HEAT AND MASS TRANSFER OF NANOFLOWD FLOW IN THE PRESENCE OF VARIABLE STREAM CONDITIONS

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Dedicated To

My brother, MUHAMMED
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ABSTRACT

The effects of variable thickness, hydromagnetic flow, Brownian motion, heat generation, Thermophoresi, thickness, chemical reaction, porous medium, Lewis number and Prandtl number on heat and mass transfer characteristics and mechanical properties of a moving surface embedded into cooling medium consists of water with nanoparticles are studied. The governing boundary layer equations are transformed to ordinary differential equations. These equations are solved numerically using fourth-fifth Runge-Kutta Fehlberg method with shooting technique. The velocity, temperature, and concentration profiles within the boundary layer are plotted and discussed in details for various values of all mentioned parameters, effect of the cooling medium and flatness on the mechanical properties of the surface are investigated to find the most effective parameters.
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Values of $f''(0)$ and $-\theta'(0)$, and $-\phi'(0)$ for various values of parameter $\gamma$ and $\delta$, when $M=n=\beta=0.5$, $Nb=Nt=\lambda=0.1$, $Le=2$, $Pr=7.0$

Values of $f''(0)$ and $-\theta'(0)$, and $-\phi'(0)$ for various values of parameter $\gamma$ and $\lambda$, when $M=n=\beta=\delta=0.5$, $Nb=Nt=0.1$, $Le=2$, $Pr=7.0$
**LIST OF SYMBOLS AND ABBREVIATIONS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>constant</td>
</tr>
<tr>
<td>$B$</td>
<td>magnetic field</td>
</tr>
<tr>
<td>$b$</td>
<td>constant</td>
</tr>
<tr>
<td>$d$</td>
<td>diameter</td>
</tr>
<tr>
<td>$C$</td>
<td>nanoparticle fraction</td>
</tr>
<tr>
<td>$C_d$</td>
<td>convection diffusivity</td>
</tr>
<tr>
<td>$C_w$</td>
<td>Surface concentration</td>
</tr>
<tr>
<td>$C_\infty$</td>
<td>ambient concentration</td>
</tr>
<tr>
<td>$D_B$</td>
<td>Brownian diffusion coefficient</td>
</tr>
<tr>
<td>$D_T$</td>
<td>thermophoretic diffusion coefficient</td>
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<tr>
<td>$f(\eta)$</td>
<td>dimensionless stream function</td>
</tr>
<tr>
<td>$h$</td>
<td>function of x</td>
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<tr>
<td>$j$</td>
<td>the flux</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity</td>
</tr>
<tr>
<td>$K$</td>
<td>permeability of the porous medium</td>
</tr>
<tr>
<td>$k_{nf}$</td>
<td>thermal conductivity of nanofluid</td>
</tr>
<tr>
<td>$k_1$</td>
<td>rate of chemical reaction</td>
</tr>
<tr>
<td>$Le$</td>
<td>Lewis number</td>
</tr>
<tr>
<td>$M$</td>
<td>Magnetic parameter</td>
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<tr>
<td>$n$</td>
<td>Shape parameter</td>
</tr>
<tr>
<td>$N_b$</td>
<td>Brownian motion parameter</td>
</tr>
<tr>
<td>$N_t$</td>
<td>thermophoresis parameter</td>
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</table>
\( p \) \hspace{1cm} \text{dimensionless fluid pressure}  \\
\( Pr \) \hspace{1cm} \text{Prandtl number}  \\
\( Q \) \hspace{1cm} \text{heat generation}  \\
\( Q_0 \) \hspace{1cm} \text{Constant of heat generated}  \\
\( r \) \hspace{1cm} \text{radius}  \\
\( Re \) \hspace{1cm} \text{Reynolds number}  \\
\( T \) \hspace{1cm} \text{Fluid temperature}  \\
\( T_w \) \hspace{1cm} \text{Surface temperature}  \\
\( T_0 \) \hspace{1cm} \text{Ambient fluid temperature}  \\
\( t \) \hspace{1cm} \text{Time}  \\
\( u, v \) \hspace{1cm} \text{Velocity components along the } x \text{ and } y \text{ directions}  \\
\( U_w \) \hspace{1cm} \text{velocity of the moving surface}  \\
\( U_\infty \) \hspace{1cm} \text{free stream Velocity}  \\
\( V \) \hspace{1cm} \text{velocity}  \\
\( x, y \) \hspace{1cm} \text{Cartesian coordinates along the plate and normal to it, respectively} \\

**Greek symbols**  \\
\( \alpha \) \hspace{1cm} \text{Thermal diffusivity of the nanofluid}  \\
\( \phi \) \hspace{1cm} \text{Dimensionless concentration}  \\
\( \lambda \) \hspace{1cm} \text{Heat source parameter}  \\
\( \eta \) \hspace{1cm} \text{Similarity variable}  \\
\( \mu \) \hspace{1cm} \text{Dynamics viscosity}  \\
\( \nu \) \hspace{1cm} \text{Kinematic viscosity}  \\
\( \theta \) \hspace{1cm} \text{Dimensionless temperature}  \\
\( \rho \) \hspace{1cm} \text{Density}  \\
\( \rho_{nf} \) \hspace{1cm} \text{Density of nanofluid}  \\
\( \psi \) \hspace{1cm} \text{Stream function}  \\
\( \beta \) \hspace{1cm} \text{Shape parameter}  \\
\( \gamma \) \hspace{1cm} \text{Chemical reaction parameter}  \\
\( \delta \) \hspace{1cm} \text{Porous medium parameter}  \\
\( \omega \) \hspace{1cm} \text{is the dissipation function}
<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
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<tbody>
<tr>
<td>$f$</td>
<td>Fluid fraction</td>
</tr>
<tr>
<td>eff</td>
<td>effective</td>
</tr>
<tr>
<td>bf</td>
<td>basefluid</td>
</tr>
<tr>
<td>$\infty$</td>
<td>Infinity</td>
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<tr>
<td>$W$</td>
<td>wall</td>
</tr>
<tr>
<td>$P$</td>
<td>particle</td>
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<tr>
<td>$HD$</td>
<td>Heat diffusion</td>
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<table>
<thead>
<tr>
<th>Superscript</th>
<th>Description</th>
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<tbody>
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<td>'</td>
<td>Differentiation with respect to $\eta$</td>
</tr>
<tr>
<td>Appendix A</td>
<td>MAPLE PROGRAMMING TO FIND VELOCITY TEMPERATURE AND CONCENTRATION PROFILE</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.0 Research Background

In recent years, the study of fluid flow with nanoparticles in base fluids has attracted the attention of several researchers due to its various applications to science and engineering problems. Recent investigations on convective heat transfer in nanofluids indicate that the suspended nanoparticles markedly change the transport properties and thereby the heat transfer characteristics compared to the base fluid which is useful in many applications such as (engine cooling, solar water heating, cooling of electronics, cooling of transformer oil, improving diesel generator efficiency, cooling of heat exchanging devices, improving heat transfer efficiency of chillers, domestic refrigerator-freezers, cooling in machining, in nuclear reactor, etc). According to American Institute of Physics 2013, in the year 2011 alone, there were nearly 700 research articles where the term nanofluid was used in the title, showing rapid growth from 2006 (175) and 2001 (10) researches. Improved heat transfer using new methods makes significant savings in energy costs and protecting the environment. Methods such as using suspensions instead of fluid factor in heat transfer equipment and the use of finned surfaces to enhance heat transfer surfaces and the use of sub-surfaces to disrupt laminar layer in the boundary layer of turbulent flow and to create secondary flows in order to better incorporate the flow factor in a heat transfer environment are the new techniques. The purpose of this study is to investigate the influence of parameters of fluid flow and heat transfer of nanofluid in a two-dimensional surface.
1.1 Problem Statement

The heat and mass transfer of nanofluid flow over horizontal plate is an important types of flow because of the heat, mass transfer occur in both engineering and science processes. Parameters like velocity, temperature, Brownian, thermophoresis, shape, magnetic, viscosity, heat source, flatness effect on the mechanical properties of the surface and many others are making difference within the boundary layer for example, according to physical properties of most of realistic fluids, the viscosity and the thermal conductivity are usually related to the temperature and may vary dramatically with temperature. Thus, it would be more reasonable to investigate the effects of parameters by mathematical modelling, so to do that the following challenges arise

1. How to generate the mathematical model?
2. What are the effects of each parameter?
3. How to reduce or increase these effects by manipulating with other parameter?
4. What are the comparisons toward the velocity, temperature and concentration profiles at every change?

1.2 Objectives

The objectives of this thesis are:

(a) determine effects of each parameter on the rate of skin friction, heat transfer and mass transfer
(b) determine the effects of these parameter on velocity, temperature and concentration profile
(c) compare the effect of parameter in different conditions.

1.3 Importance of the Study

The correlation of different stream conditions of nanofluids and the mechanical properties of fluids together with the heat transfer and flow of nanofluid is the signification of this search. Boundary layer equations are ordinary differential equations these equations can solve for general conditions. The velocity, temperature,
and concentration with various values of the different parameters effects of all above on the heat and mass transfer of nanofluid flow be analyze by solving boundary layer equations and to be controlled on these effect.

1.4 Scope of Study

This study focused on the nanofluid flow over horizontal plate. This is introduced by partial differential equation (PDE), and by using the similarity transformation to reduce partial differential equation (PDE) into ordinary differential equation (ODE). Then, we will get the approximate solution by using Maple programming.
In this study we shall focus on the effects of each parameter on heat and mass transfer of horizontal flow of nanofluids.

1.5 Thesis Outline

The first chapter establishes the purpose of the research, the statement of the problem, the scope of the research and the methods to be used. Chapter 2 presents a review of the relevant literature dealing with nanoparticles to introduce nanofluid, to introduce characteristics of nanofluid and mechanisms used to measure these characteristics. Chapter 3 presents the method that will be used along the analysis, how to reduce the governing equation to the dimensionless form. It also shows the equations that are used in the shooting technique with MAPLE 18. Chapter 4 presents the result and discussion on what finding that have been obtained from the simulation, how the prescribe parameters effects on velocity, temperature and concentration profiles. Chapter 5 presents the conclusion on the objectives that has been achieved. Some recommendations for future work are also mentioned in this chapter.
CHAPTER 2

LITERATURE REVIEW

2.0 Nanofluid

Nanofluids are stable colloidal suspensions of nano-materials (nanoparticles, nanorods, nanotubes, nanowires, nanofibers, nanosheets, other nanocomposites, or even nano-droplets and nano-bubbles) in common, base fluids, such as water, oil, ethylene-glycol mixtures (antifreeze), refrigerants, heat transfer fluids, polymer solutions, bio-fluids, and others. Nanoparticles are very small, nanometer-sized particles with their smallest dimension usually less than 100 nm (nanometers). The smallest nanoparticles, only a few nanometers in diameter, may contain a few thousand atoms. These nanoparticles can possess properties that are substantially different from their parent materials, and they may interact quite differently within their dynamic molecular structure with the base fluids, than the corresponding microparticles, and respond differently within different force-flux processes accompanied with mass energy transfers. Similarly, nanofluids may have properties that are substantially different from their base fluids, like much higher thermal conductivity, and other flow and heat transfer characteristics.

Increased thermal conductivity will result in higher heat transfer than that of the base (pure) fluid without dispersed nanoparticles. Measurements of the heat transfer coefficients of nanofluids have show that the heat transfer capability of water increased by 15% with a dispersion of less than 1 vol.% copper oxide nanoparticles and about 80% improvements in heat transfer with the dispersion of less than 3 vol.% alumina nanoparticles (Kostic M. 2013).

It should be noted that the observed heat transfer rates of nanofluids are much higher than those predicted by conventional heat transfer correlations, even when changes in thermo physical properties such as thermal conductivity, density, specific
heat, and viscosity are considered. It appears that the effect of particle size and number becomes predominant in enhancing heat transfer in nanofluids the main reasons may be listed as follows:

1. The suspended nanoparticles increase the surface area and the heat capacity of the fluid.
2. The suspended nanoparticles increase the effective (or apparent) thermal conductivity of the fluid.
3. The interaction and collision among particles, fluid and the flow passage surface are intensified.
4. The mixing fluctuation and turbulence of the fluid are intensified.
5. The dispersion of nanoparticles flattens the transverse temperature gradient of the fluid. (Xuan Y. et. al. 2000)

All of these results on thermal conductivity and heat transfer enhancement were from nanofluids containing metallic oxide nanoparticles. Even greater effects are expected for nanofluids that contain metal nanoparticles (such as Cu, Ag) rather than oxides (Sivashanmugam P. 2011). Therefore, there is great potential to “engineer” ultra-energy-efficient heat transfer fluids by choosing the nanoparticle material, as well as controlling particle size. The types of particles used in nanofluid might be categorized as follows:

- Metals, such as Au, Ag, Cu and Fe.
- Oxides, such as CuO, Cu₂O, Fe₂O₃, Al₂O₃, SiO₂ and TiO₂.
- Carbon nanotubes (CNTs), which have shown the highest conductivity enhancement: Single-walled (DWCNT) and multi-walled (MWCNT) are three types CNTs commonly used.
- Other particles such as Si compounds. (Das and Choi, 2006).

The previous research on nanofluid in a mathematical and engineering vision focused on the effects of the various parameters on the velocity, temperature and concentration profiles Table 2.1 states some of these studies with parameters had studied in these researches.
Table 2.1 some previous researches and the parameters included in these researches

<table>
<thead>
<tr>
<th>Research</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bhattacharyya, K. et. al. (2011). MHD boundary layer slip flow and heat transfer over a flat plate.</td>
<td>$M$</td>
</tr>
<tr>
<td>Qasim, Met, al. (2013). Heat transfer and mass diffusion in nanofluids over a moving permeable convective surface.</td>
<td>$Nb, Nt, Le, Pr, \gamma$</td>
</tr>
<tr>
<td>Noghrehabadi, A., et.al. (2014). Analyze of fluid flow and heat transfer of nanofluids over a stretching sheet near the extrusion slit.</td>
<td>$Nb, Pr, Nt$</td>
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<tr>
<td>Zaimi, K., et.al.. Boundary layer flow and heat transfer over a nonlinearly permeable stretching/shrinking sheet in a nanofluid.</td>
<td>$Nb, Nt, Le$</td>
</tr>
<tr>
<td>Noghrehabadi, A. et.al. (2014). Effects of variable viscosity and thermal conductivity on natural-convection of nanofluids past a vertical plate in porous media.</td>
<td>$Nb, Nt, Le$</td>
</tr>
<tr>
<td>Aziz, A et.al.. (2014). Steady Boundary Layer Slip Flow along with Heat and Mass Transfer over a Flat Porous Plate Embedded in a Porous Medium.</td>
<td>$M, Pr, \delta$</td>
</tr>
<tr>
<td>Haile, E et.al. (2014). Heat and Mass Transfer in the Boundary Layer of Unsteady Viscous Nanofluid along a Vertical Stretching Sheet.</td>
<td>$\gamma, Nb, Nt, Pr, M$</td>
</tr>
<tr>
<td>Khan, W. et.al. (2013). Boundary layer flow past a wedge moving in a nanofluid.</td>
<td>$Nt, Le, Nb$</td>
</tr>
<tr>
<td>Agarwala, P et.al. The boundary layer flow of nanofluids over an isothermal stretching sheet influenced by magnetic field.</td>
<td>$M$</td>
</tr>
<tr>
<td>Abdel-wahed, et.al. (2015). Flow and heat transfer over a moving surface with non-linear velocity and variable thickness in a nanofluids in the presence of Brownian motion.</td>
<td>$M, \lambda, Nt, Nb, Pr, Le, n, \beta$</td>
</tr>
<tr>
<td>Elbashbeshy, E. M. A et.al.. (2013). Flow and heat transfer over a moving surface with nonlinear velocity and variable thickness in a nanofluid in the presence of thermal radiation.</td>
<td>$n, \beta$</td>
</tr>
<tr>
<td>Rudraswamy, et.al. (2014). Influence of Chemical Reaction and Thermal Radiation on MHD Boundary Layer Flow and Heat Transfer of a Nanofluid over an Exponentially Stretching Sheet.</td>
<td>$\gamma, Nb, Nt$</td>
</tr>
</tbody>
</table>
2.1 Properties of Nanofluids

The four unique features observed are listed by (Das and Choi, 2006). The most important feature observed in nanofluids was an abnormal rise in thermal conductivity, far beyond expectations and much higher than any theory could predict.

- Nanofluids have been reported to be stable over months using a stabilizing agent.
- Large enhancement of conductivity was achieved with a very small concentration of particles that completely maintained the Newtonian behaviour of the fluid. The rise in viscosity was nominal; hence, pressure drop was increased only marginally.
- Particles size dependence. Unlike the situation with microslurries, the enhancement of conductivity was found to depend not only on particle concentration but also on particle size. In general, with decreasing particle size, an increase in enhancement was observed.

Das and Choi (2006) observed the following advantage of nanofluid

- High specific surface area and therefore more heat transfer surface between particles and fluids.
- High dispersion stability with predominant Brownian motion of particles.
- Reduced pumping power as compared to pure liquid to achieve equivalent heat transfer intensification.
- Reduced particle clogging as compared to conventional slurries, thus promoting system miniaturization.

- Adjustable properties, including thermal conductivity and surface wet ability, by varying particle concentrations to suit different applications.

Whereas Das and Choi (2006) observed the following drawbacks of nanofluid

- The particles settle rapidly, forming a layer on the surface and reducing the heat transfer capacity of the fluid.

- If the circulation rate of the fluid is increased, sedimentation is reduced, but the erosion of the heat transfer devices, pipelines, etc., increases rapidly.

- The large size of the particles tends to clog the flow channels, particularly if the cooling channels are narrow.

- The pressure drop in the fluid increases considerably.

- Finally, conductivity enhancement based on particle concentration is achieved (i.e., the greater the particle volume fraction is, the greater the enhancement and greater the problems, as indicated above).

2.2 Thermal Conductivity of nanofluid

Researchers have shown that the thermal conductivity of the nanofluid is a function of thermal conductivity of the base fluid and the nanoparticle material, the volume fraction, the surface area, and the shape of the nanoparticles suspended in the liquid, and distribution of the dispersed particles. (Efstathios E. 2014)

Nanofluids thermal conductivity depends on many factors, like: (1) nanoparticle type, (2) nanoparticle size, (3) nanoparticle shape, (4) nanoparticle concentration in base fluid, (5) base fluid type, (6) additives and clustering, (7) temperature, (8) possibly temperature gradient, and other unknown factors which influence nanoparticle-base fluid molecule’s interactions during the conduction heat transfer (Kostic et. al.2013).

The first models proposed for solid–liquid mixture with relatively large particles (Evans et. al.2006). It was based on the solution of heat conduction equation through a stationary random suspension of spheres. The effective thermal conductivity is given by
\[ k_{\text{eff}} = \frac{k_p + 2k_{bf} + 2\varnothing(k_p - k_{bf})k_p}{2k_{bf} - \varnothing(k_p - k_{bf})k_{bf}} \]  

(2.1)

Where

- \( k_p \) is the thermal conductivity of the particles,
- \( k_{\text{eff}} \) is the effective thermal conductivity of nanofluid,
- \( k_{bf} \) is the base fluid thermal conductivity,
- \( \varnothing \) is the volume fraction of the suspended particles.

The general trend in the experimental data is that the thermal conductivity of nanofluids increases with decreasing particle size. This trend is theoretically supported by two mechanisms of thermal conductivity enhancement; Brownian motion of nanoparticles and liquid layering around nanoparticles. (Timofeeva et. al. 2011)

2.2.1 Viscosity of nanofluid

Viscosity of nanofluid determined the effective viscosity of a suspension of spherical solids as a function of volume fraction (volume concentration lower than 5%) using the phenomenological hydrodynamic equations. This equation was expressed by

\[ \mu_{\text{eff}} = (1 + 2.5\varnothing)\mu_{bf} \]

Where

- \( \mu_{\text{eff}} \) is the effective viscosity of nanofluid,
- \( \mu_{bf} \) is the base fluid viscosity,
- \( \varnothing \) is the volume fraction of the suspended particles.

Brinkman (1952) presented a viscosity correlation that extended Einstein’s equation to suspensions with moderate particle volume fraction, typically less than 4%.

\[ \mu_{\text{eff}} = \mu_{bf} \frac{1}{(1 + \varnothing)^{2.5}} \]

For isotropic structure of suspension, the effective viscosity was given by:

(Timofeeva et. al. 2011)
\[ \mu_{\text{eff}} = (1 + 2.5\phi + 6.2\phi^2)\mu_b \]

### 2.2.2 Specific Heat and Density of Nanofluid

Using classical formulas derived for a two-phase mixture, the specific heat capacity of the nanofluid as a function of the particle volume concentration and individual properties can be computed using following equations (Eq2.5) (Xuan Y. 2000), (Kostic et. al. 2013):

\[ \rho_{\text{eff}} = (1 - \phi)\rho_b + \phi\rho_p \quad (2.2) \]

\[ (\rho C_p)_{\text{eff}} = (1 - \phi)(\rho C_p)_b + \phi(\rho C_p)_p \quad (2.3) \]

Where \( \rho \) is the density, \( \rho_{\text{eff}} \) the effective density of nanofluid, \( C_p \) specific heat

\( \rho C_p \) is a volumetric heat capacity.

### 2.3 Heat Transfer Performance Of Nanofluid

In light of all the mentioned, nanofluid property trends development of a heat transfer nanofluid requires a complex approach that accounts for changes in all important thermophysical properties caused by introduction of nonmaterials to the fluid. Understanding the correlations between nanofluid composition and thermo-physical properties is the key for engineering nanofluids with desired properties. The complexity of correlations between nanofluid parameters and properties described in the previous section and schematically presented on the figure below Figure 2.1, indicates that manipulation of the system performance requires prioritizing and identification of critical parameters and properties of nanofluids (Godson et. al. 2010)

For example the enhancements due to the high thermal conductivity of nanoparticles and the role of Brownian motion of nanoparticles on the
enhancement of thermal conductivity, which is due to the larger surface area of nanoparticles for molecular collisions. It is known that the physical properties of fluid may change significantly with each others, temperature, viscosity, velocity, density, etc. For lubricating fluids, heat generated by the internal friction and the corresponding rise in temperature affects the viscosity of the fluid and so the fluid viscosity can no longer be assumed constant. The increase of temperature leads to a local increase in the transport phenomena by reducing the viscosity across momentum boundary layer and so the heat transfer rate at the wall is also affected. Therefore, to predict the flow behaviour accurately it is necessary to take into account the viscosity variation for incompressible fluids.

![Nanofluid Parameters Diagram]

Figure 2.1 Heat transfer performance of nanofluid. (Timofeeva E. 2011)

### 2.4 Heat Diffusion of nanofluid

Diffusion of heat occurs by the net transport of heat from a higher temperature region to a lower temperature one by random molecular motion. The result of diffusion is a progressive equilibrium temperature. Conduction is the transfer of heat by direct contact of molecules. Heat is transferred by conduction when adjacent atoms vibrate against each other. Heat conduction in a stationary fluid could be simulated as the diffusion of particles into a fluid. Normally one can
estimate the rate of heat diffusion in a solid or stationary fluid by the thermal diffusivity as

$$\alpha = \frac{k}{\rho c_p} \quad (2.4)$$

Where
- $k$ is the thermal conductivity of the medium,
- $\rho c_p$ is a volumetric heat capacity.

Based on the thermal diffusivity the time required for the heat to diffuse into a liquid by a distance equal to the size of the particle is (Sivashanmugam, 2010)

$$\tau_{HD} = \frac{d^2}{6\alpha} = \frac{4\pi^2 c_p \rho}{6k_f} \quad (2.5)$$

Where, $\alpha$ is thermal diffusivity, $c_p$ Specific heat, $d$ Diameter, $r$ Radius, $\tau_{HD}$ Time scale of heat diffusion. (Efstatios E. 2014)

2.5 Nanoconvection

The energy transport due to the convection of the nanoparticles caused by the Brownian motion is called nanoconvection. Nanoconvection is the mechanism that could be responsible of the energy transport in nanofluid systems. The rate of heat transfer due to convection could be calculated by convection diffusivity. Convection diffusivity is the kinematic diffusivity ($c_d$) of the liquid, which is also known as the momentum diffusivity

$$c_d = \mu / \rho \quad (2.6)$$

Where $c_d$ is convection diffusivity, $\mu$ the viscosity of the base liquid, $\rho$ is the liquid density. Hence, the time required for the transfer of the heat due to the convection effect at a distance equal to the nanoparticle size is given by

$$\tau_{(cd)} = \frac{d^2}{v} = \frac{4r^2}{v} \quad (2.7)$$
Where $\tau_{(cd)}$ is the time scale of convective diffusion, $\nu$ Kinematic viscosity.
(Sivashanmugam, 2010)

2.6 Thermophoresis

The phenomenon of diffusion of particles under the effect of temperature gradient is known as thermophoresis. Thermophoresis or thermodiffusion is take place when a mixture of different types of motile particles is subjected to the force of a temperature gradient and particles of different types respond to it differently (Sivashanmugam, 2010)

The thermophoretic velocity, $\nu_T$ can be found as

$$\nu_T = -\beta \frac{\mu}{\rho} \cdot \frac{\nabla T}{T} \quad (2.8)$$

Where an expression for the proportionality factor $\beta$, is given by

$$\beta = 0.26 \frac{k}{2k + k_p} \quad (2.9)$$

In Eq. (2.9), $k$ and $k_p$ are the thermal conductivity of the fluid and particle materials, respectively, $\nabla T$ temperature gradient, $T$ temperature. (Lazarus et. al. 2010)

2.7 Brownian motion

Brownian motion is the random movement of microscopic particles suspended in a liquid or gas, caused by collision with molecules of the surrounding medium. In nanofluid systems, due to the size of the nanoparticles Brownian motion takes place which can affect the heat transfer properties. The Brownian motion leads to diffusion of particles in accordance with the Fick’s law i.e.

$$j = -D \frac{dc}{dx} \quad (2.10)$$
Where \( c \) is the concentration, \( J \) is the flux, and \( D \) is the diffusion coefficient (Azizian et. al. 2009). It has found that the Brownian motion of nanoparticles at the molecular and nanoscale level is a key mechanism governing the thermal behavior of nanoparticle-fluid suspensions. It was concluded that Brownian motion is mainly responsible for the anomalous enhancement in thermal conductivity. This motion controls the temperature and the concentration of the particles within the boundary layer over the surface. The Brownian motion parameter \( N_b \) is the key of this mechanism such that the increasing of \( N_b \) leads to increasing of the boundary layer temperature and decreasing of the nanoparticles concentration (John H. 2008).

### 2.8 Chemical reaction

A chemical reaction is a process involving one, two or more substances (called reactants), characterized by a chemical change and yielding one or more product which is different from the reactants. A chemical change is defined as molecules attaching to each other to form larger molecules, molecules breaking apart to form two or more, smaller molecules, or rearrangement of atoms within molecules.

In chemical reaction theory, chemical reaction parameter is often represented by \( \gamma \),

\[
\gamma = \frac{v \cdot k_1}{U^2}
\]  \hspace{1cm} (2.11)

Where, \( v \) is the kinematic viscosity, \( k_1 \) is the rate of chemical reaction and \( U \) is the characteristic velocity. Chemical reaction classified according to the phase of reactants as below Figure 2.2 (Muhaimin 2007).

![Figure 2.2 Classification of chemical reaction](image-url)
2.9 Porous Media

A porous medium or a porous material is a material containing pores (voids). The skeletal portion of the material is often called the "matrix" or "frame". The pores are typically filled with a fluid (liquid or gas). The skeletal material is usually a solid, but structures like foams are often also usefully analyzed using concept of porous media.

A porous medium is most often characterised by its porosity. Other properties of the medium (e.g., permeability, tensile strength, electrical conductivity) can sometimes be derived from the respective properties of its constituents (solid matrix and fluid) and the media porosity and pores structure, but such a derivation is usually complex. Even the concept of porosity is only straightforward for a poroelastic medium.

Often both the solid matrix and the pore network (also known as the pore space) are continuous, so as to form two interpenetrating continua such as in a sponge. However, there is also a concept of closed porosity and effective porosity, i.e., the pore space accessible to flow. (Muhammad 2007).

Many natural substances such as rocks and soil (e.g., aquifers, petroleum reservoirs), zeolites, biological tissues (e.g. bones, wood, cork), and manmade materials such as cements and ceramics can be considered as porous media. Many of their important properties can only be rationalized by considering them to be porous media. The concept of porous media is used in many areas of applied science and engineering: filtration, mechanics (acoustics, aeromechanics, soil, mechanics, rock mechanics), engineering (petroleum engineering, bio-remediation, construction engineering), geosciences (hydrogeology, petroleum geology, geophysics), biology and biophysics, material science, etc.

2.10 Fundamental equations

2.10.1 Navier–Stokes equations

The Navier-Stokes (NS) equations are the fundamental partial differential equations that describe the flow of incompressible fluids. These equations present the statement of second law of Newton for fluid flow and relate the sum of the forces acting on an element of fluid to its rate of change of momentum or acceleration (Ahmed et.al.2012)
2.10.1.1 Continuity Equation

In fluid dynamics, the continuity equation states that, in any steady state process, the rate at which mass enters a system is equal to the rate at which mass leaves the system. The differential form of the continuity equation is:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0
\]  

(2.12)

Where

- \( \rho \) is fluid density,
- \( t \) is time,
- \( u \) is the flow velocity vector field.

In this context, this equation is also one of the Euler equations (fluid dynamics). The Navier–Stokes equations form a vector continuity equation describing the conservation of linear momentum. If \( \rho \) is a constant, as in the case of incompressible flow, the mass continuity equation simplifies to a volume continuity equation.

\[
\nabla \cdot u = 0
\]

Which means that the divergence of velocity field is zero everywhere (Kuo et. al. 2005).

2.10.1.2 Energy Equation

The energy equation is constituted from first law of thermodynamics. The transient two dimensional form of thermal energy conservation of fluid with dissipation term is given by

\[
\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \omega
\]  

(2.13)

Where \( \omega \) is the dissipation function defined as.

\[
\omega = 2\mu \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right] \] (Marion et. al. 1998)
2.10.1.3 Momentum Boundary Layer Equation

Consider the case of viscous incompressible fluid over sheet stretching with velocity $U_{w}(x)$ and the steady laminar boundary layer with the free stream velocity $U_{\infty}$. Considering the body forces are negligible and, the pressure and the viscosity of the fluid to be constant. The continuity and the NS equations may be written as. (Marion et. al. 1998)

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}) = - \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]

\[
(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}) = - \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\]  \hspace{1cm} (2.14)

2.10.2 Fick's Laws

The simplest description of diffusion is given by Fick's laws, which were developed by Adolf Fick in the 19th century:

1. The molar flux due to diffusion is proportional to the concentration gradient.
2. The rate of change of concentration at a point in space is proportional to the second derivative of concentration with space.

2.10.2.1 Fick's First Law of Diffusion

Writing the first law in a modern mathematical form:

\[
N_i = -D_i \nabla C_i
\]
Where for species $i$, $N_i$ is the molar flux, $D_i$ the diffusion coefficient, and $C_i$ is the concentration.

From the continuity equation for mass:

$$\frac{\partial C_i}{\partial t} + \nabla \cdot N_i = 0$$ (2.15)

### 2.10.2.2 Fick's Second Law of Diffusion

From first law we can derive Fick’s second law directly

$$\frac{\partial C_i}{\partial t} = D_i \nabla^2 C_i$$ (2.16)

Fick's second law of diffusion is a linear equation with the dependent variable being the concentration of the chemical species under consideration. Diffusion of each chemical species occurs independently. These properties make mass transport systems described by Fick's second law easy to simulate numerically. (Efstathios E. 2014)

### 2.11 Derivation of Boundary Layer Equations

According to the previously mentioned, to derive the equations of a steady, laminar, two dimensional flow of an incompressible nanofluid over flat in the presence of magnetic field, heat generation and chemical reaction, we need to the following

**Conservation of mass:**

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$$ (2.17)

**Momentum:**

$$\frac{\partial u}{\partial t} + u \nabla = -\frac{1}{\rho} \nabla p + \nu \nabla^2 u + -B_i u - \gamma_i u$$ (2.18)
Energy:

$$\frac{\partial T}{\partial t} + u \cdot \nabla T = \alpha (\nabla^2 T - \frac{1}{k} \nabla \cdot \mathbf{Q}_s)$$

(2.19)

And Concentration:

$$\frac{\partial C}{\partial t} + u \cdot \nabla C = D \nabla^2 C$$

(2.20)

Where $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$, $u = (u, v)$ is the velocity vector, $T$ is the temperature in the boundary layer, $T_\infty$ is the temperature of ambient fluid, $C$ is the species concentration in the boundary layer $C_\infty$ is the species concentration of ambient fluid $p$ is pressure $\rho$ is the density of the fluid, $\nu$ is the kinematic coefficient of viscosity, $\kappa$ the thermal conductivity, $\alpha$ the thermal diffusivity and $D$ the diffusion coefficient, $B$ is magnetic field parameter, and

$$Q_s = -\frac{4\sigma}{3\alpha_k} \nabla(n^2T^4)$$

Where $\alpha_k = a + \sigma_s$ where $a$ is absorption coefficient, $\sigma_s$ is the scattering coefficient, $n$ the refractive index and $\sigma$ the Stefan-Boltzmann constant.

2.12 Boundary layer thickness

Boundary layer thickness, $\delta_b$, is the distance across a boundary layer from the wall to a point where the flow velocity has essentially reached the 'free stream' velocity, $u_0$. This distance is defined normal to the wall, and the point where the flow velocity is essentially that of the free stream is customarily defined as the point where:

$$u(y) = 0.99u_0$$

For laminar boundary layers over a flat plate, the Blasius solution gives:
\[ \delta_b \approx \frac{4.91x}{Re_x} \]

Where

\[ Re_x = \frac{\rho u_\infty x}{\mu} \]

\( \delta_b \) is the overall thickness (or height) of the boundary layer, \( Re_x \) is the Reynolds Number, \( \rho \) is the density, \( u_\infty \) is the free stream velocity, \( x \) is the distance downstream from the start of the boundary layer, \( \nu \) is the kinematic viscosity, \( \mu \) is the dynamic viscosity (White et. al 2006).

\[ U_\infty \quad T_\infty \quad C_\infty \]

\[ U_w \quad T_w \quad C_w \]

Fig 2.3 Boundary layer thickness

2.13 Stream function

Stream function \( \psi(x, y, t) \) in the point \( P \) with two-dimensional coordinates \( (x, y) \) and as a function of time \( t \) for an incompressible flow is defined as (White F. et. al 2006).

\[ \psi = \int_A (u \, dy - v \, dx) \]

Which is an exact differential provided

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]
This is the condition of zero divergence resulting from flow incompressibility. Since

\[ \delta \psi = \frac{\partial \psi}{\partial x} \delta x + \frac{\partial \psi}{\partial y} \delta y \]

The flow velocity components have to be

\[ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \]  \hspace{1cm} (2.21)

2.14 Some Types of Flow

2.14.1 Steady Vs Unsteady Flow

When all the time derivatives of a flow field vanish, the flow is considered to be a steady flow. Steady-state flow refers to the condition where the fluid properties at a point in the system do not change over time. Otherwise, flow is called unsteady. Whether a particular flow is steady or unsteady, can depend on the chosen frame of reference. For instance, laminar flow over a sphere is steady in the frame of reference that is stationary with respect to the sphere. In a frame of reference that is stationary with respect to a background flow, the flow is unsteady. Turbulent flows are unsteady by definition. A turbulent flow can, however, be statistically stationary. (White F. et. al 2006).

For steady flow

\[ \frac{\partial v}{\partial t} = 0 \]

For unsteady flow

\[ \frac{\partial v}{\partial t} \neq 0 \]  \hspace{1cm} (2.22)

2.14.2 Laminar vs. Turbulent

Laminar flow occurs when a fluid flows in parallel layers, with no disruption between the layers. At low velocities, the fluid tends to flow without lateral mixing,
and adjacent layers slide past one another like playing cards. There are no cross-currents perpendicular to the direction of flow, nor eddies or swirls of fluids. In laminar flow, the motion of the particles of the fluid is very orderly with all particles moving in straight lines parallel to the pipe walls. Laminar flow is a flow regime characterized by high momentum diffusion and low momentum convection.

When a fluid is flowing through a closed channel such as a pipe or between two flat plates, either of two types of flow may occur depending on the velocity of the fluid, laminar flow or turbulent flow. Laminar flow tends to occur at lower velocities. Turbulent flow is a less orderly flow regime that is characterised by eddies or small packets of fluid particles which result in lateral mixing. In non-scientific terms, laminar flow is smooth while turbulent flow is rough. (White M. 1998)

Figure 2.4 laminar and turbulent flow

2.14.3 Compressible Vs. Incompressible Flow

All fluids are compressible to some extent, that is, changes in pressure or temperature will result in changes in density. However, in many situations the changes in pressure and temperature are sufficiently small the changes in density are negligible. In this case the flow can be modelled as an incompressible flow. Otherwise the more general compressible flow equations must be use (Efstathios E. 2014). Mathematically, incompressibility is expressed by saying that the density \( \rho \) of a fluid parcel does not change as it moves in the flow field, i.e.,
This is equivalent to saying that
\[
\frac{\partial p}{\partial t} + u \cdot \nabla p = 0
\] (2.23)

2.15 Some Dimensionless Numbers

Most ideas for enhancing the heat transfer coefficient of fluids have focused on two strategies. The first is to increase the Nusselt number, which is dependent on the Reynolds number, the Prandtl number.

2.15.1 Reynolds Number

The flow in a tube, whether laminar or turbulent, depends on the relative importance of the inertia force in comparison with the viscous force. At relatively low values of the Reynolds number, the viscous force is relatively more important, and disturbances in the flow are damped out by viscosity. Thus, it is difficult for disturbances to grow and sustain themselves. On the other hand, at relatively large values of the Reynolds number, the damping of disturbances by viscosity is less effective, and inertia is more important, so that disturbances can perpetuate themselves.

\[
Re = \frac{\rho v L}{\mu} = \frac{vl}{v}
\] (2.24)

When \( Re < 2000 \) – laminar flow

When \( Re > 4000 \) – Turbulent flow

Here, \( \mu \) is the dynamic viscosity of the fluid, and \( \rho \) is the density of the fluid, \( L \) is a characteristic linear dimension. The ratio \( v = \mu/\rho \) is termed the kinematic viscosity. The experiments were conducted over a range of Reynolds numbers, \( 5 < Re < 110 \). The base fluids and the nanofluids all show increasing heat transfer coefficients as
the average flow velocity, and Reynolds numbers increase. (Sheikholeslam et. al. 2014)

2.15.2 Prandtl number

Is the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. That is, the Prandtl number is given as:

\[ Pr = \frac{\nu}{\alpha} = \frac{c_p \mu}{k} \]  \hspace{1cm} (2.25)

Where:

- \( \nu \): kinematic viscosity, \( \nu = \frac{\mu}{\rho} \), (SI units: m²/s)
- \( \alpha \): thermal diffusivity, \( \alpha = \frac{k}{\rho c_p} \), (SI units: m²/s)
- \( \mu \): dynamic viscosity, (SI units: Pa s = N s/m²)
- \( k \): thermal conductivity, (SI units: W/(m K))
- \( c_p \): specific heat, (SI units: J/(kg K))
- \( \rho \): density, (SI units: kg/m³).

When (Pr<<1, means thermal diffusivity dominates)

When (Pr>>1, means momentum diffusivity dominates)

In the presence of viscous dissipation the effect of increasing the values of Prandtl number Pr is to increase temperature distribution near the boundary and decrease everywhere away from the boundary.

Researchers show that the heat transfer rates decrease and mass transfer rates increase. As the Prandtl number Pr increase (White et. al 2006).

2.15.3 Lewis Number

(Le) is a dimensionless number defined as the ratio of thermal diffusivity to mass diffusivity. It is used to characterize fluid flows where there is simultaneous heat and mass transfer by convection. [14]
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