefs*, *55(11)*, pp. 1168–1172.

Zeidan, D., Chau, C. K., Lu, T.-T., & Zheng, W.-Q. (2020). Mathematical studies of the solution of Burgers' equations by Adomian decomposition method. *Mathematical Methods in the Applied Sciences*, *43(5)*, pp. 2171–2188.

Zhang, X. (2019). Stability and bifurcation analysis for a delayed Shimizu-Morioka model. *Miskolc Mathematical Notes*, *20*(1), pp. 585–598.

Zhou, Y., Liu, H., Cao, J., & Li, S. (2019). Composite learning fuzzy synchronization for incommensurate fractional-order chaotic systems with time-varying delays. *International Journal of Adaptive Control and Signal Processing*, *33(12)*, pp. 1739–1758.

Zourmba, K., Oumate, A., Gambo, B., Effa, J., & Mohamadou, A. (2019). Chaos in the incommensurate fractional order system and circuit simulations. *International Journal of Dynamics and Control*, *7(1)*, pp. 94–111.