MHD STAGNATION FLOW OF NANOFUID PAST
A PERMEABLE SHRINKING SURFACE
WITH VARIOUS CONDITIONS

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A project report is submitted in partial
fulfillment of the requirement for the award of the
Degree of Master of Science (Applied Mathematics)

Faculty of Applied Science and Technology
Universiti Tun Hussein Onn Malaysia

JANUARY 2018
Specially dedicated to

MY PARENTS

(Their pure love, devotion, cares and prayers had help me to achieve this target)

MY SIBLINGS AND FRIENDS

(Their love, care, encouragement and motivation made me finish this valuable work)
ABSTRACT

The purpose of the present study is to analyze the effects of slip and heat source/sink on magnetohydrodynamic stagnation-point flow and heat transfer past a permeable shrinking surface with convective boundary condition. The velocity and temperature profiles are analyzed with respect to the involved parameter such as slip parameter, magnetic parameter, heat source/sink parameter, Brownian motion parameter, thermophoresis parameter, suction parameter, shrinking parameter, Prandtl number, Biot number and Lewis number. Using a similarity transformation, the governing partial differential equations are transformed into nonlinear ordinary differential equations and the numerical solutions are solved by bvp4c in Matlab software. The numerical results indicate that the velocity increases with the increase in the values of slip parameter and suction parameter. The temperature profile increases as the increase of the Biot number and heat generation parameter whereas the increases of the heat absorption reduce the temperature.
ABSTRAK

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CHAPTER 1

INTRODUCTION

1.1 Project Background

Nowadays, the abundance of practical applications in chemical and manufacturing process have received a great attention. This study focuses on the effects of slip and heat generation/absorption on magnetohydrodynamic stagnation-point flow and heat transfer over a permeable shrinking surface, with convective boundary conditions, in the presence of nanoparticle fractions.

Different from the flow over a stretching surface that have attracted many researchers, the flow over a shrinking surface was considered only several years ago in 2006 by Wang. In this type of flow, the fluid is stretched towards a slot and the flow is quite different from the stretching case. The physical reason is that the vorticity generated due to a shrinking sheet is not confined within the boundary layer and consequently a situation appears where some other external forces are to be imposed. Miklavcic and Wang (2006) have studied the viscous flow due to a shrinking sheet. Since the flow is unlikely to exist in unsteady flow, they introduced adequate suction at the boundary, to confine the vorticity within a boundary layer.

In this project, the fluid flow and heat transfer over a permeable shrinking surface is studied. By using appropriate similarity variables, the governing partial differential equations are reduced to the ordinary differential equations and then being solved numerically by bvp4c in Matlab. The effects of suction, heat
source/sink, Biot number and slip parameters on the skin friction coefficient and the
local Nusselt number as well as on the velocity and temperature profiles are plotted
and discussed.

1.1.1 Boundary Layer

Boundary layer is the region of retarded fluid near the surface of a body which
moves through a fluid or past which a fluid moves. It has lower rate of flow than the
bulk of the fluid because of the adhesion to the solid. It can also be known as the
boundary between object and free-flowing fluid. Within the boundary layer, there are
two different types of flow condition, that is, the flow can be laminar or turbulent.
The laminar boundary is a very smooth flow, while turbulent boundary layer contains
swirls or eddies. The laminar creates less skin friction drag than the turbulent flow,
but is less stable. Hence in this study, laminar flow has been used as the boundary
layer.

1.1.2 Nanoparticle

The analysis of boundary layer flow and heat transfer of nanofluid has received great
attention. The term nanofluid has been introduced by Choi and Eastman (1995) that
represent the mixture of nanoparticle (diameter smaller than 100nm) and base fluid
such as water, ethylene glycol and oil. The base fluid has low thermal conductivity.
Thus, the addition of the nanoparticle into the base fluid is able to enhance the heat
transfer performance of the base fluid. According to Saidur, Leong and Muhammad
(2011), nanofluid has the potential heat transfer fluids with enhanced thermophysical
properties and heat transfer performance can be applied in many devices for better
performances.
1.1.3 Magnetohydrodynamic

Magnetohydrodynamic (MHD) is the study of magnetic properties of electrically conducting fluids such as plasmas, liquid metals, salt water and electrolytes. The word magnetohydrodynamic is derived from magneto- meaning magnetic field, hydro- meaning water and -dynamics meaning movement.

MHD is widely used in the fields of engineering and metallurgy. In 1832, the earliest application of MHD is the electromagnetic pump that has been used in nuclear reactors. In the field of engineering, MHD is mostly used in the use of mortar electromagnet which is by putting the liquid metal that to be stirred into a rotating magnetic field.

1.1.4 Dimensionless Number

In this study, some important dimensionless numbers that are involved directly or indirectly will be discussed.

1.1.4.1 Nusselt Number, *Nu*

The Nusselt number is the ratio of convective to conductive, heat transfer across the boundary. The Nusselt number is defined as

\[ Nu = \frac{\text{Transport heat transfer}}{\text{Conductive heat transfer}} = \frac{hL}{k} \]

where \( h \) is the convective heat transfer coefficient of the flow, \( L \) is the characteristic length, \( k \) is the thermal conductivity of the fluid. The Nusselt number close to one represents the laminar flow while the larger Nusselt number is represents the turbulent flow.
1.1.4.2 Biot Number, $Bi$

Biot number is used in heat transfer calculations. It is ratio of the heat transfer resistance inside of and at the surface of a body. The Biot number is defined as

$$Bi = \frac{L h}{k}$$

where $h$ is a film coefficient or heat transfer coefficient or convective heat transfer coefficient, $L$ is a characteristic length, defined as the volume of the body divided by the surface area of the body, $k$ is a thermal conductivity of the body. The values of Biot number smaller than 0.1 imply that the heat conduction inside the body is much faster that the heat convection away from its surface, having this smaller Biot number can be assumed to be constant throughout the material’s volume.

1.2 Problem Statement

Nowadays, the study of flow and heat transfer has made great progress. The heat transfer of fluid flow is limited based on thermal properties of fluids. Base fluids such as water, ethylene glycol and oil have low thermal conductivity, for increasing the thermal conductivity needs to improve the thermal properties of the base fluids. Nanofluids have high thermal conductivities and abilities to increase the heat transfer. Thus, in this project the presence of nanoparticle fractions is considered.

However, there are many aspects such as the velocity, temperature, and nanoparticle concentration profiles that can be analyzed with respect to the involved parameters such as slip parameter, magnetic parameter, source/sink parameter, Brownian motion parameter, thermophoresis parameter, Prandtl number, Lewis number, Biot number, shrinking parameter and suction parameter. The method that has been used for the numerical solution is bvp4c in Matlab.

This research is to investigate the MHD stagnation point flow of nanofluid over a permeable shrinking surface in the presence of slip and magnetic field with
internal heat generation or absorption. Moreover, the purpose of this research is to analyze how the flow and temperature fields of a nanofluid within boundary layer are influenced by the amount of slip, applied magnetic field, heat generation or absorption and convective boundary condition. So, the analysis on velocity, temperature profiles, skin friction coefficient and local Nusselt number will be conducted.

1.3 **Significance of Study**

In this study, the numerical results obtained can be used by the other researchers especially in the field of engineering to be compared experimentally. If the comparison is satisfactory, then this model can go further to predict the results that may be more difficult to be obtained experimentally.

1.4 **Objectives**

The objective of the present work is to study the effect of slip and heat source/sink on magnetohydrodynamic (MHD) flow over a permeable shrinking surface with convective boundary conditions in the presence of nanoparticle which is the extension of a previous paper done by Nandy, Mahapatra and Wang. The resulting equations are solved numerically by using bvp4c in Matlab. Variations of several pertinent emerging parameters are analyzed in detail.

Meanwhile, the main focuses of the analysis are:

a) Obtaining the governing non linear boundary equations.
b) Finding the numerical solution by bvp4c in the Matlab.
c) Analyzing the temperature, velocity, skin friction coefficient and Nusselt number from the data obtained
1.5 Project Scope

There are several scopes of this project. The scopes are:

a) Considering a two-dimensional steady state boundary layer flow of nanofluid.
b) The utilization of Bougiorno model.
c) The utilization of bvp4c to find the numerical solution.
d) Finding temperature, velocity, skin friction coefficient and Nusselt number.
e) The solution depends on the slip parameter, source/sink parameter, shrinking parameter, suction parameter and Biot number.

1.6 Organization of Project

This project consists of an introductory chapter and four main chapters dealing with the following problems.

In chapter 1, the introduction, the beginning of all the research works that has been done for the past years. The problems that are encountered in this present work are listed in the problem statement. The aim and the objectives that have pushed this present work with numerous findings and data are provided consecutively.

In chapter 2, a collection of previous studies in several key areas of MHD stagnation flow in a nanofluid, slip boundary condition, heat source/sink and convective boundary condition are discussed. The data from the numerous existing researches from all over the world which are used for the research work purpose are provided in this chapter.

In chapter 3, the method of this project carried out is explained. Governing equations are converted into the non linear ordinary differential equations and then transformed by similarity transformation. The numerical solutions are solved numerically by bvp4c in Matlab software. For validation of Matlab software, data are analyzed with the previous presented results.

In chapter 4, the data and graph obtained from the numerical solution are presented and discussed briefly. The effect of the involved parameters on the
velocity, temperature and nanoparticle concentration profiles as well as skin friction coefficient and Nusselt number are tabulated and illustrated in the graph.

In chapter 5, the summary of this present work is given. Further recommendations are also discussed.
CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter discusses the literature reviews related to this project. The items of discussion are about what other people have done related to the effects of slip and heat generation/absorption on MHD stagnation flow of nanofluid past a shrinking surface with convective boundary conditions. There is also a discussion on the outcome result from other journals, articles and books.

2.2 MHD Stagnation Point Flow of Nanofluid

J. Hartman was the first that study about MHD by created the electromagnet pump on 1918 (Davidson, 2001). The study about a magnetic field on the incompressible laminar flow of electrically conducting fluid have a wide uses in the manufacturing industry. Pavlov (1974) has proposed the vorticity flow over a stretching sheet in the presence of uniform magnetic field and obtain an accurate similarity solution for the MHD boundary layer equations for the two-dimensional steady flow of electrically conducting fluid that caused by a stretching elastic planar surface in the presence of uniform transverse magnetic field (Khamisah, 2013). The electromagnetic effect plays
exactly the same role as the MHD, which is to reduce the horizontal flow resulting from stretching.

Stagnation point occurs when a flow impinges on a solid object. It is a point in a flow field where the local velocity of the fluid is zero. The stagnation region encounters the highest pressure, the highest rate of heat transfer and the highest rate of mass decomposition (Wang, 2008). According to Mabood, Pochai and Shateyi (2016), the study of a stagnation point flow is traced back to Hiemenz in 1911. He analyzed the two-dimensional stagnation point flow on a stationary plate using a similarity transformation. The study was to reduce Navier Stokes equations to non-linear ordinary differential equations and presents its exact solution (Wang, 2008; Malvandi, Hedayati and Ganji, 2014). Ibrahim and Daniel (2014) have studied about the boundary layer flow near stagnation points on a vertical surface with slip in the presence of transverse magnetic field. It is observed that the presence of magnetic field increases the skin-friction coefficient and the rate of heat transfer near the surface towards the stagnation point.

Buongiorno (2006) has proposed the increase factor of convective heat transfer in nanofluid and found that it is because of the viscosity and thinning of the laminar sublayer. However, his main finding is the recognition of Brownian motion and thermophoresis as the main of slip mechanism for the nanofluid. The stability model for the nanofluid transport has been emerged by considering the Brownian motion and thermophoresis. The Buongiorno model has been used to study about the nanofluid in the various geometry and boundary conditions. In 2013, Ibrahim et al have proposed MHD stagnation point flow and heat transfer due to a nanofluid towards a stretching sheet.

For the stagnation point flow over a shrinking sheet in a nanofluid, there are several researchers studied it such as stagnation point flow and mass transfer with chemical reaction past a permeable stretching/shrinking sheet in a nanofluid (Rosca, 2012). It is found that dual solution are exists for the shrinking case. Nandy and Mahapatra (2012) have proposed the effects of slip and heat generation/absorption on MHD boundary layer stagnation point flow and heat transfer over a stretching/shrinking surface. From the studied, it is found that the flow velocity, the temperature field and the nanoparticle concentration profiles are strongly influenced by the slip parameter. In 2015, Mahatha et al have studied about MHD stagnation
point flow of a nanofluid with velocity slip, non-linear radiation and Newtonian heating. It is found that nanoparticle volume fraction behaves as an increasing function of magnetic field, velocity partial slip, thermophoretic force and convective heating while it is decreasing function of stretching velocity and Brownian diffusion.

2.3 Slip Boundary Condition

The boundary conditions with no-slip flow are one of the important principles in the Navier Stokes theory. In certain situations, the assumption of no-slip flow is not applicable and should be replaced by the partial slip boundary conditions. Partial slips occur for fluid with particulate such as emulsion suspension, foams and polymer solutions. Certain models have been developed to discuss about the slip that occur at the boundary. A short description about the models has been proposed by Rao and Rajagopal (1999).

The slip flow over a permeable wall in a viscous fluid is proposed by Beavers (1967). For the flow behaviour and the shear stress in the fluid, there is a quiet different between has a slip flow at the wall boundary and there is no-slip flow. Wang (2003) proposed the stagnation flows with slip. He found the exact similarity solutions of the Navier-Stokes equations for stagnation point flow towards a plate with slip. The fluids that exhibit the boundary slip have important technological applications such as in the polishing of the artificial heart valves and internal cavities. Due to the increase of slip parameter, the range of velocity ratio parameter where the similarity solution exists increases (Krishnendu Bhattacharyya, 2011).

2.4 Permeable Shrinking Surface

There is a different between a stretching and a shrinking surface. For the shrinking surface, the fluid is stretched towards a slot and the flow is quite different from the stretching case. Different from nonlinearly stretching sheet, it is found that the solutions for a nonlinearly shrinking sheet are non-unique (Nandy, 2015).
It is noticed that the reverse flow occurs near the surface due to the shrinking effect but the shrinking effect can be reduced by applying a strong external magnetic field (Lok, 2011). The velocity on the boundary layer for the shrinking case is towards a fixed point (Wang, 2008). There are two conditions that the flow due to shrinking sheet is likely to exist, whether an adequate suction on the boundary is imposed (Miklavcic and Wang, 2006) or a stagnation flow is added (Wang, 2008). Wang (2008) found that solutions do not exist for larger shrinking rates and may be non-unique in the two-dimensional case and convective heat transfer decreases due to an increase in boundary layer thickness.

According to Zaimi K. (2014), both the skin coefficient and the local Nusselt number increases with increasing values of the suction parameter. Bachok et al (2012) found that mass suction widen the range of the stretching/shrinking parameter for which the solution exists. Nandy (2013) has studied the effects of slip and heat source/sink on MHD stagnation point flow in a nanofluid with convective boundary condition and found that the temperature of the fluid increases with the increase in heat source parameter and for a shrinking surface the solutions are non-unique.

### 2.5 Heat Generation/Absorption

In controlling heat transfer, the effect of heat generation/absorption on the heat transfer is very important. Heat generation/absorption also may be important in a weak electrically conducting polymeric liquid due to the non-isothermal situation and also due to the cation/anion salts dissolved in them (Abel, 2011). The effect of non-uniform heat source with suction/blowing, but confined to the case of viscous fluids only was analyzed by Abo-Eladahab and El Aziz (2004).

The previous study of heat generation/absorption by deformation impacts on magnetohydrodynamic (MHD) viscoelastic fluid over a nonlinear stretching sheet shows that the internal heat generation/absorption enhances or damps the heat transfer (Hady, 2013). Hsiao obtained the numerical solutions for the flow and heat transfer of a viscoelastic fluid over a stretching sheet with electromagnetic effects.
and non-uniform heat source/sink using the combination of finite difference method, Newton’s method, and Gauss elimination method (Ramesh, 2014).

2.6 Convective Boundary Condition

The convective boundary condition is usually used in the application of heat transfer problem. Merkin and Chaudhary (1994) proposed the natural convection boundary layer flow on a vertical surface with exothermic catalytic chemical reaction by using an asymptotic analysis (Makinde and Aziz, 2010). They found that the flow is controlled by activation energy, Prandtl numbers and heat of reaction. Convective boundary condition is more realistic and general especially for the engineering and industrial processes (Makinde and Aziz, 2010).

Yao, Fang and Zhong (2011) proposed the heat transfer of a viscous fluid with a convective boundary condition flow over a stretching/shrinking sheet. They found that the convective boundary conditions results in temperature slip at the wall and this temperature slip is greatly affected by the mass transfer parameter, the Prandtl number, and the wall stretching/shrinking parameters. Alsaedi, Awais and Hayat (2012) analyzed the stagnation point flow of nanofluid near permeable stretched surface with convective boundary condition to examine the influence of heat generation/absorption towards a linear stretching surface on the stagnation point flow of nanofluid.

2.7 Conclusion

In conclusion, the present study is to extend the work of Nandy and Mahapatra (2013) for effects of slip and heat source/sink on MHD stagnation flow of nanofluid past a permeable shrinking surface with convective boundary condition. The surface is taken permeable and exhibits convective heating boundary condition.
Different from a previous study, an appropriate similarity transformations reduce the governing partial differential equation into a system of nonlinear ordinary differential equations which are then solved numerically using bvp4c method in the Matlab.
CHAPTER 3

RESEARCH METHODOLOGY

3.1 Flowchart

The aim of the present study is to analyze the effect of slip and heat generation/absorption on MHD stagnation point flow in a nanofluid. The present work is based on the nanofluid model introduced by Buongiorno. The numerical results are tabulated and shown graphically to illustrate the influence of the slip parameter, heat source/sink parameter, suction parameter and Biot number on the velocity, temperature, skin friction coefficient and Nusselt number. Figure 3.1 describes the flowchart of the present work and Figure 3.2 shows the physical model and the coordinate system.

Figure 3.1 shows the flow of the present study. This project will be conducted by a few steps. It will be started with formulating the governing equations which is the partial differential equations and then the partial differential equations are converted into the nonlinear ordinary differential equations by using suitable similarity transformations. Next, the nonlinear ordinary differential equations subject to the boundary conditions are solved numerically by bvp4c in Matlab. If the data collected is sufficient, then it will be proceed to the output, if there is insufficient data, it will back to the first step which is the governing equations.
REFERENCES


