VIBRATION ANALYSIS OF A BEAM STRUCTURE ATTACHED WITH TWO DYNAMIC VIBRATION ABSORBERS

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A THESIS SUBMITTED AS FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF MASTER OF MECHANICAL ENGINEERING

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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

SEPTEMBER, 2014
ABSTRACT

A fixed end beam is a structural element supported at both sides which carries load primarily in flexure that may experience vibration as it does carry vertical loads and gravitational forces. Its exposure to vibration can lead to excessive deflections and failure of the structure. The aim of this research is to develop the application of dynamic vibration absorber on a fixed end beam structure. A classical mathematical model based on dynamic vibration absorber theory is improved by an analytical derivation until two degree of freedoms. The theoretical model is verified by experimental works. In experimental, two vibration absorbers were fabricated to be installed to the beam in four different conditions; and subjected to a force vibration frequency loading using an exciter. The resonance frequencies of interest were 11.23Hz and 35.45Hz. The vibration level that occurred on the beam is measured by comparing the effect of absorber presence to see the reduction in its amplitudes. Based on the experimental and theoretical analysis, both shows reduction in the beam amplitudes. From those results can be concluded that the dynamic vibration absorber has an ability to reduce and suppresses the beam vibration whereas the third condition has been chosen as the best arrangement where the persistent reduction results recorded 95 and 99 percents reduction of first and second DVA respectively. The knowledge gained from this research can be used to minimize the vibration amplitude of a structures and machines, increasing their life-span simultaneously. The other benefit comes from this research in specific or potential application aspect is it could control vibration in building or bridge structure and airplane wing flutter control.
ABSTRAK

Rasuk tetap merupakan satu struktur yang disokong pada kedua-dua hujung yang mungkin menanggung beban lenturan akibat getaran yang berpunca daripada beban menegak dan daya graviti. Pendedahan rasuk ini kepada getaran dalam satu jangka masa yang lama akan menyebabkan berlakunya lenturan yang berlebihan yang boleh mengakibatkan kegagalan pada strukturnya. Tujuan utama kajian ini dijalankan adalah untuk membangunkan aplikasi penyerap getaran dinamik pada struktur rasuk tetap. Model matematik klasik berdasarkan teori penyerap getaran dinamik ditambahbaik dengan satu terbitan analisis sehingga dua darjah kebebasan. Model teori ditentusahkan melalui ujikaji eksperimen. Untuk eksperimen, dua penyerap getaran telah difabrikasi untuk dipasang kepada rasuk dalam empat keadaan yang berbeza dan dikenakan beban getaran yang berlainan frekuensi dengan menggunakan penggetar. Frekuensi resonan yang menjadi tumpuan kajian adalah pada 11.23Hz dan 35.45Hz. Tahap getaran yang terhasil pada rasuk tetap diukur dengan membandingkan kesan kehadiran penyerap getaran untuk melihat pengurangan dalam amplitudnya. Data yang diperolehi menunjukkan terdapat pengurangan pada nilai amplitud rasuk bagi kedua-dua kaedah. Hasil daripada keputusan itu dapat disimpulkan bahawa penyerap getaran dinamik mempunyai keupayaan untuk mengurangkan dan menyekat getaran di mana penyerap getaran yang diletakkan dalam keadaan ketiga dipilih sebagai yang terbaik antara semua keadaan di mana pengurangan amplitud sebanyak 95 dan 99 peratus dicatat oleh penyerap getaran pada mod pertama dan kedua. Pengetahuan yang diperolehi daripada kajian ini boleh digunakan untuk meminimumkan amplitud getaran pada struktur dan mesin, sekaligus memanjangkan jangka hayatnya. Kelebihan lain daripada kajian ini secara spesifik atau dalam aspek potensi aplikasi adalah ia boleh diguna untuk mengawal getaran pada bangunan atau struktur jambatan dan juga sayap kapal terbang.
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<tr>
<td>D, d</td>
<td>Diameter</td>
</tr>
<tr>
<td>l</td>
<td>length</td>
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<tr>
<td>m</td>
<td>mass</td>
</tr>
<tr>
<td>r</td>
<td>radius</td>
</tr>
<tr>
<td>b</td>
<td>width</td>
</tr>
<tr>
<td>t</td>
<td>thickness</td>
</tr>
<tr>
<td>k</td>
<td>linear spring stiffness</td>
</tr>
<tr>
<td>f</td>
<td>frequency (Hz)</td>
</tr>
<tr>
<td>F, f</td>
<td>force</td>
</tr>
<tr>
<td>g</td>
<td>acceleration constant</td>
</tr>
<tr>
<td>(\omega)</td>
<td>undamped circular frequency (rad/s)</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>X, x</td>
<td>amplitude</td>
</tr>
<tr>
<td>cm</td>
<td>centimeter</td>
</tr>
<tr>
<td>m</td>
<td>meter</td>
</tr>
<tr>
<td>(\infty)</td>
<td>infinity</td>
</tr>
<tr>
<td>(\Omega)</td>
<td>natural circular frequency (rad/s)</td>
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<tr>
<td>DVA</td>
<td>dynamic vibration absorber</td>
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<td>UTHM</td>
<td>Universiti Tun Hussein Onn Malaysia</td>
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CHAPTER 1

INTRODUCTION

Until early this century, machines and structures usually had very high mass and damping, because heavy beams, timbers, castings and stonework were used in their construction. Since the vibration excitation sources were often small in magnitude, the dynamic response of these highly damped machines was low. However, with the development of strong lightweight materials, increased knowledge of material properties and structural loading, and improved analysis and design techniques, the mass of machines and structures built to fulfil a particular function has decreased. Furthermore, the efficiency and speed of machinery have increased so that the sources which can create intense vibration problems. The demands made on machinery, structures and dynamic systems are also increasing, therefore the dynamic performance requirements are always rising. There have been very many cases of systems failing or not meeting performance targets because of resonance, fatigue and excessive vibration of one component. Because of the very serious effects which unwanted vibrations can have on dynamic systems, it is essential that vibration analysis be carried out as an inherent part of their design, when necessary modifications can most easily be made to eliminate vibration, or at least it as much as possible. To summarize, present-day machine and structures often contain high-energy sources which create intense vibration excitation problems, and modern construction methods resulted in systems with low mass and low inherent damping. Therefore careful design and analysis is necessary to avoid resonance or undesirable dynamic performance (Beards, 1995).
1.1 Research background

Vibration is mechanical oscillations, produced by regular or irregular period movements of a member or body from its rest position. Vibration can be a source of problem at an engineering level because resulting in damage and loss of control of equipment, and thus reducing the efficiency of operation in machines. Vibration can also cause a discomfort at a low level and at a high level it can risk the person safety (Inman, 2008).

Each vibrating structure has tendency to oscillate with larger amplitude at certain frequencies. These frequencies are known as resonance frequencies or natural frequencies of the structure. At these resonance frequencies, even a small periodic driving force can result in large amplitude vibration. When resonance occurs, the structure will start to vibrate excessively.

The primary method of eliminating vibration is at a source by designing the equipment and ensuring control over the manufacturing tolerances. Others method that can reduce the vibrations that generated by machinery is by modifying the system so that the natural frequencies are not close to the operating speed, to prevent large responses by including damping, install vibration isolating devices between adjacent sub-systems and the other way is include auxiliary mass into the equipment to reduce the response and absorb vibration (Ramamurti, 2008).

In this research, a new control strategy has been tested in order to absorb vibration. This vibration absorbing devices demonstrated as a good vibration absorber when applied on fixed-fixed end beam. In designing a good vibration absorber, the vibrations characteristic need to be studied completely.

Theoretically, every vibration system can be modelled by an equivalent mass-spring vibration system. The excessive vibration can be reduced by mean of a Dynamic Vibration Absorber (DVA). DVA is a device consisting of an auxiliary mass–spring system which tends to ‘absorb’ the vibration of a system to which it is attached (Rao, 2005; Inman, 2008). A classical DVA consists only a single pair of an auxiliary mass-spring system. This classical DVA only useful for a single degree of freedom system (Bonsel et al., 2004; Wong et al., 2007; Khazanov, 2007), hence limiting its application prospects. Figure 1.1 illustrates the result using dynamic vibration absorber.
Figure 1.1 shows the response of a system with and without the use of an absorber. In general, without absorber the primary system has a high peak at certain frequency which indicates the mode of the system. It can be seen from the figure that the absorber, while eliminating vibration at the known impressed frequency of the primary structure, introduces two resonant frequencies $\Omega_1$ and $\Omega_2$. In practice, the operating frequency must therefore be kept away from the frequencies $\Omega_1$ and $\Omega_2$. In addition, the argument states that the two invariant points in the frequency response curve of the primary system, its response amplitude is independent of the damping of the absorber system. The stiffness value that results in equal amplitudes at the invariant points is taken to be optimal.

The problem of vibration of a beam is of intrinsic interest because the beam represents the simplest of all engineering structural. The subject of vibrations is of fundamental importance in engineering and technology. In this research project, an analysis of a DVA for a multi degree of freedom system i.e. beams structures is going to be investigated. This special DVA can be used to control the vibration level of a building built in earthquake prone area, to control the vibration level of a bridge exposed to high speed or turbulence wind and to control airplanes wing flutter.
1.2 Problem statement

Excessive vibration in engineering systems are generally undesirable and therefore avoided for the sake of safety and comfort. Vibration has been known as the factor of disturbance, discomfort, damage and destruction. It could also lead to excessive deflections and failure on the machines and structures. Exposure to vibration for a long period also can be harmful where it causes disease and muscular-skeletal pain. Thus, vibration needs to be eliminated with the effective ways to prevent all the bad effects. It is possible to reduce untoward amplitudes by attaching to the main vibrating system an auxiliary oscillating system. Therefore, this study was undertaken to reduce the risk stated by producing a new design of dynamic vibration absorber (DVA) attached to a structure called fixed-fixed end beam.

Most of researches focus on the applications of absorbers in the system under harmonic excitations with a single frequency. However, many systems in real applications are excited more than two frequencies. In addition, only few research done in study the characteristics of DVA involving two DVAs in a system. Due to these circumstances, a research theoretically and experimentally using DVA which tuned up to second mode need to be conducted.

1.3 Research aim

This research aims to study on absorber system and its tuning for a fixed-fixed end beam.

1.4 Objective(s) of the research

The main objective of this research is to study the vibration characteristics of a new designed dynamic vibration absorber attached on fixed-fixed beam subjected to force vibration frequency loading.
Based on the research gaps identified in literature survey, this study embarks on the following objectives;

i) To develop a theoretical mathematical model for the development of DVA for two degree of freedom system based on classical vibration absorber equation.

ii) To verify the theoretical model of the DVA by experimental works.

iii) To determine the vibration characteristics of a vibration system attached with the developed DVA analytically and experimentally.

1.5 Scopes

The research is focused on fixed-fixed end beam subjected to two frequency loading. This research is limited according to the scopes below:

(a) Theoretical analysis to tune the DVA to the beam.
   - The analytical derivation only presented for the first two degree of freedoms.

(b) Calculation to determine the beam dimension so its do not exceed the frequencies produced by exciter motor.
   - The speed control of the motor is up to 3000 rev/min.

(c) Design and fabrication the beam holder
   - The beam rig designing process are restrained to the experimental lab equipment at UTHM vibration laboratory.
   - Fabricated in 2 pieces so that the beam attachment could be fixed at both sides.

(d) Determine the vibration levels of the beam by running a force vibration test so that a comparison before and after using DVA could be obtained.
   - This initial test run conduct until reach the motor's maximum speed.

(e) Calculation work to determine the appropriate design of DVA to get the optimum tuned mass absorber for the beam.
   - The optimum parameters such as stiffness, mass and natural frequency.
   - Calculate the $l$, length for the both sides absorber flyers.

(f) DVAs fabrication
   - Most of the absorber parts are made of aluminium material.
(g) Experimental work to measure the beam amplitude after mounting DVA.
- Two DVAs attached to the structure and vibrated using shaker.
- Conducted at vibration laboratory, UTHM.
- Check the optimum tuned mass absorber for fixed-fixed beam.

1.6 Expected outcomes

The results from this research is expected that the proposed dynamic vibration absorber will absorb the structure vibration levels at constant frequency range. Subsequently, it will achieve the objective where the vibration characteristic of a dynamic vibration absorber attached on a fixed-fixed beam is studied. Figure 1.2 describes the expected amplitude-frequency response obtained before and after using two DVAs. Refer to the graph, the important thing is that after using the absorbers the structure primary amplitudes expected to be lowered at certain frequency.

![Amplitude-frequency response](image)

Figure 1.2 : Expected amplitude-frequency response using DVA
1.7 Significant of research

In engineering history, excessive vibration has been a common problem in causing the fatigue life of structures shorter and the performance of machines reduces. The intensity of vibration sources around us is increasing and tolerances on allowable vibration levels are becoming more and more stringent. From this phenomena, we know that vibration affects the machines and structures life span. Due to this, it is necessary to come out for a solution by solving from its root.

Vibration also can be harmful and therefore should be avoided. The most effective way to reduce unwanted vibration is to suppress the source of vibration. Above this condition, this research was carried out to understand the vibration characteristic in order to design a dynamic vibration absorber due to the needs of vibration protection itself. As a result, it gave an idea on how to produce an effective absorber. The knowledge gained from this research can be used to minimize the vibration amplitude of a structures and machines, increasing their life-span simultaneously. A complete understanding of vibration is needed involves in the analysis and design of a vibration absorber devices so this are the importance why this study should be conducted.

This research also has its own novelty in theories and knowledge whereas the finding of this research is instrumental in terms of identifying key of theoretical and mathematical model in development of DVA for multi degree of freedom systems. The other benefit comes from this research in specific or potential application aspect is it could control vibration in building or bridge structure and airplane wing flutter control.
CHAPTER 2

LITERATURE STUDY

This chapter gives insight into vibration theory, vibration control, beam, dynamic vibration absorber (DVA) and previous research done on dynamic vibration absorber.

2.1 Concept of vibration

Variations in physical phenomena that take place more or less regularly and repeated themselves in respect to time are described as oscillations. In other words, any motion that repeats itself after an interval of time is called vibration or oscillation. The theory of vibration deals with the study of oscillatory motion of bodies and the associated forces. The oscillatory motion shown in Figure 2.1 below is called harmonic motion and is denoted as

\[ x(t) = X \cos \omega t \]  

The theory of vibration deals with the study of oscillatory motions of bodies and the forces associated with them (Rao, 2005). Vibration also is the study of the repetitive motion of objects relative to a stationary frame of reference or nominal position (usually equilibrium).
Any body having mass and elasticity is capable of oscillatory motion. In engineering, an understanding of the vibratory behaviour of mechanical and structural systems is important for the safe design, construction and operation of a variety of machines and structures. The failure of most mechanical and structural elements and systems can be associated with vibration. Vibration problems may be classified into the following types: (Rao, 2007)

1. Undamped and damped vibration
   - If there is no loss or dissipation of energy due to friction or other resistance during vibration of a system, the system is said to be undamped. If there is energy loss due to the presence of damping, the system is called damped as refer to Figure 2.2 and Figure 2.3.

![Figure 2.2: Undamped vibration](image)

![Figure 2.3: Damped vibration](image)
2. Free and forced vibration
- If a system vibrates due to an initial disturbance (with no external force applied after time zero), the system is said to undergo free vibration. On the other hand, if the system vibrates due to the application of an external force, the system is said to be under forced vibration.

3. Linear and nonlinear vibration
- If all basic components of a vibrating system (i.e., mass the spring and the damper) behave linearly, the resulting vibration is called linear vibration. However, if any of the basic components of a vibrating system behave nonlinearly, the resulting vibration is called nonlinear vibration. (Rao, 2007)

The physical explanation of the phenomena of vibration concerns the interplay between potential energy and kinetic energy. A vibrating system must have a component that stores potential energy and releases it as kinetic energy in the form of motion (vibration) of a mass and it is shown in Figure 2.4. The motion of the mass then gives up kinetic energy to the potential energy storing device. Vibration can occur in many directions and can be the result of the interaction from many objects (Inman, 1996).

![Figure 2.4: Elements of vibratory system where: M-mass (stores kinetic energy); K-spring (stores potential energy, support load) and C-damper (dissipates energy, cannot support load).](image)

Vibration can be caused by many types of excitation. These include:

1. Fluid flow
2. Reciprocating machinery
3. Rotating unbalanced machinery
4. Motion induced in vehicles travelling over uneven surfaces
5. Ground motion caused by earthquakes. (Palm III, 2007)

We cannot design a proper vibration suppression system without knowing what levels of vibration are harmful, or at least disagreeable. If the vibration affecting people, we need to know what levels affect health and comfort; if it affects buildings or some structure, the vibration levels at which damage may occur need to be known.

Figure 2.5: Specification of vibration levels on a monograph (Palm III, 2007)

Figure 2.5 shows a typical case for which the maximum allowable amplitudes of displacement, velocity and acceleration have been specified. The boundary formed by the lines corresponding to these maximum values defines the allowable operating region for the system. Acceleration values are often quoted as $rms$ values, which stands for root mean square.

Maximum acceleration amplitude is the limit most often specified for comfort and health, and it is often specified in terms of the gravitational acceleration constant $g$. Vibrations with frequencies above approximately 9Hz are normally beyond the threshold of perception by humans (Palm III, 2007).
2.2 Definitions and terminology

Several definitions and terminology are used to describe harmonic motion and other periodic functions. The motion of a vibrating body from its undisturbed or equilibrium position to its extreme position in one direction, then to the equilibrium position, then to its extreme position in the other direction, and then back to the equilibrium position is called a cycle of vibration.

The amplitude of vibration denotes the maximum displacement of a vibrating body from its equilibrium position. The amplitude of vibration is shown as $A$ in Figure 2.6. The period of oscillation represents the time taken by the vibrating body to complete one cycle of motion. The period of oscillation is also known as the time period and is denoted by $\tau$. In Figure 2.3, the time period is equal to the time taken by the vector $\overrightarrow{OP}$ to rotate through an angle $2\pi$. This yields

$$\tau = \frac{2\pi}{\omega} \quad (2.2)$$

Where $\omega$ is called circular frequency. The frequency of oscillation or linear frequency (or simply the frequency) indicates the number of cycles per unit time. The frequency can be presented as

$$f = \frac{1}{\tau} = \frac{\omega}{2\pi} \quad (2.3)$$

Note that $\omega$ is called circular frequency and is measured in radians per second, whereas $f$ is called the linear frequency and is measured in cycles per second (hertz).
2.3 Beams

One of the most common structural elements is a beam; it bends when subjected to loads acting transverse to its centroidal axis or sometimes to loads acting both transverse and parallel to this axis (Liew, 2005). A beam is a structural element that carries load primarily in bending (flexure). Beams generally carry vertical gravitational forces but can also be used to carry horizontal loads (i.e. loads due to an earthquake or wind). The loads carried by a beam are transferred to columns, walls, or girders, which then transfer the force to adjacent structural compression members. Beams are characterized by their profile (their shape of their cross-section), length, and material. In contemporary construction, beams are typically made of steel, reinforced concrete, or wood. One of the most common types of the steel beam is

Figure 2.6: Harmonic motion; projection of rotating vector (Rao, 2007)
the I-beam or wide flange beam. This type of beams is commonly used in steel-frame buildings and bridges.

The equation of motion for the transverse vibration of thin beams derived by Daniel Bernoulli in 1735, and the first solutions of the equation for various support conditions were given by Euler in 1744. Their approach has become known as the Euler-Bernoulli or thin beam theory. Rayleigh presented a beam theory by including the effect of rotary inertia. In 1921, Stephen Timoshenko presented an improved theory of beam vibration, which has become known as the Timoshenko or thick beam theory, by considering the effects of rotary inertia and shear deformation (Rao, 2007).

### 2.3.1 Fixed-ended beams

When ends of a beam are held so firmly that they are not free to rotate under the action of applied loads, the beam is known as a built-in or fixed-ended beam and it is indeterminate (Liew, 2005). Determining the natural frequency of any system helps us to find out how the system will behave when disturbed and left to vibrate, and to find what kind of excitation frequency to be avoided in the system.

Vibration analysis of a fixed-fixed beam system is important as it explains and help us analyse a number of real life systems. The following few figures tell us about the significance of analysing fixed-fixed beams and their relevance to the real world. For an example, Figure 2.7 shows the application fixed-fixed beam on suspension system.

![Figure 2.7: A fixed-fixed suspension system](image-url)
In machinery too, we can find parts which can be analysed as fixed-fixed beams like the following grinder and support system of a furnace in Figure 2.8 and Figure 2.9:

**Figure 2.8: Grinder**

**Figure 2.9: Furnace**

To determine the natural frequency of a fixed-fixed beam, the following parameters; stiffness and mass of the fixed-fixed beam are needed. The stiffness of fixed-fixed beam can be calculated by a simple equation from strength of materials.

\[
k = \frac{192EI}{L^3}
\]

(2.4)

Where; \( k \) = stiffness of system (N/m)

\( E \) = Young’s Modulus of the material (N/m²)

\( I \) = Area moment of inertia (m⁴)

\( L \) = Effective length of fixed-fixed beam (m)

The fundamental natural frequency \( \omega_n \) by definition is \( \omega_n = \sqrt{\frac{k}{m}} \) (radians/second)

(Rao, 2007)

### 2.3.2 Fixed-ended beam natural frequencies and mode shapes

Fixed boundary conditions are often difficult if not impossible to simulate experimentally, but they are important to consider in many applications. Figure 2.10 show the general mode shapes of beam fixed at both ends.
In this research, the natural frequency and the mode of vibration of the beam need to be found. By identified this, the parameters of the vibration absorber can be tuned to the beam natural frequency so that the beam vibration will be fully absorbed hence the vibration can be reduced. Because of the experimental scope will be conduct to a fix-fix beam, here, the formula of this type of boundary condition beam will be discussed.

![Figure 2.10: Mode shapes of fixed conditions at both ends](image)

Refer to Rao (2007), at a fixed end, the general transverse equation for free vibration solution is:

\[ w(x,t) = \sum_{n=1}^{\infty} w_n(x,t) \]

\[ = \sum_{n=1}^{\infty} \left[ (\cos \beta_n x - \cosh \beta_n x) - \frac{\cos \beta_n l - \cosh \beta_n l}{\sin \beta_n l - \sinh \beta_n l} (\sin \beta_n x - \sinh \beta_n x) \right] \times (A_n \cos \omega_n t + B_n \sin \omega_n t) \]

Then, applying the boundary condition simplified the equation to

\[ \omega_n = (\beta_n l)^2 \frac{EI}{\rho Al^4} \quad \text{Where; } \beta_n l \approx (2n + 1)\pi/2; \]

\[ \beta_1 l = 4.7300 \]
\[ \beta_2 l = 7.8532 \]
\[ \beta_3 l = 10.9956 \]
\[ \beta_4 l = 14.1372 \]

The constants \( A_n \) and \( \beta_n \) in the equation can be determined from the known initial conditions as in the case of a beam with fixed supported ends.
2.3.3 Beam system having a concentrated mass

There are many important engineering structures that can be modelled as beam carrying one, two or multi degree of freedom spring mass system. Figure 2.11 illustrates a clamped-clamped beam carrying concentrated masses. Examples of such applications include components of building and machine tools, vehicle suspensions, rotating machinery, accessories of machine structures and robotics amongst others. Because of these wide ranging applications the vibration behaviour of beams carrying discrete structural elements such as a one, two or multi degree of freedom spring-mass systems has received considerable attention for many years (Banerjee, 2003).

![Figure 2.11: A clamped-clamped beam carrying a two degree-of freedom spring mass system (Banerjee, 2003).](image)

2.4 Vibration analysis

A vibratory system is a dynamic system for which the response (output) depends on the excitations (input) and the characteristic of the system (e.g., mass stiffness and damping) as indicated in Figure 2.12 below. The excitation and response of the system are both time dependent. Vibration analysis of a given system involves
determination of the response for the excitation specified. The analysis usually involves mathematical modelling, derivation of the governing equation of motion, solution of the equations of motion, and interpretation of the response results.

![Figure 2.12](image-url) Input-output relationship of a vibratory system

The purpose of mathematical modelling is to represent all at the important characteristic of a system for the purpose of deriving mathematical equations that govern the behaviour of the system. The mathematical model is usually selected to include enough details to describe the system in terms of equations that are not too complex. The mathematical model may be linear or nonlinear, depending on the nature of the system characteristic. Once the mathematical model is selected, the principles of dynamics are used to derive the equations of motion of the vibrating system. For this, the free-body diagrams of the masses, indicating all externally applied forces (excitations), reaction forces, and inertia forces, can be used (Rao, 2007).

### 2.5 Dynamic vibration absorber

Traditional treatment methods that involve structural modifications are often time consuming and expensive. Blocking the problem frequencies in the variable frequency drive limits the use of the system by the user. One possible solution is an installation of a dynamic vibration absorber.

A dynamic vibration absorber (DVA) is a device consisting of an auxiliary mass-spring system which tends to neutralize the vibration of a structure to which it is attached. The DVA has certain advantages over other methods of vibration suppression. It is external to the structure, so no re-installation of equipment necessary. A DVA can be designed and tested before installation. In many scenarios, this offers an economical vibration reduction solution.
Figure 2.13: Two DVAs

Figure 2.13 depicts a two DVAs mounted on identical primary systems. $M_1$ and $M_2$ are the corresponding mass; $K_1$ and $K_2$ are the corresponding stiffness; $C_1$ and $C_2$ are the corresponding damping. It is assumed that identical forces with two harmonic components are applied to the primary mass of the system (Sun et al., 2007).

A vibration absorber is useful for situations in which the disturbance has a constant frequency. As opposed to a vibration isolator, which contains stiffness and damping elements, a vibration absorber is a device consisting of another mass and a stiffness element that are attached to the main mass to be protected from vibration. The new system consisting of the main mass and the absorber mass has two degrees of freedom, and thus the new system has two natural frequencies.

If the absorber is tuned so that its natural frequency coincides with the frequency of the external forcing, the steady state vibration amplitude of the main device becomes zero. From a control perspective, the absorber acts like a controller that has an internal model of the disturbance, which therefore cancels the effect of the disturbance (Eskinat et al.). If we know the frequency of the disturbing input and the natural frequency of the original system, we can select values for the absorber’s mass and stiffness so that the motion of the original mass is very small, which means that its kinetic and potential energies will be small. In order to achieve this small motion, the energy delivered to the system by the disturbing input must be “absorbed” by the absorber’s mass and stiffness. Thus the resulting motion absorber will be large.
2.6 Previous researches

The study of the dynamic and vibrations of mechanical systems is one of the important problems in industry. Suppression of unwanted vibrations is an important goal in many applications such as machines, tall buildings, bridges, offshore platforms, pipelines and aircraft cabins. A significant amount of work has been devoted to search for a suitable means to reduce the vibration level in these applications. Different concepts had been developed and employed in this research area. One of the concepts is using vibration absorber. Vibration absorber is a mechanical device, consists mainly of a mass, spring and damper, designed to have a natural frequency equal to the frequency of the unwanted vibration of the primary system.

Design of vibration absorbers has a long history. First vibration absorber proposed by Frahm in 1909 (Hartog, 1956) consists of a second mass-spring device attached to the main device, also modelled as a mass-spring system, which prevents it from vibrating at the frequency of the sinusoidal forcing acting on the main device (Eskinat et al.). According to Hartog (1956) in his book, the classical problem of damped vibration absorber that consists of a mass, spring and viscous damper attached to an undamped single degree of freedom system of which the mass is subject to harmonic forcing, has a well known solution. Eskinat et al. said that if damping is added to the absorber, the vibration amplitude of the main mass cannot be made zero at the forcing frequency but the sensitivity of the system to variations in the forcing frequency decreases. Also the vibration amplitude of the absorber mass decreases considerably with a damped absorber. In the literature, the term ‘vibration absorber’ is used for passive devices attached to the vibrating structure.

In the research by Khazanov (2007) he stated that a DVA sometimes referred as tuned mass damper. In its classical form, its natural frequency is tuned to match the natural frequency of the structure it is installed on. Because of this tuning, a DVA exerts a force on the main system that is equal and opposite to excitation force, cancelling vibration at the resonant frequency. In modern applications, the goal is to assure the performance within specifications over a wide frequency range while minimizing the size of the device (Khazanov, 2007).
Brennan (1998,2000) brought significant amount of knowledge in this field. Many of his research colleagues such as Dayou (2002), El-Khatib *et al.* (2005), Salleh (2007), Gao *et al.* (2011) and Jang (2012) also contribute on vibration absorbers.

### 2.6.1 Dynamic vibration absorber as controllers

Dynamic vibration absorbers (DVA) have been successfully used to attenuate the vibration of many structures. The DVA usually consists of a mass attached to the structures to be controlled through a spring mass-damper system. It is usually used to suppress a harmonic excitation at a given frequency (Sun *et al.*, 2007). Many researchers have considered the topic of vibration control in the last century but Ormondroyd and Den Hartog published the first mathematical treatment of passive system of vibration control 1928. The above-mentioned paper can be considered as the beginning of the systematic treatment of the problem an many works have appeared since then (Curadelli *et al.*, 2004).

The dynamic vibration absorber was invented in 1909 by Hermann Frahm, and since then it has been successfully used to suppress wind-induced vibration and seismic response in buildings. Characteristic of DVA were studied in depth by Den Hartog (Khazanov, 2007). The application of a DVA has been investigated by many authors, for example, Sun *et al.* (2007), in their research in finding a simple and effective vibration control of multi-frequency harmonic excitation by the comparison of different DVAs and frequency tuning methods. The comparisons they did are between single DVA, two DVA and state-switched absorber (SSA). In the research, they found that SSA and dual DVAs have better performance than single DVA. Another example of DVA usage by Sun *et al.* (2009), where they did implemented mathematical models of floating raft system with/without DVAs by assembling the mobility matrices of the subsystem. They investigated the vibration reduction performance of this passive DVA by numerical simulations based on vibration energy transmission and the power flow transmission characteristic of the floating raft system.

Other authors that interested in DVA research are Bonsel *et al.* (2004) and Wong *et al.* (2007). Damped and undamped DVA have been used by Bonsel *et al.* to
suppress first resonance of a piecewise linear system. They did investigate the possibility of reducing the vibrations of an archetype piecewise linear system using a linear DVA. The response from the research stated that the undamped DVA suppresses the first resonance. By calculations, it is proved that the damped DVA suppresses the harmonic resonance and also suppresses its super- and sub-harmonic resonances. The DVA produced by them was shown at Figure 2.14.

![Dynamic vibration absorber manufactured](image)

**Figure 2.14 : Dynamic vibration absorber manufactured (Bonsel et.al., 2004)**

Differs from Bonsel et al (2004), Wong (2007) proposed a new DVA combining a translational with a rotational-type absorber for isolation of beam vibration under point or distributed harmonic excitation. Both research distinctly shows vibration reduction effect. Another DVA research brought by Yang et.al. (2010). A study of control mechanism of DVA in frequency bandwidths and structure on both narrow and broadbands has been done by them. They found that insertion of the DVA into the host structure activates more structural mode and revitalizes a more active modal coupling. A modification of single DVA absorber with inertia effect shown a suppressing effect in the research done by Wu (2006). The inertia effect implemented on the spring mass and successfully reduced the maximum dynamic responses of the main research structural system. The classical vibration absorber produced was as a spring-damper-mass system and directly attached to the main system.

Aside from work mentioned above, an author named Samani with his research partner Pellicano (2009) conduct a research on beams excited by moving loads using a single DVA. The effectiveness and performance of the DVA being compared with the classical damper. Their result prove the dynamic dampers are capable of reducing the vibration amplitude in the presence of excitations due to moving loads. In addition to their analysis, Hsueh et al. (2000) did study an
analytical and closed-form vibration transmissibility of a general unidirectional multi degree of freedom system with multiple dynamic absorbers. Analytical expressions for both force and displacement transmissibility has been derived by them and did discover that it is easy to apply the method in designing the dynamic absorber for vibration reduction of mechanical system. A study that can be considered as an earlier research in this area did by Kojima (1983) when he investigated the forced vibrations of a beam with a non-linear DVA.

Fields’ major researcher Brennan (1998,2000) had done many works about absorber. His absorber named Tuned Vibration Neutraliser used to control the flexural waves on a beam structure. The tuned vibration neutraliser designed as a beam-like with small brass masses attached and with fixed characteristic and being fitted to steel beam. The investigation includes analytical and experimental works on the dynamic behaviour of the dynamic system (neutraliser) when the neutraliser is tuned. The research successful reduced the motion of the beam. He then continued that topic of research by conducting an investigation to know the parameters of the neutralizer that influence the control of the vibrational kinetic energy of a structure. The mathematical model of the structure and the neutralisers that he had investigated were in terms of receptances and dynamic stiffness. Simplified model is developed uses only a single tuneable vibration neutraliser shown in diagram in Figure 2.15. The frequencies are the natural frequencies of the host structure when its pinned at the neutralizer attachment point. He stated that if the frequencies coincide with a frequency of interest then they can be shifted to other frequencies by changing the position of the neutralizer.

![Neutralizers of dynamic stiffness attached to a structure which has M modes (Brennan,2000)](image-url)
A research conducted by El-Khatib et al. (2005) used tuned vibration absorber to control flexural waves in a beam. The absorber was made of steel beam with block of brass attached at each end. The absorber was attached at its centre to a straight steel beam suspended at 4 points along its length. The ends of the beam were embedded in sand boxes to reduce reflections.

2.6.2 Structure tested

As we know, this research approaches to reduce a vibration on a structure with fixed-fixed end boundary condition. From the literature review, only few researcher conducted a research on a fixed beam. Most of them were interested on cantilever beam types.

Major researcher in this field; Brennan (1998,2000,2011) in almost of his paper studied the using of his tuned vibration neutraliser on cantilever beam structure. In Gao et al. (2011) paper, he used infinite beam to investigate wave suppression using multiple neutraliser.

Dayou (2003) in his paper had experimented his tunable vibration neutraliser also on cantilever beam. He said that the reasons of using cantilever beam selection as host structure was because it is relatively easy to set-up experimental rig without losing generality. The structure tested in Dayou paper shown in Figure 2.16 below:

![Figure 2.16: Cantilever beam tested (Dayou, 2003)](image-url)
REFERENCES


D.A. Pape & S. Andhikari. A statistical analysis of modal parameters for uncertainty qualification in structural dynamics. Department of Engineering and Technology, Central Michigan University. USA.


Dr. S. Talukdar. Vibration of continuous system. Profesor, Department of Civil Engineering. Indian Institute of Technology Guwahati-781039.


Safa Bozkurt Coskun, Mehmet Tarik Atay & Baki Ozturk. Transverse Vibration analysis of Euler-Bernoulli beams using analytical approximate techniques. Department of Civil Engineering. Nigde University, Turkey.


Yuri Khazanov. Dynamic vibration absorber-application with variable speed machines. President and Principal Engineer of InCheck technologies Inc. 2007.